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MHD CASSON FLUID FLOW PAST AN UPRIGHT PLATE UNDER THE IMPACT OF HEAT SINK AND CHEMICAL REACTION

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Abstract

The present study considers an analytical investigation of unsteady MHD free convection flow of a Casson fluid past a vertical porous plate through a porous medium with time dependent oscillatory permeability and suction in presence of a uniform transverse magnetic field. Then effects of radiation, heat absorption, radiation absorption and homogeneous chemical reaction are considered. The coupled nonlinear partial differential equations are turned to ordinary by super imposing solutions with steady and time dependent transient part. Finally, the set of ordinary differential equations are solved with a perturbation method to meet the inadequacy of boundary condition. Impact of Casson parameter leads to decrease the fluid velocity. The heavier species with low conductivity reduces the flow within the boundary layer.

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1. Introduction

The study of characteristics of non-Newtonian fluids is one of the important and interesting topics for scholars and scientists having direct or indirect affiliation with the field of fluid science. Non-Newtonian fluids have nonlinear relationship between strain rate and stress while in Newtonian fluid it is in linear mode. On the other hand, fluid flow properties are different in any way from that of the Newtonian fluid is called a non-Newtonian fluid. Some examples of non-Newtonian fluids are salt solutions, molten polymers, ketchup, custard, toothpaste, starch suspensions, paints, blood and shampoo. Emmanuel et al. [1] investigated that analysis of Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field. Mustafa et al. [2] have studied that unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate. Kirubushan kumar et al. [3] has studied thermal effects on magneto hydrodynamic Casson liquid stream between electrically conducting plates. Kirubushan kumar et al. [4] examined that Casson fluid flow and heat transfer over an unsteady porous stretching surface. Arthur et al. [5] analyzed that Analysis of Casson fluid flow over a vertical porous surface with a chemical reaction in the presence of a magnetic field. Pramanik et al. [6] have studied that Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation. Pushpalatha et al. [7] have studied that heat and mass transfer in unsteady MHD Casson fluid flow with convective boundary conditions. Chandra Reddy et al. [8, 9] considered and analyzed MHD boundary layer flows of a visco-elastic fluid as well as Rivlin-Ericksen fluid past a porous plate with different parameters and boundary conditions. Rama Mohan Reddy et al. [10] studied thermal diffusion and Joule-heating effects on magnetohydrodynamic, free-convective, heat-absorbing/generating, viscous-dissipative Newtonian fluid with variable temperature and concentration. Sidda Reddy et al. [11] examined thermal diffusion and Joule heating effects on MHD radiating fluid embedded in porous medium. Some more recent publications by Zhang et al. [12, 13, 14] are also studied.

Motivated by the above studies (in most of the studies a Newtonian fluid was considered), in this article we have considered an unsteady MHD free convection flow of an incompressible, electrically conducting Casson fluid past a vertical porous plate through a porous medium with time dependent oscillatory permeability and suction in presence of a uniform transverse magnetic field. The novelty of the present study is to analyze the effect of time defendant fluctuate suction and permeability of the medium on a Casson fluid flow in the presence of radiation, heat absorption, radiation absorption and chemical reaction. The contribution is the implementation of Casson parameter by replacing the visco-elastic parameter in the study of Rama Mohan Reddy et al. [10]. The Casson parameter is taken due to the significance of non-Newtonian fluids in real time applications in chemical industries and petroleum refineries.

2. Formulation of the Problem

The unsteady free convective flow of a radiative, chemically reactive, heat absorbing, Casson fluid past an infinite vertical porous plate in a porous medium with time dependent oscillatory suction as well as permeability in presence of radiation absorption and a transverse magnetic field is considered. Let y^*- axis be along the plate in the direction of the flow and - axis normal to it. Let us consider the magnetic Reynolds number is much less than unity so that induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t^* < 0,$ the plate as well as fluids are at the same temperature and also concentration as the species is very low so that the Soret and Dofour effects are neglected. When, $t^*,$ the temperature of the plate is instantaneously raised to T_W^* and the concentration of the species is to $C_W^*.$ Let the permeability of the porous medium and the suction velocity be considered in the following forms respectively.

$$K^*(t^*) = K_p^*(1 + \varepsilon e^{n^* t^*}),\ v^*(t^*) = -v_0(1 + \varepsilon e^{n^* t^*}), \tag{1}$$

where $v_0 > 0$ and $\varepsilon \leq 1$ are positive constants.

