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FUZZY EIGEN VALUES AND EIGEN VECTORS OF FUZZY MATRIX

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Abstract

In this paper we compute fuzzy eigen values and fuzzy eigen vectors of a generalized triangular fuzzy matrix within the context of a fuzzy markov property. Using this ideology generalized triangular fuzzy linear system is ameliorated and can find the optimal results using an unique arithmetic operations. We have given some properties and an example was given to exemplify the methodology.

1. Introduction

Numerous uses of matrices in both engineering and science use eigen values and eigen vectors are indispensable. Real eigen values and eigen vectors assume a conspicuous part in the study of Vibrant Analysis, Electric Circuits and Quantum Mechanics. In fuzzy Linear algebra, the concept of eigen values and eigen vectors plays a vital role. Markov property says that whatever happens next in a process only depends on how it is right now at the state it.

The idea of fuzzy sets were introduced by Zadeh [12]. Buckley [2]

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discussed fuzzy eigen values with fuzzy positive elements. Theodoroua et al. [11] investigated fuzzy eigen values with triangular fuzzy numbers. Allahviranloo and Laleh Hooshangian [1] demonstrate how do the obtained 2 spreads lead to derive maximal and minimal eigen values. Salahshour et al. [5] studied fuzzy eigen value and fuzzy eigen vector by using the maximal and minimal symmetric spreads.

Sanjibmondal and Madhumangalpal [6] found inversibility and eigen values of intutionistic fuzzy matrix. Indirani and Saranya [3] they discussed some properties of eigen values. Ramakrishna Mankena [4] proved few properties and results of eigen value and eigen vectors of m- polar fuzzy matrices. Stephen Dinagar and Hetciyal S. P. [7] studied the concept of generalized triangular fuzzy matrix. Stephen Dinagar and Hetciyal S. P. [8] discussed the fuzzy LU decomposition method with the aid of GTFNs. Stephen Dinagar and Hetciyal S. P. [9, 10] explored how the GTFLS applied in economy and find a optimal solution.

This paper has been arranged as follows, In section 2, some basic definitions are collected which are needful for this work. In section 3, some properties of generalized triangular fuzzy linear system has been given. In section 4, algorithm for finding generalized triangular fuzzy eigen values and generalized triangular fuzzy eigen values are marked. In section 5, numerical example is presented. Finally this paper ends with conclusion and references.

2. Preliminaries

Definition 1. A fuzzy matrix $\widetilde{H} = (h_{ij}^{(1)}, h_{ij}^{(2)}, h_{ij}^{(3)}; w_{ij})$ of order $m \times n$ is said to be generalized triangular fuzzy matrix if the elements of the matrix are generalized triangular fuzzy numbers.

Definition 2. Let $\widetilde{H} = (h_{ij}^{(1)}, h_{ij}^{(2)}, h_{ij}^{(3)}; w_{ij}) n \times n, i, j \in N$ be a generalized triangular fuzzy square matrix of order n.

The characteristic equation of $\widetilde{H}$ is $|\widetilde{H} - \widetilde{\gamma}\widetilde{I}| = \widetilde{0}$. The roots of the characteristic equation are called generalized triangular fuzzy eigen value.

Definition 3. Let $\widetilde{H} = (h_{ij}^{(1)}, h_{ij}^{(2)}, h_{ij}^{(3)}; w_{ij}) n \times n, i, j \in N$ be a generalized triangular fuzzy square matrix.

If there exists a non-zero generalized triangular fuzzy vector

$$\widetilde{U} = \begin{bmatrix} (u_1^{(1)}, u_1^{(2)}, u_1^{(3)}; w_1) \\ (u_2^{(1)}, u_2^{(2)}, u_2^{(3)}; w_2) \\ \vdots \\ (u_n^{(1)}, u_n^{(2)}, u_n^{(3)}; w_n) \end{bmatrix} \text{ such that}$$

$\widetilde{H}\widetilde{U} = \widetilde{\gamma}\widetilde{U}$, then the vector $\widetilde{U}$ is called generalized triangular fuzzy eigen vector of H corresponding to the generalized triangular fuzzy eigen value $\widetilde{\gamma}$.

3. Properties of Eigen Values and Eigen Vectors of Fuzzy Matrix

Property 1.

