Advances and Applications in Mathematical Sciences
Volume 21, Issue 7, May 2022, Pages 3695-3707

ADMISSION CONTROL PROBLEM IN A SERVICE FACILITY WITH INVENTORY MANAGEMENT

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2020 Mathematics Subject Classification: 34Bxx, 76-10, 80A30.

Keywords: Admission control, Service facility, Replenishment, Poisson demand, Exponential service time, Markov Decision Process.

Received December 22, 2021; Accepted February 3, 2022

Abstract

In this article, we address the problem of optimally controlling the admission of customers to a service facility system with inventory and two types of customer viz. ordinary and priority customers, compete for service. We consider a service facility system having finite waiting space and Poisson arrivals. For the given values of maximum inventory and reorder level s, the lead times are assumed to be exponentially distributed. We determine the optimal order quantity or reorder level, so that the long run expected cost is minimized. We prove the existence of a stationary optimal policy and solve it by employing LP technique. Numerical examples for different instances are provided to get insight into the system behavior.

1. Introduction

In this article, we discuss the problem of optimally controlling the admission of customers to a service facility system with inventory and two types of customer viz. ordinary and priority customers, compete for service. We consider a service facility system having finite waiting space.

For the given values of maximum inventory and reorder level s, the lead times are assumed. For example, in real life situation where customer service is becoming more and more important. An important aspect of service is product availability, which is related to service levels in inventory systems. In many cases, not all customer demand for a single product requires the same service level. The type of inventory that, we wish to study are spare parts inventory in the airline or shipping industries and spare parts for refinery equipment.

In all these cases, equipment is categorized in many different classes and different service levels are defined for each type. Some of the equipment is very critical for the smooth running of the operations and needs to be serviced on a priority basis, while other equipment is less critical and will have lower priority. Typically, the equipment is ranked as vital, essential or auxiliary. Hence spare parts are rationed and when inventory levels are low, only the vital equipment is serviced and the other equipment has to wait for a fresh supply of spare parts.

We analyzed from previous articles an integrated approach like Markov Decision Process model is most appropriate to study service facility system (Queues-Inventory) and Maintenance systems. Sapna, K. P., and Berman, O., [1] studied one such system under MDP structure using LPP method to control the service rates. So for in the literature only admission control and service rate control problems are studied under MDP regime. Hild Mohamed et al. [6] analyzed a Markov decision problem: Optimal control of servers in a service facility holding perishable inventory with impatient customers.

Dekker, Hill, and Kleijn [5] considered a lost sales $(S-1, S)$ inventory system with priority demand classes. Sapna [12] considered a lost-sales (s, Q) inventory system with demand classes – ordinary and priority. The demands of these two classes arrive according to independent Poisson processes with different parameters. The distribution of the lead time is assumed to be exponential. This model was extended by Sivakumar and Arivarignan [13] by assuming MAP for two types of customers.

Recently Karthick et al. [7] considered a (s, S) inventory system with two types of arriving customers (Type-1 or Type-2). The arrival of customers is assumed to follow a MAP and the lead time is assumed to have a phase-type distribution.

Veinott [16] was the first to consider the problems of several demand classes in inventory systems. He analyzed a periodic review inventory model with multiple demand classes and zero lead time and introduced the concept of a critical level policy.

Nahmias and Demmy [11] introduced multiple demand classes. They considered two demand classes, Poisson arrival for demands, backordering and fixed lead time for supply of orders and derived approximate expressions for total cost.

Kim [8] considered the admission control and the inventory management problem of a make-to-order facility with a common component, which is purchased from a supplier under stochastic lead time processes and setup costs.

The rest of the paper is organized as follows. We provide a formulation of our Markov Decision model in the next section. Analysis part of the model is given in section 3. In section 4, we present a procedure to apply long – run expected cost rate criteria to get the optimal vales of the system parameters.

2. Problem Formulation

In this Service facility system model we assume the following:

- Customers arrive the service facility according to a Poisson process with parameter $\lambda(>0)$ and are served according to a FCFS queue discipline.
- One unit (item) from inventory is used upto serve one customer.
- The capacity of the system is finite (N), any customer sees N customer has to leave the system compulsory.
- There are two types of customers - ordinary and priority customers compete for service.
- The service times follow an exponential distribution with parameter $\mu_1(>0)$ for priority customer and $\mu_2(>0)$ for a ordinary customer.
- The maximum capacity of inventory is S. Whenever the inventory level reaches to a prefixed level s $(0 \leq s < S)$, an order for $Q = S - s > s$ items placed and the lead time is exponentially distributed with parameter $\gamma(>0)$. The size of the order is adopted at the time of replenishment.
- At each state of the system we check the inventory level to take decisions whether accept or reject the ordinary customer.