Under the above assumption with usual Boussinesq's approximation and

Rama Mohan Reddy et al. [10], the governing equations and boundary conditions are given by

$$\frac{\partial u^*}{\partial t^*} = \left(1 + \frac{1}{\beta}\right) v \frac{\partial^2 u^*}{\partial t^{*2}} + g\beta(T^* - T^*_\infty) + g\beta^*(C^* - C^*_\infty)$$

$$- \sigma B_0^2 \frac{u^*}{\rho} - \left(1 + \frac{1}{\beta}\right) v \frac{u^*}{k^*} \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} \rho c_p = K \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q^*}{\partial y^*} - Q^*(T^* - T^*_\infty) + Q_l^*(C^* - C^*_\infty) \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - C^*_\infty) \quad (4)$$

$$u = f(t) = 1,\ T^* = T_\infty(T_W - T_\infty)e^{n^* t^*},\ C^* = C_\infty + \varepsilon(C_W - C_\infty)e^{n^* t^*} \text{ at } y = 0$$

$$u \to 0,\ T^* \to T_\infty,\ C^* \to C_\infty, \text{ as } y \to \infty. \quad (5)$$

Introducing the non-dimensional quantities,

$$y = \frac{v_0 t^*}{v},\ t = \frac{v_0^2 t^*}{4v},\ w = \frac{4\vartheta w^*}{v_0^2},\ u = \frac{u^*}{v_0},\ T = \frac{T^* - T^*_\infty}{T_W - T_\infty},\ C = \frac{C^* - C^*_\infty}{C_W - C_\infty},$$

$$S = \frac{\vartheta S^*}{v_0^2},\ K_p = \frac{v_0^2 K_p^2}{v^2},\ M^2 = \sigma \frac{B_0^2 v}{v_0^2 \rho},\ P_r = \frac{v}{K},\ S_c = \frac{v}{D},\ R_c = \frac{v_0^2 K_0}{v^2 \rho},$$

$$G_c = \frac{vg\beta^*(C_W - C_\infty)}{v_0^3},\ G_r = \frac{vg\beta(T_W - T_\infty)}{v_0^3},\ F = \frac{4I_1 v}{v_0^2 \rho C_p},\ S = \frac{Qv}{v_0^2 \rho C_p},$$

$$R = \frac{Q_l v(C_W^* - C_\infty)}{v_0^3 \rho (T_W^* - T_\infty)},\ K_c = \frac{k_r v}{v_0^2},\ H = F + S. \quad (6)$$

The equations (2)-(5) reduce to following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + G_r T + G_c C - \left(M^2 + \frac{1}{K_p}\right)u \quad (7)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - HT + RC \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \tag{9}$$

$$u = f(t) = 1,\ T = 1 + \varepsilon e^{nt},\ C = 1 + \varepsilon e^{nt}\ y = 0$$

$$u \to 0,\ T \to 0,\ C \to 0,\ \text{as}\ y \to \infty. \tag{10}$$

3. Method of Solution

In view of periodic suction, temperature and concentration at the plate let us assume the velocity, temperature, concentration the neighborhood of the plate be

$$u(y, t) = u_0(y) + \varepsilon e^{nt} u_1(y),\ T(y, t) = T_0(y) + \varepsilon e^{nt} T_1(y),$$

and

$$C(y, t) = C_0(y) + \varepsilon e^{nt} C_1(y). \tag{11}$$

Substituting equations (11) into (7-9) and comparing the no harmonic and harmonic terms we get

$$\left(1 + \frac{1}{\beta}\right)u_0^{11} - \left(M^2 + \frac{1}{K_p}\right)u_0 = -G_r T_0 - G_c C_0 \tag{12}$$

