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SOME ISOMORPHISM PROPERTIES OF INTUITIONISTIC L-FUZZY GRAPHS

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Abstract

In this paper we define and study homomorphisms, isomorphisms and weak isomorphisms of intuitionistic L-fuzzy graphs. We establish some of their properties. Further, we show that isomorphism of ILFGs is an equivalence relation, whereas weak isomorphism is a partial order relation. Another interesting structure that comes up consequently is the collection of automorphisms-we establish that it is a group. We discuss the subgroup of this group using (s, t) level set of its intuitionistic L-fuzzy subgroup. We also derive some interesting properties here.

1. Introduction

In 1965, as an extension of classical notion of sets, L. A. Zadeh [12] introduced fuzzy sets for describing the situations which are imprecise or vague. Since its inception the theory of fuzzy sets has advanced in a variety of

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ways and in many disciplines. In 1975, A. Rosenfeld [7] considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs. As an extension of fuzzy graphs, K. T. Atanassov [1] introduced intuitionistic fuzzy graph theory in 1999. As a special case of Atanossov's intuitionistic fuzzy graphs, M. G. Karunambigai, R. Parvathy and R. Buvaneswari [3] introduced intuitionistic fuzzy graphs. K. R. Bhutani [2] studied isomorphism of fuzzy graphs and its properties. A. Nagoor Gani and S. Shajitha Begum [4] discussed isomorphism on strong intuitionistic fuzzy graphs. N. Palaniappan, S. Naganathan and K. Arjunan [5] discussed about intuitionistic L-fuzzy subgoups and its properties. As another generalization of fuzzy graphs, R. Pramada and K. V. Thomas [6] introduced L-fuzzy graphs. Intuitionistic L-fuzzy graphs being a relatively new concept, studies on this structure just begun to appear. The greater volume of the work has been contributed by the authors [8, 9, 10].

The vast possibilities in the study of ILFGs remain unexplored and offer much scope for theoretical research. This paper is an attempt in studying some special mappings of ILFGs and also some structural properties of these mappings, specifically automorphisms.

In this paper, we introduce the notion of homomorphisms and different types of isomorphisms of ILFGs and study some properties. We discuss the automorphism group of ILFGs and obtain some results.

The highlight of this paper is that a systematic study has been carried out on various types of mappings on ILFGs, establishing a strong structural foundation, just as in crisp graphs on ILFGs.

Throughout this paper, the lattice L is a complete lattice and the underlying graph of G^L is a simple graph.

2. Preliminaries

In this section, we review some of the necessary basic definitions required for the development of some new concepts in intuitionistic L-fuzzy graphs.

Definition 2.1. Let L be a complete lattice with an involutive order reversing operation $c : L \to L$. An intuitionistic L-fuzzy set A in X is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle / x \in X\}$ where $M_A : X \to L$

and $N_A : X \to L$ such that $M_A(x) \leqslant c(N_A(x)), \forall x \in X$. Here $M_A(x)$ and $N_A(x)$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively.

Definition 2.2 [8]**.** Let L be a complete lattice with an involutive order reversing operation $c : L \to L$. An intuitionistic L-fuzzy graph (ILFG) G^L with underlying set V is defined to be $G^L = (V, \sigma, \mu)$ where $\sigma = (M_\sigma, N_\sigma)$ and $\mu = (M_\mu, N_\mu)$ such that

(i) the functions $M_\sigma : V \to L$ and $N_\sigma : V \to L$ should satisfy $M_\sigma(x) \leqslant c(N_\sigma(x)), \forall x \in V$. Here $M_\sigma(x)$ and $N_\sigma(x)$ denote membership value and non-membership value of the vertex $x \in V$ respectively.