If the waiting space is full, no customer is allowed to enter the system.

If there are some places in the waiting space, we look for inventory level:

If inventory level i is such that $s+1 \leq i \leq S$, then both type customers are allowed, otherwise if $i \leq s$, then only type priority customers are allowed.

Let $I(t)$ and $X(t)$ denote the inventory level and the number of customers in the system at time t. Then $\{(I(t), X(t)) : t \geq 0\}$ is a two dimensional stochastic process with state space, $E_1 \times E_2$, where $E_1 = \{0, 1, 2, \ldots, S\}$ and $E_2 = \{0, 1, 2, \ldots, N\}$.

Decision Rule. Let $A_i (i = 1, 2)$ denotes the set of possible actions where, $A_1 = \{0\}$, $A_2 = \{1\}$, $A = A_1 \cup A_2$.

Suppose S denote the class of all stationary policies, then a policy f (sequence of decisions) can be defined as a function $f : E \to A,$ given by

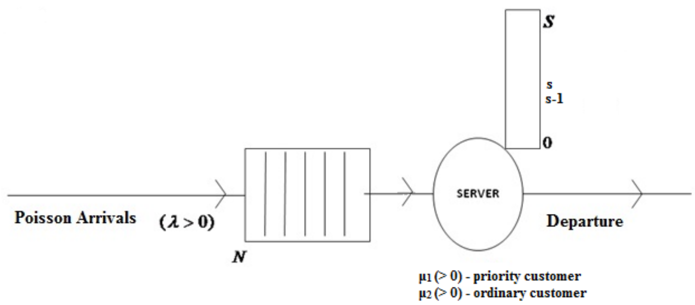
$$f(i, q) = \{(k) : (i, q) \in E_i, k \in A_i, i = 1, 2, 3\}$$

Let $E_1 = \{(i, q) \in E / f(i, q) = 0\}$.

$$E_1 = \{(i, q) \in E / f(i, q) = 1\}. \; E_1 \cup E_2 = E$$

0 represents 'reject' the ordinary customer when an inventory level is below the prefixed level 's' and 1 represents 'accept' the ordinary customer for service when an inventory level i is above $s(s < i \leq S)$.

Objective of the problem is to find the optimal order quantity Q or reorder level for controlling the admission of customers to a service facility with inventory, so that the long run expected total cost rate is minimum.



Notations and Assumptions

1. $E_1 \times E_2 = E$ is the state space of the Stochastic Process $\{(I(t), X(t)) : t \geq 0\}$, where $E_1 = \{0, 1, 2, \ldots, S\}$ and $E_2 = \{0, 1, 2, \ldots, N\}$

2. $A_s -$ decision set corresponding to state $s \in E$.

3. $\underset{(i, q)}{C}(a) -$ cost occurred when action a is taken at state (i, q).

4. $p_{(i, q)}^{(j, r)}(a) -$ the transition probability from state (i, q) to state (j, r) when action a is taken at state $(i, q) \in E$.

5. Inventory levels are reviewed at the time of service completion epoch.

6. Reordering policy is $(s, S) : Q = S - s$ items ordered when the inventory level reaches s (prefixed level), where $0 \leq s \leq S$.

7. F-the class of stationary policies.

3. Analysis

Let R denote the stationary policy, which is deterministic time invariant and Markovian Policy (MD). From our assumptions it can be seen that $\{(I(t), X(t)) : t \geq 0\}$ is denoted as the controlled process $\{(I^R(t), X^R(t) : t \geq 0\}$ when policy R is adopted. Since the process $\{(I^R(t), X^R(t) : t \geq 0\}$ is a Markov Process with finite state space E. The process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov chain.

