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THE INFLUENCE OF SLIP ON A JEFFREY FLUID FLOW THROUGH NARROW TUBES

SANTHOSH NALLAPU and P. RAJESHWAR RAO

School of Science and Humanities
Department of Mathematics
Sreenidhi Institute of Science
and Technology, Yamnampet, Ghatkesar
Hyderabad, Telangana-50130, India
E-mail: santhoshn@sreenidhi.edu.in
rajeshwarraop@sreenidhi.edu.in

Abstract

A two-fluid model for the flow of Jeffrey fluid through a small tube has been studied under the influence of slip condition. The model consists of Jeffrey fluid in the core region and Newtonian fluid in the peripheral region. It is assumed that the viscosity in the core region differs from the peripheral region. The effective viscosity is observed to decrease with the Jeffrey parameter and Darcy number but increase with the slip parameter and tube hematocrit. The model's effective viscosity is found to be in good agreement with the tolerable range. Further, the effective viscosity rises with tube radius, this effect is known as the Fahraeus-Lindqvist effect.

Introduction

The flow through the smallest blood vessels such as arterioles, capillaries, and venules is referred to as microcirculation. It encompasses vessels less than 200microns in diameter. It transports oxygenated blood to the capillaries, and blood exits the capillaries via venules into veins. Its major role is to provide nutrients and oxygen to all parts of the body while also removing carbon dioxide.

It has been studied that when blood is pumped through smaller tubes, the diameter of the red blood cell (RBC) becomes comparable and the two-phase

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nature of blood becomes more important. Further, flow through tiny blood vessels is accompanied by several effects, like Fahraeus - Lindqvist effect, the Fahraeus effect, and so on. Several investigators (Seshadri and Jaffrin [1] and Whitmore [2]) have confirmed these effects. Srivastava and Rashmi [3] considered a two-phase macroscopic model for blood and studied the effects of hematocrit on increased flow resistance and other flow characteristics. Saran and popel [4] presented a two-phase blood flow model in narrow arteries assuming that the central core consists of suspended erythrocytes and a cell-free layer surrounded by the core. Bugliarello and Sevilla [5] investigated a two-fluid model in which both the regions are occupied by Newtonian fluids but with different viscosities. Chaturani and Upadhya [6, 7] analyzed a two-fluid model in a narrow tube where a cell-free layer is occupied by Newtonian fluid and the core region by polar fluids.

Santhosh et al. [8] recently investigated a two-fluid model of a Jeffrey fluid flow through a porous medium in narrow tubes. Jeffrey fluid is one such non-Newtonian fluid that has captivated the interest of many researchers due to its discovery as a better model for physiological fluids (Hayat et al. [9]). Vajravelu et al. [10] have explored the peristalsis and heat transfer on the physiological flow of a two-fluid model occupying the core region with a Jeffrey fluid and the peripheral region with a Newtonian fluid in a symmetric channel bounded by permeable walls. Eswara Rao and Sreenadh [11] have considered the two-phase flow of MHD Jeffrey fluid in presence of porous media over a stretching/shrinking sheet. Krishna Murthy et al. [12] have studied a numerical solution on the hydromagnetic flow of Jeffrey fluid over a deformable porous channel with slip conditions. Recently, Santhosh et al. [13, 14], have been studied a two-fluid model through small diameter tubes under various conditions.

In view of the above researchers, an attempt has been made in this paper to investigate the effect of slip at the wall of a two-fluid model consisting of Jeffrey fluid in the core and Newtonian in the peripheral region. Following the analysis of Santhosh et al. [13, 14], analytical solutions were found for the linearized equations of motion. The effects of effective viscosity, core hematocrit, and bluntness are assessed. The results obtained throw light on the impact of tube radius in the circulation of blood in small blood vessels. Further, it be may be useful in understanding the effect of Fahraeus-Lindqvist effect.

2. Mathematical Model

Let us consider the effect of slip at the walls of an incompressible Jeffrey fluid occupying in the core region of radius '*b*' with viscosity μ_c and the surrounding outer cell-free layer of a thickness $(\varepsilon)(a - b = \varepsilon)$ with a Newtonian fluid of viscosity μ_p. We analyze the flow in half the diameter 'a' of the tube as illustrated in Figure 1 for symmetry and simplicity. The cylindrical polar coordinate system (r, θ, z) is chosen so that the *z*-axis is parallel to the tube's axis.