$$\left(1 + \frac{1}{\beta}\right)u_1^{11} - \left(M^2 + \frac{1}{K_p} + \frac{n}{4}\right)u_1 = -G_c C_1 - G_r T_1 \tag{13}$$

$$T_0^{11} - P_r H T_0 = -RP_r C_0 \tag{14}$$

$$T_1^{11} - \left(H + \frac{n}{4}\right)P_r T_1 = -RP_r C_1 \tag{15}$$

$$C_0^{11} - K_c S_c C_0 = 0 \tag{16}$$

$$C_1^{11} - \left(K_c + \frac{n}{4}\right)S_c C_1 = 0. \tag{17}$$

The boundary conditions now reduce to

$$u_0 = 1, u_1 = 0, T_0 = T_1 = 1, C_0 = C_1 = 1, \text{ at } y = 0$$

$$u_0 = u_1 \to 0, T_0 = T_1 \to 0, C_0 = C_1 \to 0, \text{ as } y \to \infty. \tag{18}$$

Solving these differential equations (12)-(18) with the help of boundary conditions we get

$$u(y, t) = (1 - b_3 - b_4)e^{-\sqrt{a_5}y} + b_3 e^{-\sqrt{a_3}y} + b_4 e^{-\sqrt{a_1}y}$$
$$+ \varepsilon e^{nt}\{(-b_5 - b_6)e^{-\sqrt{a_8}y} + b_5 e^{-\sqrt{a_4}y} + b_6 e^{-\sqrt{a_2}y}\} \tag{19}$$

$$T(y, t) = (1 - b_1)e^{-\sqrt{a_3}y} + b_1 e^{-\sqrt{a_1}y} + \varepsilon e^{nt} + \{(1 - b_2)e^{-\sqrt{a_4}y} + b_2 e^{-\sqrt{a_2}y}\} \tag{20}$$

$$C(y, t) = e^{-\sqrt{a_1}y} + \varepsilon e^{nt}\{e^{-\sqrt{a_2}y}\}. \tag{21}$$

The skin friction at the plate in terms of amplitude and phase angle is given by

$$\tau = \frac{\partial u_0}{\partial y} + \varepsilon e^{nt}\frac{\partial u_0}{\partial y}, \text{ at } y = 0$$

$$\tau = \lfloor -(1 - b_3 - b_4)\sqrt{a_5} - b_3\sqrt{a_3} - b_4\sqrt{a_1} \rfloor$$
$$+ \varepsilon e^{nt}\lfloor (b_5 - b_6)\sqrt{a_8} - b_5\sqrt{a_4} - b_6\sqrt{a_2} \rfloor. \tag{22}$$

The rate of heat transfer. That is heat flux at the N_u in terms of amplitude and phase is given by

$$N_u = -\left[\frac{\partial T_0}{\partial y} + \varepsilon e^{nt}\frac{\partial T_1}{\partial y}\right] \text{ at}$$

$$y = 0;\ N_u = \lfloor (1 - b_3)\sqrt{a_3} + b_1\sqrt{a_1} \rfloor + \varepsilon e^{nt}\lfloor (1 - b_2)\sqrt{a_4} + b_2\sqrt{a_2} \rfloor. \tag{23}$$

The mass transfer coefficient, that is the Sherwood number S_h at the plate in terms of amplitude and phase is given by

$$S_h = -\left[\frac{\partial C_0}{\partial y} + \varepsilon e^{nt}\frac{\partial C_1}{\partial y}\right] \text{ at } y = 0;\ S_h = \lfloor \sqrt{a_1} \rfloor + \varepsilon e^{nt}\lfloor \sqrt{a_2} \rfloor. \tag{24}$$