(ii) the functions $M_\mu : E \to L$ and $N_\mu : E \to L$ where $E \subseteq V \times V$ should satisfy

$$M_\mu(x, y) \leqslant M_\sigma(x) \wedge M_\sigma(y)$$

$$N_\mu(x, y) \geqslant N_\sigma(x) \vee N_\sigma(y)$$

$M_\mu(x, y) \leqslant c(N_\mu(x, y)), \forall (x, y) \in E$. Here $M_\mu(x, y)$ and $N_\mu(x, y)$ denote the membership value and non-membership value of the edge $(x, y) \in E$ respectively.

Definition 2.3. Let $A = \{\langle x, M_A(x), N_A(x)\rangle / x \in X\}$ be an intuitionistic L-fuzzy subset of a set X and $s, t \in L$. Then (s, t)-level set of A is denoted by $\mathfrak{L}_{s,t}(A)$ and is defined by $\mathfrak{L}_{s,t}(A) = \{x \in X / M_A(x) \geqslant s \text{ and } N_A(x) \leqslant t\}$.

Definition 2.4. Let G be a group. An intuitionistic L-fuzzy subset A of G is said to be an intuitionistic L-fuzzy subgroup(ILFSG) of G if

$$M_A(xy) \geqslant M_A(x) \wedge M_A(y),\ M_A(x^{-1}) = M_A(x) \text{ and}$$

$$N_A(xy) \leqslant N_A(x) \wedge N_A(y),\ N_A(x^{-1}) = N_A(x),\ \forall\, x, y \in G.$$

3. Isomorphism Properties of ILFGs

In this section, we study homomorphisms, isomorphisms and weak isomorphisms of ILFGs and come across certain interesting properties.

Definition 3.1. Let $G_1^{L_1} = (V_1, \sigma_1, \mu_1)$ and $G_2^{L_2} = (V_2, \sigma_2, \mu_2)$ be any two ILFGs. Then a homomorphism from $G_1^{L_1}$ to $G_2^{L_2}$ is a mapping $f : V_1 \to V_2$ together with the mapping $l : L_1 \to L_2$, which satisfies

(i) $l(M_{\sigma_1}(x)) \leqslant M_{\sigma_2}(f(x))$

$l(N_{\sigma_1}(x)) \geqslant N_{\sigma_2}(f(x)), \forall x \in V_1$

(ii) $l(M_{\mu_1}(x, y)) \leqslant M_{\mu_2}(f(x), f(y))$

$l(N_{\mu_1}(x, y)) \geqslant N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$

Remark 3.2. If the lattices L_1 and L_2 in the above definition are identical, then take l as the identity function.

Definition 3.3. A homomorphism of an ILFG G^L onto itself is known as an endomorphism.

Definition 3.4. Let $G_1^{L_1} = (V_1, \sigma_1, \mu_1)$ and $G_2^{L_2} = (V_2, \sigma_2, \mu_2)$ be any two ILFGs. Then an isomorphism from $G_1^{L_1}$ to $G_2^{L_2}$ is a bijective mapping $f : V_1 \to V_2$ together with the bijective mapping $l : L_1 \to L_2$ which satisfies

(i) $l(M_{\sigma_1}(x)) = M_{\sigma_2}(f(x))$

$l(N_{\sigma_1}(x)) = N_{\sigma_2}(f(x)), \forall x \in V_1$

(ii) $l(M_{\mu_1}(x, y)) = M_{\mu_2}(f(x), f(y))$

$l(N_{\mu_1}(x, y)) = N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$

If there exists an isomorphism from $G_1^{L_1}$ to $G_2^{L_2}$, then we say that $G_1^{L_1}$ is isomorphic to $G_2^{L_2}$ and we write $G_1^{L_1} \cong G_2^{L_2}$.

Remark 3.5. If the lattices L_1 and L_2 in the above definitions are identical, then take l as the identity function.

Theorem 3.6. *The isomorphism relation on the set* $\mathfrak{G}^L$ *of all intuitionistic L-fuzzy graphs on a lattice L is an equivalence relation.*

Proof. Let $G^L \in \mathfrak{G}^L$. By taking f as the identity map, we can easily seen that $G^L \cong G^L$. Hence the reflexivity.