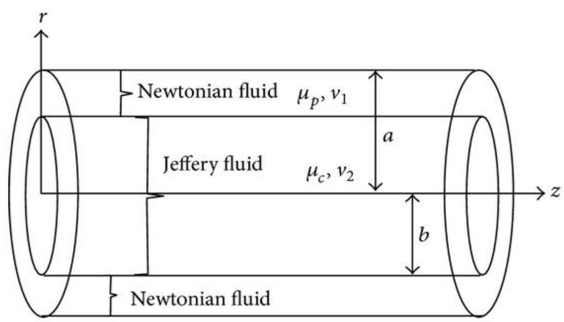

Figure 1. Geometry of the problem.

The equation governing the flow of a steady, laminar and axisymmetric flow of a Jeffrey fluid in the core region and Newtonian fluid in the peripheral region for the present problem are given by (Santhosh and Radha Krishnamacharya [8]).

$$-\frac{\partial p}{\partial z} + \frac{\mu_c}{1+\lambda_1}\frac{\partial}{\partial r}\left(r\frac{\partial v_1}{\partial r}\right) = 0, \text{ for } 0 < r \leq b \tag{1}$$

$$-\frac{\partial p}{\partial z} + \frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_2}{\partial r}\right) = 0, \text{ for } b < r < a \tag{2}$$

where v_1, v_2 are velocity components in the core and peripheral regions respectively, μ_c and μ_p are viscosities of Jeffrey fluid and Newtonian fluid, λ_1 is the ratio of relaxation to retardation times and p is the pressure.

The following boundary conditions apply to the flow:

(a) The velocity gradient along the tube's axis disappears due to symmetry:

$$\frac{\partial v_2}{\partial r} = 0 \text{ at } r = 0 \tag{3a}$$

(b) Saffman's slip boundary conditions (Saffman [14]) is:

$$a\frac{\partial v_1}{\partial r} = -\frac{\alpha}{\sqrt{D\alpha}} v_1 \text{ at } r = a \tag{3b}$$

(c) At the plasma-core interface, velocity and shear stress are constant:

(i) $v_1 = v_2$ at $r = b$ (3c)

(ii) $\mu_c \frac{\partial v_2}{\partial r} = \mu_p \frac{\partial v_1}{\partial r}$ at $r = b$ (3d)

Solving equations (1) and (2), in the presence of the boundary conditions (3a) → (3d), we get:

$$v_1(\xi) = \frac{Pa^2}{4\mu_p}\left[\left(\frac{2\sqrt{Da}}{\alpha} + 1\right) + (1 + \lambda_1)\mu'(d^2 - \xi^2) - d^2\right], 0 \le \xi \le d \tag{4}$$

$$v_2(\xi) = \frac{Pa^2}{4\mu_p}\left[\left(\frac{2\sqrt{Da}}{\alpha} + 1\right) - \xi^2\right], d \le \xi \le 1 \tag{5}$$

where

$$P = -\frac{\partial p}{\partial z}, \xi = \frac{r}{d}, d = \frac{b}{a}, \mu' = \frac{\mu_p}{\mu_c}, \tag{6}$$

The non-dimensional core radius is denoted by d.

The core region velocity can be written as:

$$v_2(\xi) = v_{\max}[1 - B\xi^2] \tag{7}$$

Where

$$v_{\max} = \frac{Pa^2}{4\mu_p}\left[\left(\frac{2\sqrt{Da}}{\alpha} + 1\right) - d^2 + (1 + \lambda_1)\mu' d^2\right] \tag{8}$$

$$B = \frac{(1+\lambda_1)\mu'}{\left[\left(\frac{2\sqrt{Da}}{\alpha}\right) - d^2 + (1+\lambda_1)\mu' d^2\right]} \tag{9}$$

The deviation from the parabolic flow is represented by parameter B, which is the bluntness of the velocity profile. When $\mu_c \to \mu_p$, $B \to 1$, during the whole cross-section of the tube, the velocity profile becomes parabolic.

