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EXISTENCE OF COLORING IN VARIOUS GRAPHS AND FRACTAL GRAPHS USING CHROMATIC NUMBER

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Abstract

The aim of this paper evaluates the chromatic number of various type of Graphs and self-similarity fractal graph. Fractal Graph is the world famous new development and modern technology graph. It is developed in many areas and has more application in various engineering fields such as Computer Science, Physical, Medical, Architecture etc. Coloring of Graph is the advanced tool in the Graph Theory and it has growing in various fields. Coloring the vertices is nothing but give the color to each vertices, edges or regions. Adjacent vertices or edges are not colored by the same. Chromatic Number is the number of colors needed to the graph for coloring the vertices. It analyses the chromatic number of various graphs and self-similarity fractal graph like Cantor Set, Von Koch Curve and Hilbert Curve. This concludes the new study about the concept of Coloring combining with Fractal Graphs.

1. Introduction

Graph Theory [2] is the advanced modern tool of Mathematics Area. Graph Coloring is the most development area of Research Methodologies. It

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has million number of real life applications in day to day life. Computer Architecture has worked in Data mining, Clustering, image capturing by the rule of Graph Coloring. Graph Coloring [1] has characterized into three categories Vertex Coloring, Edge Coloring and Region Coloring. This paper applies the vertex coloring into various types of Graphs. It analyses in how many colors are needed for coloring the whole graph [8]. This way of calculation induces the Chromatic Number of the corresponding graph. This Formula satisfies not only the single graph, it is recommended for all higher level of iterations of the corresponding Graph. Small to Large level of Iteration also follows the same formulae for vertex coloring [12].

Fractals [3] can be characterized by the property of self-similarity. It has been characterized into three categories. They are Exact Self Similarity, Accurate Self Similarity and Statistical self-similarity. Fractal Graphs. The whole body of the graph can be partitioned into small particles. Each particle resembles the same structures [4]. It is called as Exact Self Similarity Fractal Graph. Sometimes, it has loosely similar to a part of itself. A small particle has projected into the whole of the body of the object but not accurately measured. It has placed in natures like clouds, mountains, leaves, rivers and etc. Topological Dimension is not fit to measured roughness and complicated Graphs. Hausdroff dimension are implemented for the measurement of roughness or complicated Fractal Graphs. It cannot be described in proper Euclidean Geometric Language. It is well shape accomplished structure at random scales. Fractal Graphs are not easily described in Proper Euclidean Geometric Language. A fractal dimension is measured by the ratio of configuring out the complexity of a system given its measurement.

2. Preliminaries

2.1 Coloring. It is the way of assigning colors to the graph. It may be assigning the color to vertex or edges or region area of the graph. It can be characterized into three types Vertex Coloring, Edge Coloring or Region Coloring, Vertex Coloring [13] is the way of assigning different colors to adjacent vertices. Same color is not assign into terminal vertices of one edge. Edge Coloring assigns a different color to adjacent edges in the given graph. In the planar graph, it is subdivided into regions. Assign a different color to adjacent regions of the planar graph. It is called as Region Coloring [20].

These regions are partitioned into different connected graphs.

2.2 Region Coloring [17]**.** It is an assignment of colors to the different region of the planar graph. These regions are converted into graph. The regions are connected in the planar graph. Non adjacent regions are assigned by same color.

2.3 Chromatic Number [19]**.** Chromatic number is nothing but the minimum number of colors required to color the non-adjacent vertices of the graph. It is denoted by $CR(G)$. For Example, Figure 1 is Star Graph. It has colored by two different colors Blue and Red. Centre node is adjacent with all other vertices it is colored by Red. The remaining vertices are non-adjacent. So they are all colored by blue. Chromatic number of Figure 1 is two only.

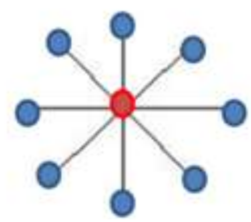


Figure 1. Example of 2 - Chromatic Graph.

3. Results: Calculating Chromatic Number to Various Types of Graphs

Chromatic Number $CR(G)$ is calculated from the following Graphs Null Graph, Trivial Graph, Complete Graph, Regular Graph, Star Graph, Cycle Graph and Most World level Famous Fractal Graph like Cantor Set, Von Koch Curve and Hilbert Curve by using the number of vertices of the corresponding graph. Chromatic Number depends on their structures and implementation of the corresponding graphs.

3.1 1-Chromatic Graph. Null Graph [6] may have single vertex or not connected without edge of many vertices. They are all vertices are non-adjacent. It is colored by single color. Therefore Chromatic Number is only one. Trivial Graph is single vertex graph. It also has one Chromatic Number.