Let $\{(I^R(t), X^R(t) : t \geq 0\}$ denote the Markov process. $\{(I(t), X(t) : t \geq 0\}$ when R is the policy adopted from our assumptions made in the previous section. The controlled process $\{I^R, \mathrm{X}^{\mathrm{R}}\}$, where, R is the randomized Markovian policy is a Markov process. Under the randomized policy $\prod$, the expected long run total cost rate is given by

$$C^{\Pi} = h\bar{I}^{\Pi} + c_1\overline{w}^{\Pi} + c_2\alpha_a^{\Pi} + g\alpha_b^{\Pi} + \beta\alpha_c^{\Pi} \tag{1}$$

h – holding cost / unit item / unit time

c_1 – waiting cost / customer / unit time

c_2 – reordering cost / order

g – balking cost / customer

β – service cost / customer

$\bar{I}^{\Pi}$ – mean inventory level

$\overline{w}^{\Pi}$ – mean waiting time in system

α_a^{Π} – reordering rate

α_b^{Π} – balking rate

α_c^{Π} – service completion rate

Our objective is to find an optimal policy Π^* for which $C^{\Pi^*} \leq C^{\Pi}$ for every MD policy in Π^{MD} For any fixed MR policy $\Pi \in \Pi^{MD}$ and $(i, q), (j, r) \in E$, define

$$P_{iq}^{\Pi}(j, r, t) = \Pr\{I^{\Pi}(t) = j, L^{\Pi}(t) = r \mid I^{\Pi}(0) = i, L^{\Pi}(0) = q\},$$

$$(i, q), (j, r) \in E. \quad (2)$$

Now $P_{iq}^{\pi}(j, r, t)$ satisfies the Kolmogorov forward differential equation $P_i'(t) = P(t)A,$ where A is an infinitesimal generator of the Markov process $\{(I^{\pi}(t), X^{\pi}(t)) : t \geq 0\}$.

For each MD policy π, we get an irreducible Markov chain with the state space E and actions space A which are finite, $P^{\pi}(j, r) = \lim_{t \to \infty} P_{iq}^{\pi}(j, r; t)$ exists and is independent of initial state conditions. This implies the balance equations (5) – (16) given below. Transitions in and out of a states give a system of equations.

Consider the typical state (j, r) that lies in the range $s + 1 \leq j \leq S - 1; 1 \leq r \leq N - 1.$ When (j, r) lies in this range, there is no order pending and no restriction to allow the customers and hence transition out of this state can be due to either by demand or a service completion. The corresponding balance equation is given by equation (7).

If the typical state (j, r) that lies in the range $0 \leq j \leq s; 0 \leq r \leq N - 1,$ then there is an order to increase the inventory and we reject the ordinary customer only.

Now the system of equations can be written in order as follows,

$$\lambda P^{\pi}(S, 0) = \gamma \sum_{j=0}^{s} P^{\pi}(j, 0), \quad (3)$$

$$(\lambda + \mu_1 + \mu_2)P^{\pi}(S, r) = \gamma\sum_{j=0}^{s} P^{\pi}(j, r) + \lambda P^{\pi}(S, r-1), 1 \le r \le N-1, \tag{4}$$

$$(\mu_1 + \mu_2)P^{\pi}(S, N) = \gamma\sum_{j=0}^{s} P^{\pi}(j, N) + \lambda P^{\pi}(S, N-1), \tag{5}$$

$$\lambda P^{\pi}(j, 0) = (\mu_1 + \mu_2)P^{\pi}(j, 1, 1), s+1 \le j \le S-1, \tag{6}$$

$$(\lambda + \mu_1 + \mu_2)P^{\pi}(j, r) = (\mu_1 + \mu_2)P^{\pi}(j+1, r+1) + \lambda P^{\pi}(j, r-1),$$

$$s+1 \le j \le S-1; 1 \le r \le N-1, \tag{7}$$

$$(\mu_1 + \mu_2)P^{\pi}(j, N) = \lambda P^{\pi}(j, N-1), s+1 \le j \le S-1, \tag{8}$$

$$(\lambda + \gamma)P^{\pi}(s, 0) = (\mu_1 + \mu_2)P^{\pi}(s+1, 1) \tag{9}$$

$$(\gamma + \mu_1 + \gamma)P^{\pi}(s, r) = (\mu_1 + \mu_2)P^{\pi}(s+1, r+1)$$

$$+ \lambda P^{\pi}(s, r-1), 1 \le r \le N-1 \tag{10}$$

$$(\mu_1 + \lambda)P^{\pi}(j, N) = \lambda P^{\pi}(j, N-1), 1 \le j \le s, \tag{11}$$