The volumetric flow in the core region and peripheral region is given by:

$$Q = 2\pi a^2 \int_0^d v_1(\xi)\xi d\xi + 2\pi a^2 \int_d^1 v_2(\xi)\xi d\xi. \tag{10}$$

Substituting equations (4) and (5) in equation (10), we get:

$$Q = \frac{P\pi a^4}{8\mu_P}\left[\left(2\frac{2\sqrt{Da}}{\alpha}+1\right) - 1 - d^4 + (1+\lambda_1)\mu' d^4\right] \tag{11}$$

Equation (11) can be rewritten as:

$$Q = \frac{P\pi a^4}{8\mu_{eff}} \tag{12}$$

By comparing (12) with (11), the effective viscosity is calculated as

$$\mu_{eff} = \frac{\mu_p}{\left[2\left(\frac{2\sqrt{Da}}{\alpha}+1\right) - 1 - d^4 + (1+\lambda_1)\mu' d^4\right]} \tag{13}$$

In case if we use the no slip condition, i.e., $v_1 = 0$ at $r = a$ instead of equation 3b, we get equation (13) as

$$\mu_{eNS} = \frac{\mu_p}{1 - d^4 + \mu'(1+\lambda_1)d^4} \tag{14}$$

Further, if we substituted $\lambda_1 = 0$ in equation (14), we obtained outcomes for Newtonian fluids, i.e.,

$$\mu_{eN} = \frac{\mu_p}{1 - d^4 + \mu' d^4} \tag{15}$$

Chaturani and Upadhyaya [10] arrived at the same conclusion.

2.1 Core Hematocrit For Cell - Free Wall Layer

The cell overall mass balance in the tube is defined as:

$$QH_0 = 2\pi a^2 \int_0^1 V(\xi) h(\xi) \xi d\xi \tag{16}$$

Here H_0 denotes the hematocrit of the discharge tube, and

$$h(\xi) = \begin{cases} H_C, \ 0 \le \xi \le d \\ 0, \ d \le \xi \le 1 \end{cases} \tag{17}$$

Using equations (4), (5) and (17) in equation (16), we obtained

$$QH_0 = \frac{P\pi a^4}{8\pi_P} H_C \left[\left(\frac{2\sqrt{Da}}{\alpha} + 1 \right) 2d^2 - 2d^4 + (1 + \lambda_1)\mu' d^4 \right] \tag{18}$$

Notice that as $d \to 1$, $\mu_{eff} \to \mu_c$ this corresponds to the blood's bulk viscosity in a large tube with a high shear rate.

The tube hematocrit H_T is defined:

$$H_T = 2\int_0^1 h(\xi) \xi d\xi \tag{19}$$

which yields:

$$H_T = d^2 H_C \tag{20}$$

By removing Q from equations (11) and (18), we obtain

$$\frac{H_C}{H_0} = \frac{\left(\frac{4\sqrt{Da}}{\alpha} + 1 \right) - d^4 + (1 + \lambda_1)\mu' d^4}{(1 + \lambda_1)\mu' d^4 + \frac{4\sqrt{4Da}}{\alpha} d^2 + 1 - d^4 - (1 - d^2)^2} \tag{21}$$

From (19), the ratio $H_C / H_0 \ge 1$ if and only if $d \le 1$. To ensure that the core hematocrit must be greater than the discharge hematocrit if the core diameter is less than the tube diameter.

Results and Discussion

The effects of various parameters like, Jeffrey parameter, Darcy number, slip, and tube hematocrit on effective viscosity, core hematocrit, and bluntness [13, 21 and 9] were computed numerically, and the results are graphically depicted in figures 2 to 13.

Figures 2 to 5 clearly show the influence of different parameters on effective viscosity. Effective viscosity is displayed to decrease with the Jeffrey parameter [Figure 2] and Darcy number [Figure 3], but increase as for the slip parameter [Figure 4] and tube hematocrit [Figure 5]. This is understood that the calculated effective viscosity values for the current problem are qualitatively similar to the comparable effective viscosity values derived in the theoretical models of Santhosh et al. [6], Chaturani and Upadhyaya [11]. Further, the effective viscosity increases with tube radius for the given values of Jeffrey parameter, Darcy number, slip parameter, and tube hematocrit [Figures 2 to 5], indicating that the flow shows the Fahraeus-Lindqvist effect.