3.2 n-Chromatic Graph. Perfect example of n-Chromatic Graph is

Complete Graph [7]. Each vertex is adjacent with all the vertices of the given graph. A complete Graph has n vertices and degree $n-1$. It is denoted by K_n. Each vertex is colored by distinct colors. A complete Graph has n Chromatic Number. i.e., $CR(G = n)$.

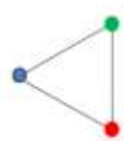
3.3 2-Chromatic Star Graph. This type of Graph has one Centre vertex [9]. All other vertices of this Graph are mutually joined to that center vertex through the edge. Parallel edges are not allowed here [24]. All the vertices except center node are not joined mutually through the edges. They are not touching together. Assign the first color to the center vertex and assign the second color to the other vertices of the corresponding Graph. Figure 1 is the example of 2-Chromatic Star Graph. Centre vertex is colored by red and all other vertices are colored by blue.

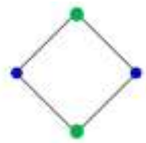
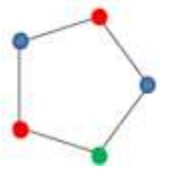
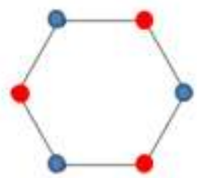
3.4 Chromatic number of Cycle Graph.

It is the connected and closed graph [11]. It has equal number of vertices and edges. It is closed path. All the nodes are joined by non-intersecting edges. It has common degree itself. It has degree 2. Initial Iteration of cycle graph started with 3 vertices and 3 edges. In the upcoming Iteration, number of edges and vertices are increased by one. It is denoted by C_n. It is shown as Table 1. Chromatic number [14] is calculated for all the iteration of cycle graph by the following formulae in Table 1

$$CR(G) = \begin{Bmatrix} 2 & if\ n\ is\ odd \\ 3 & if\ n\ is\ even \end{Bmatrix}.$$

Table 1. Chromatic number of cycle graph.

S. No.	number of vertices (n)	cycle graph C_n	$CN(G) = \begin{Bmatrix} 2 & if\ n\ is\ odd \\ 3 & if\ n\ is\ even \end{Bmatrix}$
1	3		3

2	4		2
3	5		3
4	6		2

3.5 Chromatic number of regular graph.

Let the graph has same degree for all the vertices in the given graph then it is called as regular graph [15]. The degree of the graph is mentioned in their names. If the degree of the vertices is m, it is called as m-regular graph [18]. Let us start the coloring of each vertex of the graph. Number of colors needed for the graph is called as Chromatic Number of the graph [24]. The Chromatic Number of Regular Graph is calculated in the tables 2 and 3. The Chromatic number of the regular graph is calculated by the formulae $CR(G) = \begin{cases} 2 & if\ n\ is\ odd \\ 3 & if\ n\ is\ even \end{cases}$.

Table 2. Chromatic number of 2 – regular graph.

S. NO	Degree of the graph m	Number of vertices $V(G)$	m - regular graph	$CR(G)$

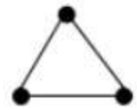
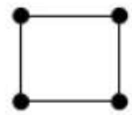
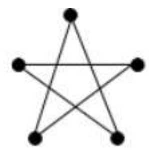
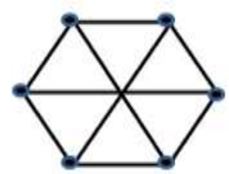
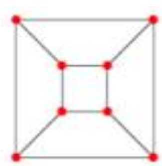
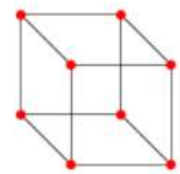
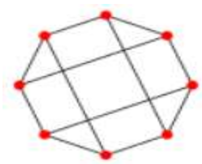
1	2	3		3
2	2	4		2
3	2	5		3

Table 3. For 3 – Rregular Graph.

S. NO.	Degree of the Graph	Number of Vertices V(G)	m - REGULAR GRAPH	CN(G)
	3	7		3
	3	8		2

	3	8		2
	3	8		2

3.6. Chromatic number of fractal graph. This Chapter finds out the Chromatic Number of Some Self Similarity Fractal Graph one by one in the following. There is a lot of Self Similarity Fractal Graphs are available in Fractal Theory [4]. Few of them only has used for the calculation of Chromatic Number. This Paper determines that All Path Fractal Graph has only 2 Chromatic Number. It is explained as follows one by one.

3.6.1 Cantor Set. George Cantor invented Cantor Set [5]. It is the basic type of Fractal Graphs. It is started at a single line that means topological dimension [21]. It has unit interval [0, 1]. A single line has splitted into three segments with the interval 1/3. Middle segment has eliminated. In the first iteration, a single edge has partitioned into two non-adjacent edges [20]. Continuing the same process in the upcoming iteration, number of edges has increased into twice. Only two colors are needed for coloring this type of the graph.