$$(\lambda + \gamma)P^{\pi}(j, 0) = \mu_1 P^{\pi}(j, 1, 1), 0 \le j \le s-1 \tag{12}$$

$$(\gamma + \mu_1 + \gamma)P^{\pi}(j, r) = \mu_1 P^{\pi}(j+1, r+1) + \lambda P^{\pi}(j, r-1),$$

$$1 \le j \le s-1, 1 \le r \le N-1 \tag{13}$$

$$(\lambda + \gamma)P^{\pi}(0, r) = \mu_1 P^{\pi}(1, r+1) + \lambda P^{\pi}(0, r-1), 1 \le r \le N-1, \tag{14}$$

$$\gamma P^{\pi}(0, N) = \lambda P^{\pi}(0, N-1). \tag{15}$$

Together with the above set of equations, the total probability condition

$$\sum_{(j, r)\in E} P^{\pi}(j, r) = 1 \tag{16}$$

gives steady state probabilities $\{P^{\Pi}(j, r), (j, r) \in E\}$ uniquely.

3.2 System Performance Measures.

The average inventory level in the system is given by

$$\bar{I}^{\pi} = \sum_{j=1}^{S} j \sum_{r=0}^{N} P^{\pi}(j, r). \tag{17}$$

Mean waiting time in the system is given by

$$\overline{W}^{\pi} = \sum_{r=1}^{N} \left[\frac{r}{\mu_1} \sum\nolimits_{j=0}^{S} P^{\pi}(j, r) + \frac{r}{\mu_2} \sum\nolimits_{j=s+1}^{S} P^{\pi}(j, r) \right]$$

$$+ \sum_{m=1}^{[N/S]} \frac{1}{\gamma} \sum_{r=1}^{S} \sum_{j=0}^{s} m P^{\pi}(j, r). \tag{18}$$

The reorder rate is given by

$$\alpha_a^{\pi} = (\mu_1 + \mu_2) \sum_{r=0}^{N} P^{\pi}(s+1, r). \tag{19}$$

The balking rate is given by

$$\alpha_b^{\pi} = \lambda \sum_{j=0}^{S} P^{\pi}(j, N). \tag{20}$$

The service completion rate is given by

$$\alpha_c^{\pi} = \mu_1 \sum_{r=0}^{N} \sum_{j=1}^{S} P^{\pi}(j, r) + \mu_2 \sum_{r=1}^{N} \sum_{j=s+1}^{S} P^{\pi}(j, r). \tag{21}$$

Now the long run expected cost rate is given by

$$C^{\pi} = h \sum_{j=1}^{S} j \sum_{r=1}^{N} P^{\pi}(j, r) + c_1 \sum_{r=1}^{N} \left[\frac{r}{\mu_1} \sum\nolimits_{j=0}^{S} P^{\pi}(j, r) + \frac{r}{\mu_2} \sum\nolimits_{j=s+1}^{S} P^{\pi}(j, r) \right]$$

$$+ c_1 \sum_{m=1}^{[N/S]} \frac{1}{\gamma} \sum_{r=1}^{S} \sum_{j=0}^{s} m P^{\pi}(j, r) + c_2 (\mu_1 + \mu_2) \sum_{j=s+1}^{S} P^{\pi}(s+1, r).$$

$$+ g\lambda \sum_{j=0}^{S} P^{\pi}(j, N) + \beta\mu_1 \sum_{r=1}^{N} \sum_{j=0}^{S} P^{\pi}(j, r) + \beta\mu_2 \sum_{r=1}^{N} \sum_{j=s+1}^{S} P^{\pi}(j, r). \quad (22)$$

4. Linear Programming Problem

4.1 Formulation of LPP

In this section we propose a LPP model within a MDP framework.