(i) The sum of the fuzzy eigen value of a generalized triangular fuzzy matrix is the sum of the elements of the main diagonal of the generalized triangular fuzzy matrix.

(ii) Product of the fuzzy eigen values is equal to the determinant of the generalized triangular fuzzy matrix.

Proof. Let $\widetilde{H} = (h_{ij}^{(1)}, h_{ij}^{(2)}, h_{ij}^{(3)}; w_{ij})n \times n, i, j \in N$ be a generalized triangular fuzzy square matrix of order n.

The characteristic equation of $\widetilde{H}$ is $|\widetilde{H} - \widetilde{\gamma}\widetilde{I}| = \widetilde{0}$. .i.e., $(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^n - \widetilde{M}_1(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^{n-1} + \widetilde{M}_2(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^{n-2}$ $- \ldots + (-1)^n \widetilde{M}_n = \widetilde{0}(1)$ where, $\widetilde{M}_1 = (m_1^{(1)}, m_1^{(2)}, m_1^{(3)}; w) =$ Sum of the diagonal elements of GTFM $\widetilde{H} = \widetilde{M}_n(m_n^{(1)}, m_n^{(2)}, m_n^{(3)}; w) =$ Determinant of GTFM $\widetilde{H}$. Fuzzy eigen values are the roots of the characteristic equations of the given GTFM.

Solving (1), we get $(\gamma_1', \gamma_2', \gamma_3'; w), (\gamma_1'', \gamma_2'', \gamma_3''; w), \ldots, (\gamma_1^{n'}, \gamma_2^{n'}, \gamma_3^{n'}; w)$
$(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^n -$ (Sum of the roots) $(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^{n-1}$
(Sum of the product of the roots taken two at a time) $(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w)^{n-2}$

$- \ldots + (-1)^n (\text{Product of the roots}) = \widetilde{0} (2)$

From (1) and (2), Equating the co-efficient, i.e.,

$$(\gamma_1', \gamma_2', \gamma_3'; w) + (\gamma_1'', \gamma_2'', \gamma_3''; w) + \ldots + (\gamma_1^{n'}, \gamma_2^{n'}, \gamma_3^{n'}; w) = (h_{ii}^{(1)}, h_{ii}^{(2)}, h_{ii}^{(3)}; w_{ii})$$

Sum of the fuzzy eigen value of a GTFM = Sum of the main diagonal element of GTFM

S_n = Product of the fuzzy eigen values $= |\widetilde{H}|$.

Property 2. A generalized triangular fuzzy square matrix $\widetilde{H}$ and its transpose $\widetilde{H}^T$ have the same fuzzy eigen values.

Proof. Let $\widetilde{H}$ be a generalized triangular fuzzy square matrix of order n. The characteristic equations are $|\widetilde{H} - \widetilde{\gamma}\widetilde{I}| = 0$ and $|\widetilde{H}^T - \widetilde{\gamma}\widetilde{I}| = 0$ Since, the determinant values remain same by the interchange of rows and columns.

$$\widetilde{\Delta} = \widetilde{\Delta}^T.$$

Hence both are identical. Therefore fuzzy eigen values of $\widetilde{A}$ and its transpose $\widetilde{H}^T$ remains same.

Property 3. Eigen values of generalized triangular diagonal (upper or lower generalized triangular fuzzy matrix) fuzzy matrix are its leading diagonal elements.

Proof. Let us consider the triangular fuzzy matrix

$$\widetilde{H} = \begin{bmatrix} (h_{11}^{(1)}, h_{11}^{(2)}, h_{11}^{(3)}; w_{11}) & (0, 0, 0; 1) & \ldots & (0, 0, 0; 1) \\ (h_{21}^{(1)}, h_{21}^{(2)}, h_{21}^{(3)}; w_{21}) & (h_{22}^{(1)}, h_{22}^{(2)}, h_{22}^{(3)}; w_{22}) & \vdots & (0, 0, 0; 1) \\ \vdots & \vdots & \ddots & \vdots \\ (h_{n1}^{(1)}, h_{n1}^{(2)}, h_{n1}^{(3)}; w_{n1}) & (h_{n2}^{(1)}, h_{n2}^{(2)}, h_{n2}^{(3)}; w_{n2}) & \ldots & (h_{nn}^{(1)}, h_{nn}^{(2)}, h_{nn}^{(3)}; w_{nn}) \end{bmatrix}$$