4. Results and Discussion

In order to assess the effects of the dimensionless thermo physical parameters on the regime, calculations have been carried out on velocity field, temperature field and concentration field for varies physical parameters like magnetic parameter, Prandtl parameter, Grash of number, modified Grash of number, chemical reaction parameter etc. The graphs and tables are drawn by using the MAT Lab program. The Schmidt number embodies the ratio of the momentum diffusivity to the species (mass) diffusivity. It physically relates to the comparative thickness of the hydrodynamic boundary layer and mass-transfer boundary layer. A decay effect is noticed from figure 1 that in the presence of Schmidt number the velocity decreases. From figure 2 it is noticed that the velocity decreases with an increase in m. From figure 3 it is seen that the velocity increases with an increase in K_p. From figure 4 it is observed that velocity decreases with the increasing values of R. Since divergence of the radiative heat flux increases, k^* decreases which in turn causes to increase the rate of radiative heat transfer of the fluid and so the fluid velocity increases. From figure 5 shows that velocity decreases for increasing values of chemical reaction parameter. From figure 6 shows that temperature decreases for the increasing values of S_c number. From figure 7, it is concluded that the concentration decreases as K_c increases. Effect of Schmidt number parameter on concentration is presented in figure 8, which witnesses that concentration decreases as the values of S_c increase.

Effects of various parameters on skin friction, the rate of heat transfer and also the rate of mass transfer are presented in tables 1-3. From table 1 it is noted that skin friction increase due to an increase in Grashof number G_r. But modified Grashof number has a different effect on skin friction. Skin friction decreases due to an increase in M. From this table it is also observed that the skin friction increases due to an increase in porosity parameter. From table 2 it is observed that skin friction increases for increasing values of R where as Nusselt number decreases with the increasing values of R. Of course skin friction, as well as Nusselt number increase for increasing values of P_r and also heat source parameter H. From table 3, it is found that skin friction and Sherwood number both increase for increasing values of S_c whereas skin friction decreases with increase values of K_c, but a reverse effect is noticed in the case of Sherwood number.

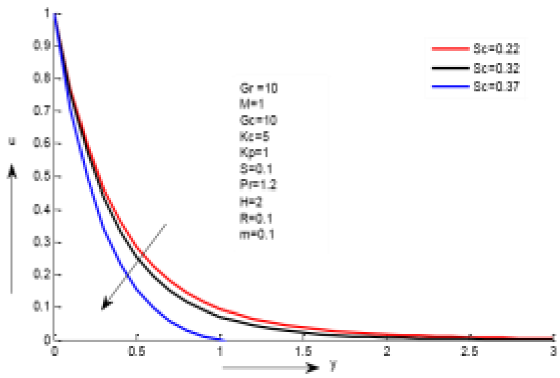

Figure 1. Effect of Sc on Velocity.

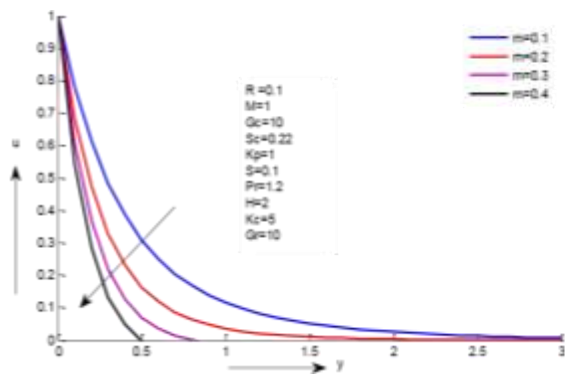

Figure 2. Effect of m on Velocity.

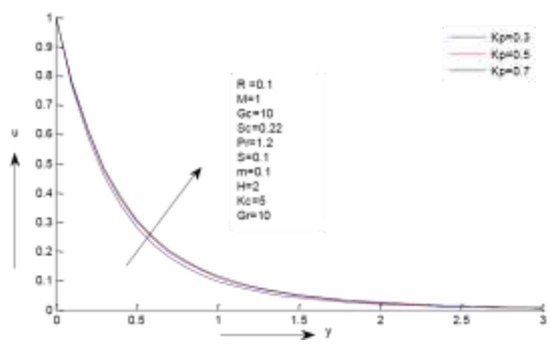

Figure 3. Effect of K_p on Velocity.