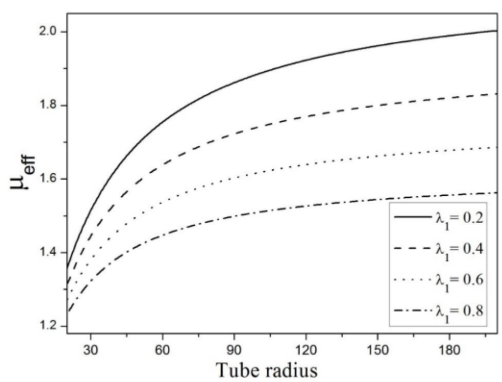

Figure 2. Variation of μ_{eff} for the Jeffrey parameter λ_1 and tube radius $(Da = 0.0001,\ \alpha = 0.2$ and $H_0 = 40\%)$.

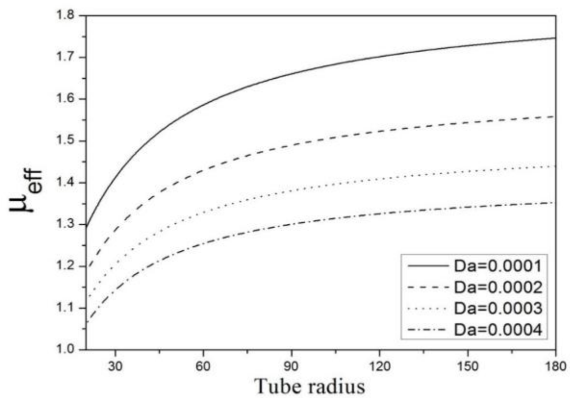

Figure 3. Variation of μ_{eff} with Darcy number Da and tube radius ($\lambda_1 = 0.5$, $\alpha = 0.2$ and $H_0 = 40\%$).

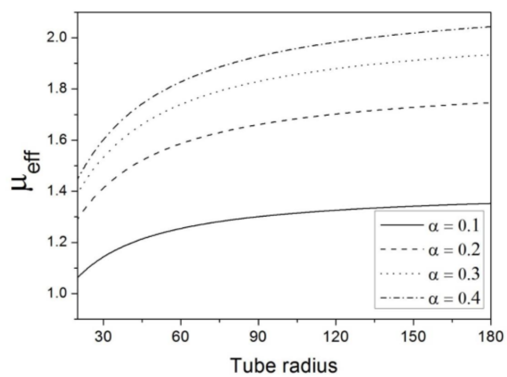

Figure 4. Variation of μ_{eff} with Slip parameter α and tube radius ($\lambda_1 = 0.5$, $Da = 0.0001$ and $H_0 = 40\%$).

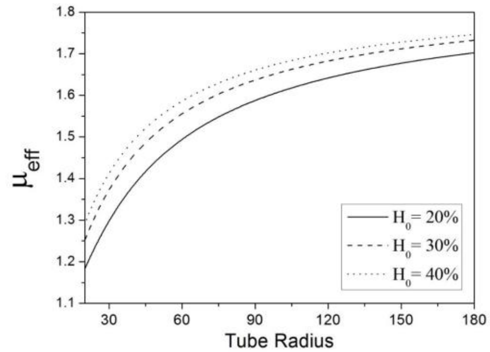

Figure 5. Variation of μ_{eff} with tube hematocrit H_0 and tube radius ($\lambda_1 = 0.5$, $Da = 0.0001$ and $\alpha = 0.2$).

Figures 6 to 9 show the effects of different parameters on core hematocrit. When the Jeffrey parameter [Figure 6], slip parameter [Figure 4], and tube hematocrit [Figure 5] diminish, and core hematocrit increases with Darcy number [Figure 7]. This is understood that the computed values of the core hematocrit for the present problem are the analysis approving within the tolerable scope, with respect to the corresponding values of the core hematocrit derived in the analytical models. Further, for the fixed H_0 the core hematocrit falls as the radius of the tube increases and it is observed that by increasing the tube radius core hematocrit approaches the tube hematocrit.