Characteristic equation of $\widetilde{H}$ is $|\widetilde{H} - \widetilde{\gamma}\widetilde{I}| = \widetilde{0}$ i.e.,

$$\begin{vmatrix} (h_{11}^{(1)}, h_{11}^{(2)}, h_{11}^{(3)} w_{11}) - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_\gamma) & (0, 0, 0;1) & \ldots & (0, 0, 0;1) \\ (a_{21}^{(1)}, a_{21}^{(2)}, a_{21}^{(3)}; w_{21}) & (h_{22}^{(1)}, h_{22}^{(2)}, h_{22}^{(3)}; w_{22}) - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_\gamma) & \cdots & (0, 0, 0;1) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{n1}^{(1)}, a_{n1}^{(2)}, a_{n1}^{(3)}; w_{n1}) & (a_{n1}^{(1)}, a_{n1}^{(2)}, a_{n1}^{(3)}; w_{n1}) & \cdots & \begin{matrix} (h_{nn}^{(1)}, h_{nn}^{(2)}, h_{nn}^{(3)}; w_{nn}) \\ - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_\gamma) \end{matrix} \end{vmatrix}$$

$= 0$

on expansion it gives,

$$((h_{11}^{(1)}, h_{11}^{(2)}, h_{11}^{(3)}; w_{11}) - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_{\gamma}))((h_{22}^{(1)}, h_{22}^{(2)}, h_{22}^{(3)}; w_{22}) - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_{\gamma}))$$

$$((h_{nn}^{(1)}, h_{nn}^{(2)}, h_{nn}^{(3)}; w_{nn}) - (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_{\gamma}))$$

$$= \tilde{0}$$

i.e., $(\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}; w_{\gamma}) = (h_{11}^{(1)}, h_{11}^{(2)}, h_{11}^{(3)}; w_{11}), (h_{22}^{(1)}, h_{22}^{(2)}, h_{22}^{(3)}; w_{22}), \ldots, (h_{nn}^{(1)}, h_{nn}^{(2)}, h_{nn}^{(3)}; w_{nn})$.

Which are the diagonal elements of the generalized triangular fuzzy matrix.

4. Generalized Triangular Fuzzy Eigen Value and Generalized Triangular Fuzzy Eigen Vector

Algorithm:

Step 1. This method is used to determine numerically largest fuzzy eigen value and the corresponding fuzzy eigen vector of generalized triangular fuzzy matrix $\tilde{H}$.

Let $\tilde{H} = (h_{ij}^{(1)}, h_{ij}^{(2)}, h_{ij}^{(3)}; w_{ij}) n \times n,\ i, j \in N$ generalized triangular fuzzy square matrix and let $(\gamma_1', \gamma_2', \gamma_3'; w), (\gamma_1'', \gamma_2'', \gamma_3''; w), \ldots, (\gamma_1^{n'}, \gamma_1^{n'}, \gamma_1^{n'}; w)$ be the distinct generalized triangular fuzzy eigen value of $\tilde{H}$.

Let $(u_1', u_2', u_3'; w), (u_1'', u_2'', u_3''; w), \ldots, (u_1^{n'}, u_1^{n'}, u_1^{n'}; w)$ be their corresponding generalized triangular fuzzy eigen vectors.

$\therefore \tilde{H}\tilde{U} = \tilde{\gamma}\tilde{U}$ and the vectors are linearly independent.

Step 2. Let $\tilde{Q}^{(0)}$ be any vector, then $\tilde{Q}^{(1)} = \tilde{H}\tilde{Q}^{(0)}$

Similarly, $\widetilde{Q}^{(2)} = \widetilde{H}\widetilde{Q}^{(1)}$

Continuing this process, $\widetilde{Q}^{(r)} = \widetilde{H}\widetilde{Q}^{(r-1)}$, using this obtain $\widetilde{\gamma}_H^D$ is the dominant generalized triangular fuzzy eigen value of $\widetilde{H}$ and generalized triangular fuzzy eigen vector.

Step 3. Then find $\widetilde{B} = \widetilde{H} - \gamma_{\widetilde{H}}^D \widetilde{I}$ and find the dominant generalized triangular fuzzy eigen value $\widetilde{\gamma}_{\widetilde{B}}^D$.