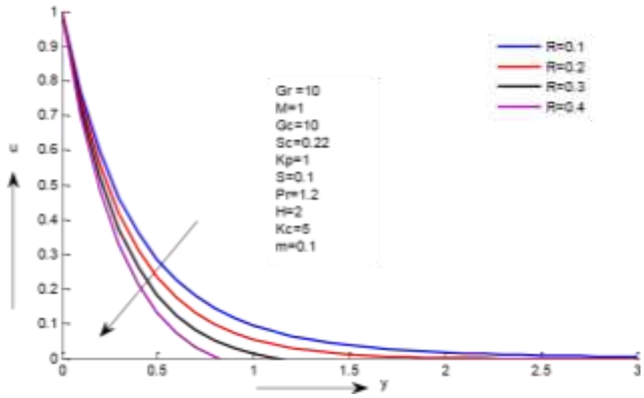

Figure 4. Effect of Ron Velocity.

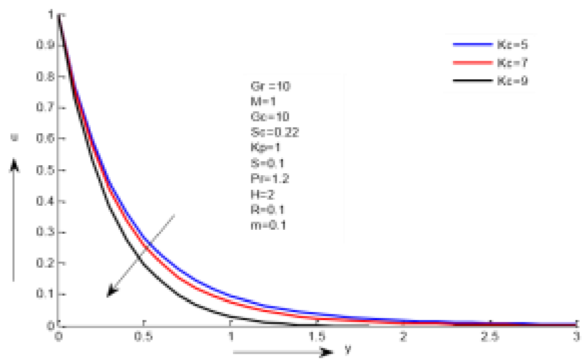

Figure 5. Effect of K_c on Velocity.

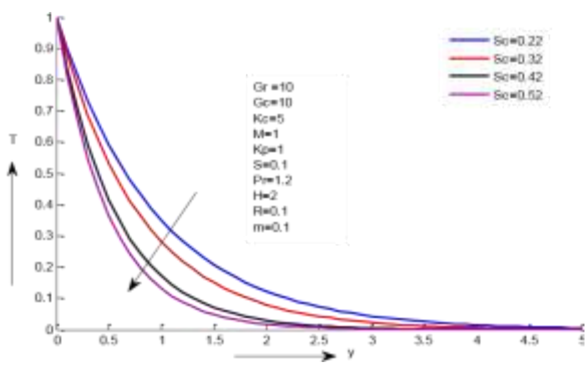

Figure 6. Effect of S_c on Temperature.

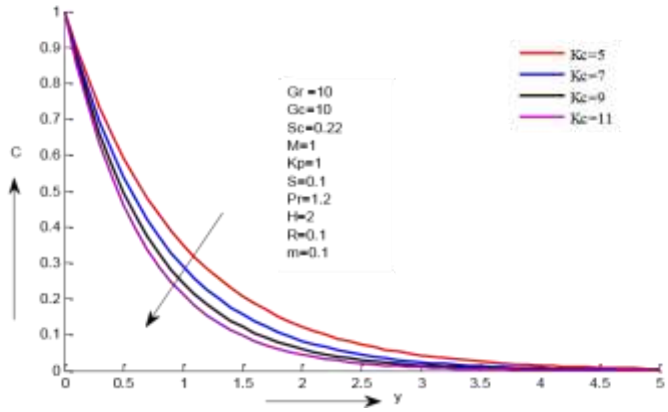

Figure 7. Effect of (K_c) on Concentration.

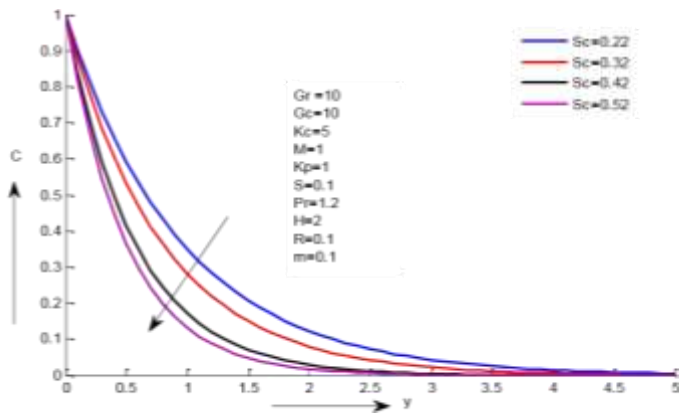

Figure 8. Effect of Schmidt number (S_c) on Concentration.