Let $G_1^L, G_2^L \in \mathfrak{G}^L$ in such a way that $G_1^L \cong G_2^L$ with the bijective mapping $f : V_1 \to V_2$. Then clearly $G_2^L \cong G_1^L$ with the bijective mapping f^{-1}. Hence symmetry.

Let $G_1^{L_1} = (V_1, \sigma_1, \mu_1), G_2^{L_2} = (V_2, \sigma_2, \mu_2)$ and $G_3^{L_3} = (V_3, \sigma_3, \mu_3)$ belongs to $\mathfrak{G}^L$ such that $G_1^L \cong G_2^L$ with the bijection $f : V_1 \to V_2$ and $G_2^L \cong G_3^L$ with the bijection $g : V_2 \to V_3$. Then $g \circ f$ is a bijective mapping from V_1 to V_3. Also $M_{\sigma_1}(x) = M_{\sigma_2}(f(x)) = M_{\sigma_3}[g(f(x))] = M_{\sigma_3}[(g \circ f)(x)]$ and

$$
\begin{aligned}
M_{\mu_1}(x, y) &= M_{\mu_2}[f(x), f(y)] \\
&= M_{\mu_3}[g(f(x)), g(f(y))] \\
&= M_{\mu_3}[(g \circ f)(x), (g \circ f)(y)]
\end{aligned}
$$

Similarly $N_{\sigma_1}(x) = N_{\sigma_3}[(g \circ f)(x)]$ and $N_{\mu_1}(x, y) = N_{\mu_3}[(g \circ f)(x), (g \circ f)(y)]$ i.e., $G_1^L \cong G_3^L$. Hence transitivity.

i.e., The isomorphism relation on the set $\mathfrak{G}^L$ is an equivalence relation. □

Theorem 3.7. *Let f be an isomorphism from the ILFG* $G_1^{L_1} = (V_1, \sigma_1, \mu_1)$ *to the ILFG* $G_2^{L_2} = (V_2, \sigma_2, \mu_2)$. *Then f will be an isomorphism between their underlying crisp graphs* $(\sigma_1^*, \mu_1^*) = (V_1, E_1)$ *and* $(\sigma_2^*, \mu_2^*) = (V_2, E_2)$.

Proof. Let $x \in \sigma_1^*$. This means that $M_{\sigma_1}(x) > 0$ and $N_{\sigma_1}(x) > 0$ or

$M_{\sigma_1}(x) = 0$ and $N_{\sigma_1}(x) > 0$ or $M_{\sigma_1}(x) > 0$ and $N_{\sigma_1}(x) = 0$. But since f is an isomorphism from G_1^L to G_2^L, we have $M_{\sigma_1}(x) = M_{\sigma_2}(f(x))$ and $N_{\sigma_1}(x) = N_{\sigma_2}(f(x))$. Hence $M_{\sigma_2}(f(x)) > 0$ and $N_{\sigma_2}(f(x)) > 0$ or $M_{\sigma_2}(f(x)) = 0$ and $N_{\sigma_2}(f(x)) > 0$ or $M_{\sigma_2}(f(x)) > 0$ and $N_{\sigma_2}(f(x)) = 0$ This implies $f(x) \in \sigma_2^*$. i.e., $x \in \sigma_1^* \Rightarrow f(x) \in \sigma_2^*$. Similarly we can show that $(x, y) \in \mu_1^* \Rightarrow (f(x), f(y)) \in \mu_2^*$. i.e., f is a bijection from V_1 to V_2 that preserves the adjacency. Hence f is an isomorphism between the underlying crisp graphs (σ_1^*, μ_1^*) and (σ_2^*, μ_2^*). □