$\therefore$ Smallest generalized triangular fuzzy eigen value $\widetilde{\gamma}_{\widetilde{H}}^l$ of $\widetilde{H} = \widetilde{\gamma}_{\widetilde{H}}^D + \widetilde{\gamma}_{\widetilde{B}}^D$.

Step 4. By property (1), The sum of the fuzzy eigen value of a generalized triangular fuzzy matrix = sum of the elements of the main diagonal of the generalized triangular fuzzy matrix.

From this we will discover the corresponding eigen values of $\widetilde{H}$

5. Numerical Example

	Sunny	Cloudy	Rainy
Sunny	(1,2,3;0.2)	(0.375,0.750,1.125;0.4)	(0.084,0.167,0.252;0.6)
Cloudy	(1,2,3;0.2)	(0.375,0.750,1.125;0.4)	(0.6,1.2,1.8;0.5)
Rainy	(0.334,0.667,1.002;0.3)	(1,2,3;0.2)	(0.375,0.750,1.125;0.4)

Today (day 0) is cloudy,

$$\widetilde{Q}^{(0)} = \begin{pmatrix} (0,\ 0,\ 0;1) \\ (1,\ 1,\ 1;1) \\ (0,\ 0,\ 0;1) \end{pmatrix}$$

Probabilities for the weather for a week, using markov process

$\tilde{Q}^{(0)}$	$\tilde{Q}^{(1)}$	$\tilde{Q}^{(2)}$	$\tilde{Q}^{(3)}$
$\begin{pmatrix}(0,0,0;1)\\(1,1,1;1)\\(0,0,0;1)\end{pmatrix}$	$\begin{pmatrix}(0.214,0.429,0.643;0.7)\\(0.218,0.436,0.655;0.688)\\(0.333,0.667,1;0.6)\end{pmatrix}$	$\begin{pmatrix}(0.228,0.456,0.686;0.549)\\(0.430,0.860,1.289;0.524)\\(0.309,0.617,00926;0.486)\end{pmatrix}$	$\begin{pmatrix}0.274,0.551,0.825;0.481\\(0.454,0.910,1.364;0.456)\\0.382,0.765,1.149;0.419\end{pmatrix}$

$\tilde{Q}^{(4)}$	$\tilde{Q}^{(5)}$	$\tilde{Q}^{(6)}$
$\begin{pmatrix}(0.293,0.588,0.883;0.448)\\0.502,1.004,1.505;0.423\\(0.410,0.818,1.231;0.385)\end{pmatrix}$	$\begin{pmatrix}(0.307,0.616,0.925;0.431)\\0.520,1.041,1.564;0.406\\(0.429,864,1.296;0.368)\end{pmatrix}$	$\begin{pmatrix}(0.313,0.628,0.943;0.422)\\(0.531,1.068,1.062;0.397)\\(0.445,0.882,1.330;0.360)\end{pmatrix}$

$\tilde{Q}^{(7)}$
$\begin{pmatrix}(0.319,0.635,0.953;0.418)\\(0.540,1.080,1.620;0.393)\\(0.448,0896,1346;0.355)\end{pmatrix}$

Today (day 0) is Sunny,

$$\tilde{Q}^{(0)} = \begin{pmatrix}(1,1,1;1)\\(0,0,0;1)\\(0,0,0;1)\end{pmatrix}$$

Probabilities for the weather for a week, using markov process

$\tilde{Q}^{(0)}$	$\tilde{Q}^{(1)}$	$\tilde{Q}^{(2)}$	$\tilde{Q}^{(3)}$
$\begin{pmatrix}(1,1,1;1)\\(0,0,0;1)\\(0,0,0;1)\end{pmatrix}$	$\begin{pmatrix}(0.296,0.593,0.889;0.675)\\(0.302,604,0.906;0.663)\\(0.154,0.308,0.462;0.650)\end{pmatrix}$	$\begin{pmatrix}(0.272,0.541,0.811;0.555)\\(0.377,0.755,1.132;0.530)\\(0.305,0.610,0.916;0.493)\end{pmatrix}$	$\begin{pmatrix}(0.279,0.557,0.836;0.485)\\(0.457,0.914,1.373;0.460)\\(0.367,0.735,1.102;0.422)\end{pmatrix}$