Expressing $P^{\pi}(j, r)$ in terms of $y(j, r, k)$, the expected total cost rate function (21) is given by

Minimize

$$C = h \sum_{k \in A} \sum_{j=1}^{S} j \sum_{r=0}^{N} y(j, r, k) + c_1 \sum_{k \in A} \sum_{r=1}^{N} \left[\frac{r}{\mu_1} \sum\nolimits_{j=0}^{S} P^{\pi}(j, r) + \frac{r}{\mu_2} \sum_{j=s+1}^{S} P^{\pi}(j, r) \right]$$

$$+ \frac{c_1}{\gamma} \sum_{k \in A} \sum_{m=1}^{[N/s]} \sum_{r=1}^{ms} \sum_{j=0}^{S} m y(j, r, k) + c_2(\mu_1 + \mu_2) \sum_{k \in A} \sum_{r=1}^{N} y(s+1, r, k)$$

$$+ g\lambda \sum_{k \in A} \sum_{j=0}^{S} y(j, N, k) + \sum_{k \in A} \beta\mu_1 \sum_{r=1}^{N} \sum_{j=0}^{S} y(j, r, k)$$

$$+ \beta\mu_2 \sum_{r=1}^{N} \sum_{j=s+1}^{S} y(j, r, k). \quad (23)$$

Subject to the constraints,

(1) $y(j, r, k) \geq 0$, $(j, r) \in E$, $k \in A_l$, $l = 1, 2$,

(2) $\sum_{l=1}^{2} \sum_{(j, r) \in E_l} \sum_{k \in A_l} y(j, r, k) = 1$,

and the balance equations (3) – (14) obtained by replacing

$P^{\pi}(j, r)$ by $\sum_{k \in A} y(j, r, k)$.

4.3 Lemma. *The optimal solution of the above Linear Programming Problem yields a deterministic policy.*

5. Numerical illustration and Discussion

In this system we consider a problem to illustrate the method described in section 4, through numerical examples.

We implemented TORA software to solve LPP by simplex algorithm. We intuitively proposed a conjecture that the reordering rate (γ) to be employed depends only on the inventory level.

This conjecture can be proved for zero lead time and reorder is made at inventory levels and the order quantity is adjusted at the time of replenishment. Sapna, K. P., and Berman, O. [5] proved that the expected cost rate,

$$C(\gamma) = h\left(\frac{S+1}{2}\right) + \sum_{j=0}^{s} \frac{c_1}{s\left[m + \frac{\alpha(1)}{\lambda}\right]} + c_2 \sum_{k\in\{0,\,1,\,2\}} \sum_{j=1}^{s} kmp(j,\,k)$$

$$+\, g\sum_{j=0}^{s} p(j,\,N) + \beta(\gamma)\sum_{j=1}^{s}\sum_{r=1}^{N} p(j,\,r) + p\sum_{k\in A}\sum_{j=1}^{S}\sum_{r=1}^{N} y(j,\,r,\,k)$$

where $m = \frac{1}{(\mu_1 + \mu_2)}$, $\alpha(1) = \frac{1 - \frac{\lambda}{(\mu_1+\mu_2)}}{1 - \left(\frac{\lambda}{(\mu_1+\mu_2)}\right)^N}$.

For $0 \le j \le S$, $1 \le r \le N$

$$p(j,\,r) = \left(\frac{\lambda}{(\mu_1+\mu_2)}\right)^{r-1} P(j,\,0).$$

For $0 \le j \le S$,

$$p(j,\ 0) = \left(\frac{1}{S}\right)\left(\frac{1 - \frac{\lambda}{(\mu_1 + \mu_2)}}{1 - \left(\frac{\lambda}{(\mu_1 + \mu_2)}\right)^N}\right).$$

Consider the MDP problem with the following parameters:

$S = 3,\ s = 2,\ N = 4,\ \lambda = 2,\ \mu_1 = 3,\ \mu_2 = 2,\ \gamma = 4,\ h = 0.1,\ c_j = 3j;\ j = 0,$ $1,\ 2,\ g = 5,\ p = 2,\ (\gamma) = 2\gamma.$ This particular case of the problem gives optimal solution R* after application of LPP method. That is whenever the inventory level reaches the reorder level $s(> 0)$ the optimal decision is to refill the inventory with $Q = S - s$ items.

5. Conclusions

Analysis of inventory control at service facility is fairly recent system study. Most of previous work determined optimal ordering policies or system performance measures. We approached the problem in a different way with two type of customers given a service rate we determine the optimal controlling the admission of customers to determine the optimal order quantity or reorder level to be employed to minimize the long – run expected cost rate. Thus the admission control using inventory in service facility is established. In future we may extend this model to multi type of customer with discrete time MDP.

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