Table1. Effects of Gr, Gc, M and Kp on skin friction coefficient

Gr	Gc	M	Kp	τ
12	6	0.7	1.7	12.2435
13	6	0.7	1.7	13.7654
14	6	0.7	1.7	14.9876
15	6	0.7	1.7	15.4356
10	7	0.7	1.7	13.7689
10	8	0.7	1.7	11.3456

10	9	0.7	1.7	10.5467
10	10	0.7	1.7	10.3456
10	6	2.5	1.7	12.5637
10	6	3.0	1.7	12.3452
10	6	3.5	1.7	11.4567
10	6	4.0	1.7	11.1034
10	6	0.7	0.4	8.6789
10	6	0.7	0.5	9.8976
10	6	0.7	0.6	9.9067
10	6	0.7	0.7	9.9156

Table 2. Effect of R and H on skin friction and Nusselt number.

	Pr	H	τ	Nu
1	0.71	1	98.5648	0.6784
2	0.71	1	127.9863	0.4532
3	0.71	1	180.5674	0.3286
4	0.71	1	230.8521	0.2341
1.5	1.71	1	14.8954	2.4724
1.5	2.71	1	21.8532	2.5342
1.5	3.71	1	32.9845	2.6282
1.5	4.71	1	37.6283	2.7845
1.5	0.71	2	7.6382	1.6784
1.5	0.71	3	8.8352	1.8745
1.5	0.71	4	9.7642	1.9564
1.5	0.71	5	0.7342	2.6745

Table 3. Effect of Sc and Kc on skin friction and Sherwood number.

Sc	Kc	τ	Sh
1.33	2	11.6754	3.6753
2.33	2	12.8643	4.6734
3.33	2	13.9743	5.6342
4.33	2	14.9345	6.5463
0.33	5	73.8453	1.3456
0.33	6	54.8542	1.4674
0.33	7	43.8645	1.6435
0.33	8	23.8956	1.8456

5. Conclusion

We have considered an unsteady MHD free convection flow of a viscoelastic, incompressible, electrically conducting fluid past a vertical porous plate through a porous medium with time dependent oscillatory permeability and suction in presence of a uniform transverse magnetic field. Some of the notable conclusions are given below.

(a) Impact of Casson parameter leads to decrease the fluid velocity.

(b) The heavier species with low conductivity reduces the flow within the boundary layer.

(c) Concentration decreases as the values of Sc increases.

(d) Skin friction increases for increasing values of R where as Nusselt number decreases with the increasing values of R.

Appendix

$$a_1 = S_c K_c,\ K_c,\ a_2 = \left(K_c + \frac{n}{4}\right)S_c,\ a_3 = P_r H,\ a_4 = \left(H + \frac{n}{4}\right)P_r;$$

$$a_5 = \left(M^2 + \frac{1}{R_p}\right)\Big/\left(1 + \frac{1}{p}\right);\ a_6 = -Gr\Big/\left(1 + \frac{1}{p}\right);\ a_7 = -Gr\Big/\left(1 + \frac{1}{\beta}\right);$$

$$a_8 = \left(M^2 + \frac{1}{K_p} + \frac{n}{4}\right)/\left(1 + \frac{1}{\beta}\right);\ b_1 = \frac{-RP_r}{a_1 - a_3};\ b_2 = \frac{-RP_r}{a_2 - a_4}$$

$$b_3 = \frac{a_6(1 - b_1)}{a_3 - a_5},\ b_4 = \frac{a_6 b_1 - a_7}{a_3 - a_5},\ b_5 = \frac{a_6(1 - b_2)}{a_4 - a_8},\ b_6 = \frac{a_6 b_2 + a_7}{a_2 - a_8}.$$

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