$\tilde{Q}^{(4)}$	$\tilde{Q}^{(5)}$	$\tilde{Q}^{(6)}$
$\begin{pmatrix}(0.296, 0.590, 0.886;0.449)\\(0.495, 0.991, 1.486;0.424)\\(0.406, 0.815, 1.221;0.387)\end{pmatrix}$	$\begin{pmatrix}(0.306, 0.612, 0.915;0.432)\\(0.519, 1.037, 1.55;0.407)\\(0.428, 0.856, 1287;0.369)\end{pmatrix}$	$\begin{pmatrix}(0.312, 0.625, 0.937;0.423)\\(0.531, 1.062, 1.592;0.398)\\(0398, 0.797, 1.195;0.397)\end{pmatrix}$

$\tilde{Q}^{(7)}$
$\begin{pmatrix}(0.309, 0.618, 0.931;0.428)\\(0.524, 1.051, 1.578;0.403)\\(0.433, 0.868, 1.304;0.365)\end{pmatrix}$

Today (day 0) is Rainy,

$$\tilde{Q}^{(0)} = \begin{pmatrix}(0, 0, 0;1)\\(0, 0, 0;1)\\(1, 1, 1;1)\end{pmatrix}$$

Probabilities for the weather for a week, using markov process.

$\tilde{Q}^{(0)}$	$\tilde{Q}^{(1)}$	$\tilde{Q}^{(2)}$	$\tilde{Q}^{(3)}$
$\begin{pmatrix}(1, 1, 1;1)\\(0, 0, 0;1)\\(0, 0, 0;1)\end{pmatrix}$	$\begin{pmatrix}(0.069, 0.139, 0.208;0.725)\\(0.429, 0.857, 1.286;0.7)\\(0.226, 0.453, 0.679;0.663)\end{pmatrix}$	$\begin{pmatrix}(0.219, 0.441, 0.664;0.570)\\(0.367, 0.735, 1.104;0.545)\\0.345, 0.690, 1.036;0.507\end{pmatrix}$	$\begin{pmatrix}(0.260, 0.521, 0.781;0.492)\\(0.461, 0.924, 1.387;0.467)\\(0.368, 0.737, 1.105;0.429)\end{pmatrix}$

$\tilde{Q}^{(4)}$	$\tilde{Q}^{(5)}$	$\tilde{Q}^{(7)}$
$\begin{pmatrix}(0.292, 0.581, 0.873;0.453)\\(0.494, 0.987, 1.481;0.428)\\(0.410, 0.821, 1.228;0.390)\end{pmatrix}$	$\begin{pmatrix}(0.305, 0.610, 0.917;0.433)\\(0.520, 1.039, 1559;0.408)\\(0.426, 0.858, 1.287;0.371)\end{pmatrix}$	$\begin{pmatrix}(0.312, 0.626, 0.942;0.424)\\(0.529, 1.064, 1, 598;0.399)\\(0.440, 0.881, 1.321;0.361)\end{pmatrix}$

$\widetilde{Q}^{(7)}$
$\begin{pmatrix} (0.318,\ 0.636,\ 0.953;0.419) \\ (0.541,\ 1.080,\ 1.621;0.394) \\ (0.447,\ 0.896,\ 1.343;0.356) \end{pmatrix}$

Fuzzy eigen value

$$\widetilde{H} = \begin{bmatrix} (1,2,3;0.2) & (0.375,0.750,1.125;0.4) & (0.084,0.167,0.252;0.6) \\ (1,2,3;0.2) & (0.375,0.750,1.125;04) & (0.6,1.2,1.8;0.5) \\ (0.334,0.667,1.002;0.3) & (1,2,3;0.2) & (0.375,0.750,1.125;0.4) \end{bmatrix}$$

$$\widetilde{Q}^{(1)} = \begin{bmatrix} (1,1,1;1) \\ (1,1,1;1) \\ (1,1,1;1) \end{bmatrix}$$

$$\widetilde{H}\widetilde{Q}^{(1)} = \begin{bmatrix} (0.552,\ 1.103,\ 1.657;0.725) \\ (0.929,\ 1.857,\ 2.786;0.7) \\ (0.679,\ 1.358,\ 2.0.38;0.663) \end{bmatrix}$$