Definition 3.8. A weak isomorphism from $G_1^{L_1}$ to $G_2^{L_2}$ is a bijective mapping $f : V_1 \to V_2$ together with the bijective mapping $l : L_1 \to L_2$, which satisfies

(i) $l(M_{\sigma_1}(x)) = M_{\sigma_2}(f(x))$

$l(N_{\sigma_1}(x)) = N_{\sigma_2}(f(x)), \forall x \in V_1$

(ii) $l(M_{\mu_1}(x, y)) \leqslant M_{\mu_2}(f(x), f(y))$,

$l(N_{\mu_1}(x, y)) \geqslant N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$

Theorem 3.9. *The weak isomorphism on the set $\mathfrak{G}^L$ of all intuitionistic L-fuzzy graphs on a lattice L is a partial order relation.*

Proof. By taking f as the identity map, we can easily seen that $G^L \cong G^L$. Hence the reflexivity.

Now let $G_1^L, G_2^L \in \mathfrak{G}^L$, where $G_i^L = (V_i, \sigma_i, \mu_i)$ for $i = 1, 2$. Also suppose that f be a weak isomorphism from G_1^L to G_2^L and g be a weak isomorphism from G_2^L to G_1^L. i.e., f is a bijective map from V_1 to V_2 satisfying $M_{\sigma_1}(x) = M_{\sigma_2}(f(x)), N_{\sigma_1}(x) = N_{\sigma_2}(f(x)), \forall x \in V_1$ and $M_{\mu_1}(x, y) \leqslant M_{\mu_2}(f(x), f(y)), N_{\mu_1}(x, y) \geqslant N_{\mu_2}(f(x), f(y)), \forall (x, y) \in V_1 \times V_1$. Also g be a bijective map from V_2 to V_1 satisfying $M_{\sigma_2}(x) = M_{\sigma_1}(g(x)), N_{\sigma_2}(x) =$

$N_{\sigma_1}(g(x)), \forall x \in V_2$ and $M_{\mu_2}(x, y) \leqslant M_{\mu_1}(g(x), g(y)), N_{\mu_2}(x, y) \geqslant N_{\mu_1}(g(x), g(y)), \forall (x, y) \in V_2 \times V_2$.

All the above inequalities holds on V_1 and V_2 only when G_1^L and G_2^L have same number of edges and the corresponding edges have the same weight. Hence G_1^L and G_2^L are identical. Hence the relation weak isomorphism is an antisymmetric relation.

Now let $G_1^L = (V_1, \sigma_1, \mu_1), G_2^L = (V_2, \sigma_2, \mu_2)$ and $G_3^L = (V_3, \sigma_3, \mu_3)$ belongs to $\mathfrak{G}^L$ such that G_1^L is weakly isomorphic to G_2^L with the bijection $f : V_1 \to V_2$ and G_2^L is weakly isomorphic to G_3^L with the bijection $g : V_2 \to V_3$. Then $g \circ f$ is a bijective mapping from V_1 to V_3. Also $M_{\sigma_1}(x) = M_{\sigma_2}(f(x)) = M_{\sigma_3}[g(f(x))]$ and $M_{\mu_1}(x, y) \leqslant M_{\mu_2}[f(x), f(y)] \leqslant M_{\mu_3}[g(f(x)), g(f(x))]$.

i.e., $M_{\sigma_1}(x) = M_{\sigma_3}[(g \circ f)(x)]$ and $M_{\mu_1}(x, y) \leqslant M_{\mu_3}[(g \circ f)(x), (g \circ f)(y)]$. Similarly $N_{\sigma_1}(x) = N_{\sigma_3}[(g \circ f)(x)]$ and $N_{\mu_1}(x, y) \geqslant N_{\mu_3}[(g \circ f)(x), (g \circ f)(y)]$. i.e., G_1^L is weakly isomorphic to G_3^L. Hence transitivity.

i.e., The relation weakly isomorphism on the set $\mathfrak{G}^L$ is a partial order relation.