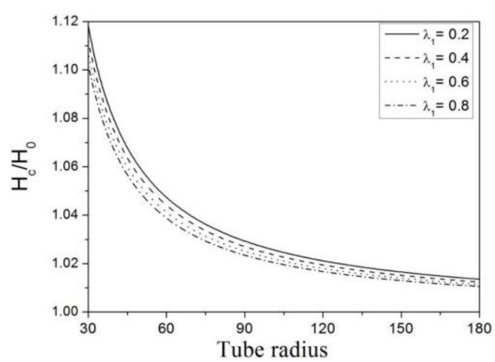

Figure 6. Variation of core hematocrit with Jeffrey parameter λ_1 and tube radius $Da = 0.0001$, $\alpha = 0.2$ and $H_0 = 40\%$).

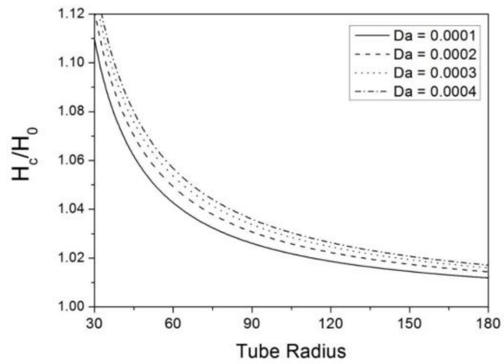

Figure 7. Variation of core hematocrit with Darcy number Da and tube radius ($\lambda_1 = 0.5$, $\alpha = 0.2$ and $H_0 = 40\%$).

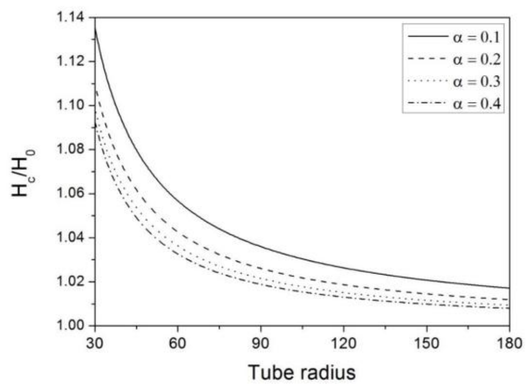

Figure 8. Variation of core hematocrit with slip parameter α and tube radius ($\lambda_1 = 0.5$, $Da = 0.0001$ and $H_0 = 40\%$).

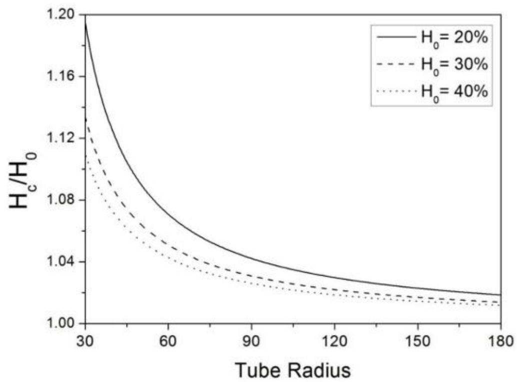

Figure 9. Variation of core hematocrit with tube hematocrit H_0 and tube radius $(\lambda_1 = 0.5,\ Da = 0.0001$ and $\alpha = 0.2)$.

The velocity profile's bluntness in the core is represented by parameter B in Equation (9) and $1 - B$ denotes the variance from the parabolic flow. The cell-free layer's thickness, as well as the viscosities of the core and outer layers, influence this parameter. The velocity profile becomes parabolic when the viscosities in the core and plasma layer are the same and equal to the bulk viscosity. The effects of various parameters on bluntness are shown in figures 10 to 13. It can be seen that bluntness increases when the Jeffrey parameter [Figure 10], slip parameter [Figure 11] and tube hematocrit [Figure 13] increases and Darcy number decrease [Figure 12].