$$= (0.929,\ 1.857,\ 2.786;0.7) \begin{bmatrix} (0.432,\ 0.863,\ 1.297;0.713) \\ (0.715,\ 1.429,\ 2.143;0.7) \\ (0.508,\ 1.016,\ 1.525;0.682) \end{bmatrix}$$

$$= (0.929,\ 1.857,\ 2.786;7)\, \widetilde{Q}^{(2)}$$

$$\widetilde{H}\widetilde{Q}^{(2)} = \begin{bmatrix} (0.538,\ 1.074,\ 1.614;0.573) \\ (0.879,\ 1.757,\ 2.637;0.548) \\ (0.716,\ 1.435,\ 2.153;0.510) \end{bmatrix}$$

$$= (0.879,\ 1.757,\ 2.637;0.548) \begin{bmatrix} (0.432,\ 0.863,\ 1.297;0.713) \\ (0.715,\ 1.429,\ 2.143;0.7) \\ (0.508,\ 1.016,\ 1.525;0.682) \end{bmatrix}$$

$$= (0.879,\ 1.757,\ 2.637;0.548)\, \widetilde{Q}^{(3)}$$

$$\widetilde{H}\widetilde{Q}^{(3)} = \begin{bmatrix} (0.636,\ 1.275,\ 1.913;0.497) \\ (1.073,\ 2.148,\ 322;0472) \\ (0.871,\ 1.744,\ 2.615;0.434) \end{bmatrix}$$

$$= (1.073,\ 2.148,\ 3.222;0.472)\begin{bmatrix}(0.643,\ 1.289,\ 1.935;0485)\\(1.059,\ 2.119,\ 3.179;0.472)\\(0.823,\ 1.647,\ 2.470;0.453)\end{bmatrix}$$

$$= (1.073,\ 2.148,\ 3.222;0.472)\,\tilde{Q}^{(4)}$$

$$\tilde{H}\tilde{Q}^{(4)} = \begin{bmatrix}(0.683,\ 1.365,\ 2.050;0.459)\\(1.151,\ 2.304,\ 3.458;0.434)\\(0.944,\ 1.894,\ 2.841;0.396)\end{bmatrix}$$

$$= (1.151,\ 2.304,\ 3.458;0.434)\begin{bmatrix}(0.702,\ 1.403,\ 2.108;0.447)\\(1.152,\ 2.304,\ 3.456;0.434)\\(0.901,\ 1.807,\ 2.711;0.415)\end{bmatrix}$$

$$= (1.151,\ 2.304,\ 3.458;0.434)\,\tilde{Q}^{(5)}$$

$$\tilde{H}\tilde{Q}^{(5)} = \begin{bmatrix}(0.714,\ 1.427,\ 2.141;0.440)\\(1.209,\ 2.420,\ 3.628;0.415)\\(0.997,\ 1.995,\ 2.989;0377)\end{bmatrix}$$

$$= (1.209,\ 2.420,\ 3.628;0.415)\begin{bmatrix}(0.735,\ 1.469,\ 2.204;0.428)\\(1.205,\ 2.410,\ 3.614;0.415)\\(0.49,\ 1.899,\ 2.846;0.396)\end{bmatrix}$$

$$= (1.209,\ 2.420,\ 3.628;0.415)\,\tilde{Q}^{(6)}$$

$$\tilde{Q}^{(6)} = \begin{bmatrix}(0.735,\ 1.469,\ 2.204;0.428)\\(1.205,\ 2.410,\ 3.614;0.415)\\(0.949,\ 1.899,\ 2.846;0.396)\end{bmatrix}$$

$\tilde{\gamma}^{D}_{\tilde{H}} = (1.209,\ 2.420,\ 3.628;0.415)$ is the dominant generalized triangular fuzzy eigen value of $\tilde{H}$ with respect to the generalized triangular fuzzy eigen vector

$$\tilde{Q}^{(6)} = \begin{bmatrix}(0.735,\ 1.469,\ 2.204;0.428)\\(1.205,\ 2.410,\ 3.614;0.415)\\(0.949,\ 1.899,\ 2.846;0.396)\end{bmatrix}$$

To find the least generalized triangular fuzzy eigen value:

Let $\widetilde{B} = \widetilde{H} - \widetilde{\gamma}_{\widetilde{H}}^{D}\widetilde{I}$

$$\widetilde{B} = \begin{bmatrix} (-2.879, -1.333, 0.216; 0.454) & (0.271, 0542, 0.812; 0.554) & (0.077, 0.153, 0.231; 0.654) \\ (0.441, 0.881, 1.322; 0.454) & (-2.449, -1.273, -0.094; 0.554) & (0.497, 0.993, 1.490; 0.604) \\ (0.199, 0.397, 0.596; 0.504) & (0.441, 0.881, 1.322; 0.454) & (-2.449, -1.273, -0.094; 0.554) \end{bmatrix}$$

Let $\widetilde{R}^{(1)} = \begin{bmatrix} (1, 1, 1, ;1) \\ (1, 1, 1, ;1) \\ (1, 1, 1, ;1) \end{bmatrix}$

$$\widetilde{B}\widetilde{R}^{(1)} = \begin{bmatrix} (-1.402, -0.258, 0.885; 0.790) \\ (-1.103, 380, 1.864; 0.777) \\ (-1.392, -0.137, 1.119; 0.759) \end{bmatrix}$$

$$= (-1.402, -0.258, 0.885; 0.790) \begin{bmatrix} (-4.342, 1.267, 6.879; 0.790) \\ (-9.053, -1.847, 5.356; 0.784) \\ (-5377, 0.658, 6.687; 0.775) \end{bmatrix}$$

$$= (-1.402, -0.258, 0.885; 0.790)\, \widetilde{R}^{(2)}$$

$$\widetilde{B}\widetilde{R}^{(2)} = \begin{bmatrix} (-2.843, -1.453, -0.062; 0.681) \\ (0.642, 2.587, 4.530; 0.668) \\ (-2.251, -1.139, -0.026; 0.649) \end{bmatrix}$$

$$= (-2.843, -1.453, -0.062; 0.681) \begin{bmatrix} (0.062, 1.469, 2.875; 0.681) \\ (-4.537, -2.591, -0643; 0.675) \\ (0.026, 1.124, 2.222; 0.665) \end{bmatrix}$$

$$= (-2.843, -1.453, -0.062; 0.681)\widetilde{R}^{(2)}$$

$$\widetilde{B}\widetilde{R}^{(3)} = \begin{bmatrix} (-3.291, -1.688, -0.083; 0.626) \\ (0.839, 3.367, 5.896; 0.623) \\ (-3.302, 1.728, -0.148; 0.595) \end{bmatrix}$$

$$= (-3.291, -1.688, -0.083; 0.626) \begin{bmatrix} (0.079, 1.597, 3.114; 0.626) \\ (-5.565, -3.178, -0.792; 0.625) \\ (3.043, 1.591, 0.136; 0.611) \end{bmatrix}$$

Dominant eigen value of $\widetilde{B} = \widetilde{\gamma}_{\widetilde{B}}^{D}\widetilde{I} = (-3.291, -1.688, -0.083; 0.626)$

$\therefore$ Least generalised triangular fuzzy eigen value of $\widetilde{H} = \widetilde{\gamma}^{l}_{\widetilde{H}}$ $= (-2.993, -0.102, 2.793; 0.521)$

Trace of generalised triangular fuzzy matrix $\widetilde{H} = (1.429, 2.857, 4.286; 0.350)$ By property (1),

The sum of the fuzzy eigen value of a generalized triangular fuzzy matrix = sum of the elements of the main diagonal of the generalized triangular fuzzy matrix.

$\widetilde{\gamma}^{(3)}_{\widetilde{H}} = (-6.017, 0.120, 6.252; 0.409).$

All the three generalized triangular fuzzy eigen values are,

$\widetilde{\gamma}^{(1)}_{H} = (1.209, 2.420, 3.628; 0.415)$, $\widetilde{\gamma}^{(2)}_{\widetilde{H}} = (-2.993, -0.102, 2.793; 0.521)$, $\widetilde{\gamma}^{(3)}_{\widetilde{H}} = (-6.017, 0.120, 6.252; 0.409)$

Here we have rank of the trace of generalized triangular fuzzy matrix $\widetilde{H}$ is equal to the rank of sum of all the three eigen values.

6. Conclusion

Computing eigen values and eigen vectors has numerous utilization of practically all parts of science. Fuzzy eigen value and fuzzy eigen vector is discovered to the generalized triangular fuzzy linear system. Markov process is used to predict the weather. The proposed method is illustrated with numerical illustration.

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