4. The Automorphism Group of an ILFG

Studies on automorphism are but a natural extension of studies on isomorphism. We do the same here and come up with some interesting results.

Definition 4.1. An isomorphism of an ILFG G^L onto itself is known as an automorphism.

Theorem 4.2. *A bijective endomorphism of an ILFG* $G^L = (V, \sigma, \mu)$ *is an automorphism of* G^L.

Proof. Let f be the bijective endomorphism of G^L onto itself. i.e., The bijective function $f: V \to V$ satisfies the following conditions

$$M_\sigma(x) \leqslant M_\sigma(f(x)),\ N_\sigma(x) \geqslant N_\sigma(f(x)),\ \forall\, x \in V$$

$$M_\mu(x,\ y) \leqslant M_\mu(f(x),\ f(y)),\ N_\mu(x,\ y) \geqslant N_\mu(f(x),\ f(y)),\ \forall\,(x,\ y) \in V \times V$$

So we have

$$M_\sigma(x) \leqslant M_\sigma(f(x)) \leqslant M_\sigma(f(f(x))) = M_\sigma(f^2(x)) \leqslant M_\sigma(f^3(x)) \leqslant \ldots \leqslant M_\sigma(f^n(x)),\quad \forall\, x \in V$$

Since f is a bijective map from V to itself, $f^n(x) = x$ for some n. Therefore from the above inequality, we have $M_\sigma(x) \leqslant M_\sigma(f(x)) \leqslant M_\sigma(x)$

Hence $M_\sigma(x) = M_\sigma(f(x)),\ \forall\, x \in V$.

Similarly it can be showed that $N_\sigma(x) = N_\sigma(f(x)),\ \forall\, x \in V$ and $M_\mu(x,\ y) = M_\mu(f(x),\ f(y)),\ N_\mu(x,\ y) = N_\mu(f(x),\ f(y)),\ \forall(x,\ y) \in V \times V$.

Hence the theorem. □

Theorem 4.3. *The collection* G_a^L *of all automorphisms of an ILFG* $G^L = (V,\ \sigma,\ \mu)$ *is a group under composition of maps.*

Proof. Let $f_1,\ f_2 \in G_a^L$.

Now consider

$$\begin{aligned} M_\sigma[(f_1 \circ f_2)(x)] &= M_\sigma[f_1(f_2(x))] \\ &= M_\sigma(f_2(x)) \text{ since } f_1 \in G_a^L \\ &= M_\sigma(x) \text{ since } f_2 \in G_a^L \end{aligned}$$

Similarly it can be showed that

$$N_\sigma[(f_1 \circ f_2)(x)] = N_\sigma(x)$$

$$M_\sigma[(f_1 \circ f_2)(x),\ (f_1 \circ f_2)(y)] = M_\mu(x,\ y)$$

$$N_{\mu}[(f_1 \circ f_2)(x), (f_1 \circ f_2)(y)] = N_{\mu}(x, y), \forall (x, y) \in V \times V$$

Also since f_1 and f_2 are bijective functions from V to V, $f_1 \circ f_2$ is also bijective i.e., $f_1 \circ f_2 \in G_a^L$. Hence the closure property.

Obviously the composition of maps is associative.

Now consider the identity map I from V to V. It is a bijection and

$$M_{\sigma}(I(x)) = M_{\sigma}(x), N_{\sigma}(I(x)) = N_{\sigma}(x), \forall x \in V$$

$$M_{\mu}[I(x), I(y)] = M_{\mu}(x, y), N_{\mu}[I(x), I(y)] = N_{\mu}(x, y), \forall (x, y) \in V \times V$$

i.e., $I \in G_a^L$. Further we have $f \circ I = I \circ f = f$. Hence it acts as an identity element. Now let $f \in G_a^L$. Since f is bijective, $f^{-1} : V \to V$ exists and it is bijective.