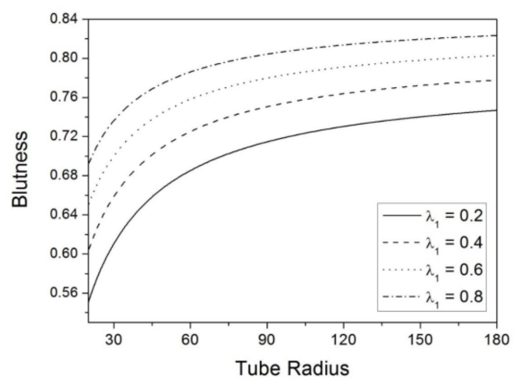

Figure 10. Variation of bluntness parameter (B) with Jeffrey parameter λ_1 and tube radius $(Da = 0.0001,\ \alpha = 0.2$ and $H_0 = 40\%)$.

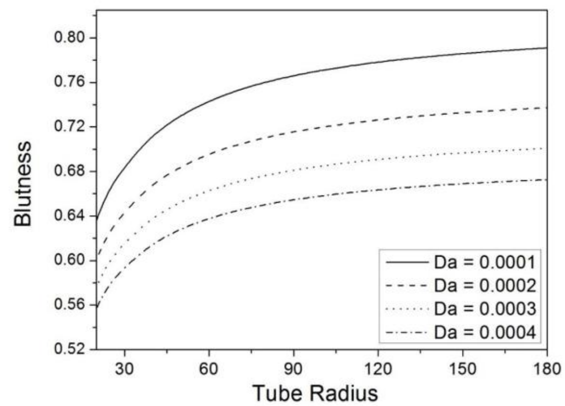

Figure 11. Variation of bluntness parameter (B) with Darcy number Da and tube radius $(\lambda_1 = 0.5, Da = 0.0001$ and $H_0 = 40\%)$.

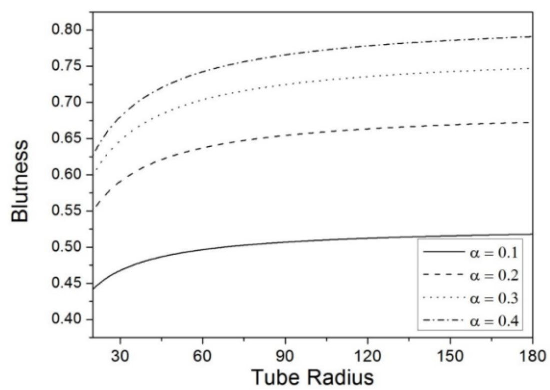

Figure 12. Variation of bluntness parameter (B) with slip parameter α and tube radius $(\lambda_1 = 0.5, Da = 0.0001$ and $H_0 = 40\%)$.

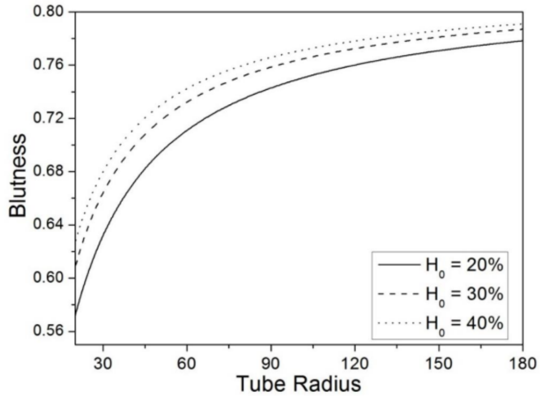

Figure 13. Variation of bluntness parameter (B) with tube hematocrit H_0 and tube radius $(\lambda_1 = 0.5,\ Da = 0.0001$ and $H_0 = 40\%)$.

Conclusion

In the present paper, a two-fluid model for the flow of Jeffrey fluid through small tubes with a slip effect is investigated. Based on the assumption that Jeffery fluid exists in the core and Newtonian fluid exists in the peripheral region, the linearized equations of motion have been solved, and analytical solutions for velocity, bluntness, flow flux, and core hematocrit have been obtained. The effective viscosity is shown to increases with the tube radius. Further, for the fixed H_0 the core hematocrit falls as the radius of the tube increases and it is observed that by increasing the tube radius core hematocrit approaches the tube hematocrit.

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