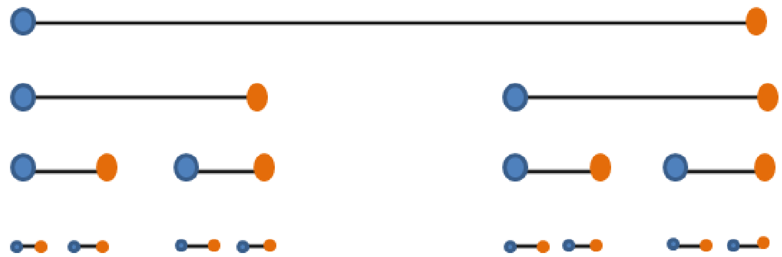


Figure 2. Graph Coloring of Cantor Set.

3.6.2 Coloring Graph of Von Koch Curve. It is the earliest fractal curve in the world. Hedge Von Koch discovered it [10]. He gave geometric definition about fractal Graphs in his paper. It has started at topological dimension [16]. It has partitioned into three segments. Middle segment has bend outward straightened and make two sides of equilateral triangle. The same process is applied in the each edge of previous iteration for the upcoming iteration. It looks like a path. The edges are not intersecting together. The path passes through all non-intersecting edges one by one. Each edge is colored by two distinct colors. These two colors are enough for all the iteration of Von Koch Curve. Few of the iteration of Von Koch Curve are given below. It resembles Von Koch Curve is 2-Chromatic Fractal Graph.

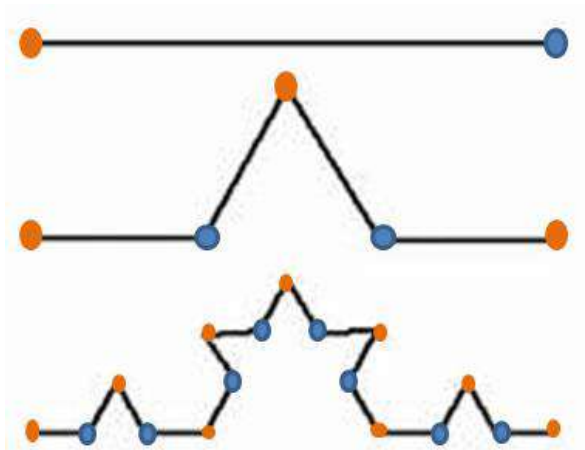


Figure 3. Graph Coloring of Von Koch Curve.

3.6.3 Hilbert Curve [16]**.** It is another example of 2-chromatic Fractal Graph. It is the most famous structure of Fractal Antenna. This is self-similarity Fractal Graph. It is open path Fractal Graph. It is non intersecting edge graph. This Fractal Graph has many more application in real life. It is used in Fractal Antenna. In the Initial Iteration, it has three edges and four vertices. Alternative vertices are colored by distinct color [23]. In the upcoming iteration, number of vertices and edges are increased in constant ratio. In the nth iteration of Hilbert Curve has $4n$ vertices. For example, Figure 4 is second iteration of Hilbert Curve. It is 2 Chromatic Graph.

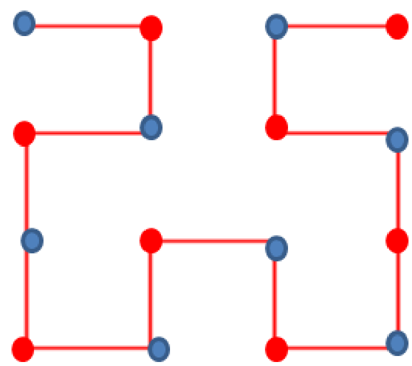


Figure 4. Graph Coloring of Hilbert Curve.

4. Conclusion

This Paper applies the Graph Coloring into few of the Graphs and Fractal Graph. Coloring has world wide application in our life. Vertex coloring is a hard combinatorial optimization problem. Coloring is used to differentiate the terms to visible. Many applications involving coloring are evaluating the minimum number of colors required. For example, in our country map has used coloring to differentiate the adjacent states. Adjacent states have colored into different colors. In this paper finds the Chromatic Number of the Fractal Graphs like Cantor set, Von Koch Curve and Hilbert Curve. This Fractal Graphs are Self Similarity Graphs. All the Iteration of this Fractal Graph has 2-Chromatic Number. It is very useful to find chromatic number in a simple way of very large iteration of Fractal Graph. In the future work, Chromatic number has been calculated in the complicated Fractal Graph also.

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