Now let $x \in V$. We have

$$M_{\sigma}(f^{-1}(x)) = M_{\sigma}(f(f^{-1}(x))) \text{ since } f \in G_a^L$$

$$= M_{\sigma}(x)$$

Similarly

$$N_{\sigma}(f^{-1}(x)) = N_{\sigma}(x)$$

$$M_{\mu}(f^{-1}(x), f^{-1}(y)) = M_{\mu}(x, y)$$

$$N_{\mu}(f^{-1}(x), f^{-1}(y)) = N_{\mu}(x, y)$$

i.e., $f^{-1} \in G_a^L$ such that $f \circ f^{-1} = f^{-1} \circ f = I$.

i.e., Inverses exists for every $f \in G_a^L$.

Hence G_a^L is a group under the composition of maps.

Remark 4.4. Let $G^L = (V, \sigma, \mu)$ be an ILFG and G_a^L be the group of all automorphisms of G^L. Then $A = \{\langle f, M_A(f), N_A(f)\rangle / f \in G_a^L\}$, where

$M_A(f) = \vee\{M_\mu(f(x), f(y))/x, y \in V\}$ and $N_A(f) = \wedge\{N_\mu(f(x), f(y))/x, y \in V\}$ is an intuitionistic L-fuzzy subset of G_a^L.

This may be established by proceeding as follows:

We have

$$M_A(f) = \vee\{M_\mu(f(x), f(y))/x, y \in V\}$$

$$= \vee\{M_\mu(x, y)/x, y \in V\} \text{ since } f \text{ is an automorphism}$$

$$N_A(f) = \wedge\{N_\mu(f(x), f(y))/x, y \in V\}$$

$$= \wedge\{N_\mu(x, y)/x, y \in V\} \text{ since } f \text{ is an automorphism}$$

Now

$$c(N_A(f)) = c(\wedge \{N_\mu(x, y)/x, y \in V\})$$

$$= \vee\{c(N_\mu(x, y))/x, y \in V\} \text{ by De Morgan's law}$$

But $M_\mu(x, y) \leqslant c(N_\mu(x, y)), \forall (x, y) \in V \times V$ since G^L is an ILFG

Hence $c(N_A(f)) \geqslant M_A(f), \forall f \in G_a^L$

Hence $A = \{\langle f, M_A(f), N_A(f)\rangle / f \in G_a^L\}$ is an intuitionistic L-fuzzy subset of G_a^L.

Theorem 4.5. *The intuitionistic L-fuzzy subset*

$A = \{\langle f, M_A(f), N_A(f)\rangle / f \in G_a^L\}$ *of* G_a^L *is an intuitionistic L-fuzzy subgroup of* G_a^L.

Proof. Let $f_1, f_2 \in G_a^L$. Then

$$M_A(f_1 \circ f_2) = \vee\{M_\mu[(f_1 \circ f_2)(x), (f_1 \circ f_2)(y)]/x, y \in V\}$$

$$= \vee\{M_\mu(x, y)/x, y \in V\} \text{ since } f_1 \circ f_2 \in G_a^L$$

Now

$$M_A(f_1) = \vee\{M_\mu[(f_1(x), (f_1)(y))]/x, \ y \in V\}$$

$$= \vee\{M_\mu(x, \ y)/x, \ y \in V\} \text{ since } f_1 \in G_a^L$$

and

$$M_A(f_2) = \vee\{M_\mu[(f_2)(x), (f_2)(y)]/x, \ y \in V\}$$

$$= \vee\{M_\mu(x, \ y)/x, \ y \in V\} \text{ since } f_2 \in G_a^L$$

Hence

$$M_A(f_1) \wedge M_A(f_2) = \vee\{M_\mu(x, \ y)/x, \ y \in V\} = M_A(f_1 \circ f_2)$$

Now let $f \in G_a^L$. Then

$$M_A(f^{-1}) = \vee\{M_\mu(f^{-1}(x), f^{-1}(y))/x, \ y \in V\}$$

$$= \vee\{M_\mu(x, \ y)/x, \ y \in V\} \text{ since } f^{-1} \in G_a^L$$

and

$$M_A(f) = \vee\{M_\mu(f(x), f(y))/x, \ y \in V\}$$

$$= \vee\{M_\mu(x, \ y)/x, \ y \in V\} \text{ since } f \in G_a^L$$

i.e., $M_A(f^{-1}) = M_A(f)$

In a similar manner, we can show that $N_A(f_1 \circ f_2) = N_A(f_1) \vee N_A(f_2)$ and $N_A(f^{-1}) = N_A(f)$

Hence $A = \{\langle f, M_A(f), N_A(f)\rangle / f \in G_a^L\}$ of G_a^L is an intuitionistic L-fuzzy subgroup of G_a^L. □

Theorem 4.6. *Let $A = \{\langle f, M_A(f), N_A(f)\rangle / f \in G_a^L\}$ be an intuitionistic L-fuzzy subgroup of a group G_a^L and $s, t \in L$. Then the (s, t)-level set of A, $\mathfrak{L}_{s,t}(A) = \{f \in G_a^L / M_A(f) \geqslant s$ and $N_A(f) \leqslant t\}$ is a subgroup of the group G_a^L.*

Proof. Let $f_1, f_2 \in \mathfrak{L}_{s,t}(A)$. We are required to prove that $f_1 \circ f_2^{-1} \in \mathfrak{L}_{s,t}(A)$. Now

$$f_1 \circ f_2^{-1} \in \mathfrak{L}_{s,t}(A) \Rightarrow M_A(f_1) \geqslant s,\ N_A(f_1) \leqslant t \text{ and } M_A(f_2) \geqslant s,\ N_A(f_2) \leqslant t$$

$$\Rightarrow M_A(f_1) \wedge M_A(f_2) \geqslant t \text{ and } N_A(f_1) \vee N_A(f_2) \leqslant t$$

Since A is an intuitionistic L-fuzzy subgroup of G_a^L,

$$M_A(f_1 \circ f_2) \geqslant M_A(f_1) \wedge M_A(f_2),$$

$$N_A(f_1 \circ f_2) \leqslant N_A(f_1) \vee N_A(f_2)$$

$$M_A(f_i^{-1}) = M_A(f_i),\ N_A(f_i^{-1}) = N_A(f_i),\ i = 1, 2$$

Therefore

$$\begin{aligned} M_A(f_1 \circ f_2^{-1}) &\geqslant M_A(f_1) \wedge M_A(f_2^{-1}) \\ &= M_A(f_1) \wedge M_A(f_2) \\ &\geqslant s \end{aligned}$$

$$\begin{aligned} N_A(f_1 \circ f_2^{-1}) &\leqslant N_A(f_1) \vee N_A(f_2^{-1}) \\ &= N_A(f_1) \vee N_A(f_2) \\ &\leqslant t \end{aligned}$$

This implies $f_1 \circ f_2^{-1} \in \mathfrak{L}_{s,t}(A)$.

i.e., $f_1, f_2 \in \mathfrak{L}_{s,t}(A) \Rightarrow f_1 \circ f_2^{-1} \in \mathfrak{L}_{s,t}(A)$. Hence the (s, t)-level set of A, $\mathfrak{L}_{s,t}(A)$ is a subgroup of the group G_a^L. □

5. Conclusion

In this paper, we have introduced the concept of isomorphism of intuitionistic L-fuzzy graphs and derived some interesting results. We have constructed the automorphism group of an ILFG and studied some properties. The establishment that the collection of automorphisms is a group is arguably the most important result in this paper. It paves the way for research in several directions. Some of which has been examined here. There is scope for much work, as indicated in [2], but dealing with a new structure namely ILFGs.

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