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EXPECTED TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM WITH WASTAGE AS A GEOMETRIC PROCESS WHEN THE BREAKDOWN THRESHOLD HAS THREE COMPONENTS

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Abstract

In this paper, the problem of time to recruitment is analyzed for a single grade manpower system in which attrition takes place due to two types of policy decisions where this classification is done according to intensity of attrition. Assuming (i) policy decisions and exits occur at different epochs (ii) the inter-decision times form an ordinary renewal process (iii) wastage of manpower due to exits form a geometric process and (iv) breakdown threshold for the cumulative wastage of manpower in the system has three components which are independent exponential random variables. Stochastic model as constructed and the variance of the time to recruitment is obtained using an univariate CUM policy of recruitment. Employing a different probabilistic analysis, analytical results in closed form for system characteristics are derived.

1. Introduction

Authors in [1] and [2] are the pioneers in the study on manpower models for a system with one or more grades. In [14] the authors have initiated the study on the problem of time to recruitment for the single grade manpower system which is subject to attrition with instantaneous exits, using the

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univariate policy of recruitment based on shock model approach for replacement of systems in reliability theory. In [8], the author has obtained the system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequences. For the study of this problem corresponding to correlated inter-decision times under different policies of recruitment, one can refer to [10], [11], [12] and [13]. In [9] the author has studied this problem assuming geometric process and order statistics for inter-decision times. In [7] the authors have studied the problem of time to recruitment by assuming that the attrition is generated by a geometric process of inter-decision times using a different probabilistic analysis. In [3] the authors have considered the single grade manpower system with non-instantaneous exits and obtained variance of the time to recruitment when the wastage of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables and breakdown threshold is an exponential random variable. While in [4] the models in [3] are studied when the policy decisions are classified into two types, in [5], the models in [4] are studied when breakdown threshold for the cumulative wastage of manpower in the system has two components. The present paper extends the research work in [4] when the breakdown threshold level for the cumulative wastage in the manpower system is the sum of three components namely an exponential threshold for cumulative wastages due to exits, an exponential threshold for cumulative wastage due to frequent breaks of existing workers and an exponential threshold for cumulative wastage due to backup sources.

2. Model Description and Analysis

Deviating from the conventional method of using Laplace transform, a different probabilistic analysis is employed here.

By the recruitment policy, recruitment is done whenever the cumulative wastage exceeds the threshold $Y_1 + Y_2 + Y_3$. When the first decision is taken, recruitment would not have been done for B_1 units of time. If the wastage $X_1 = (S_1)$ in the first exit epoch is greater than $Y_1 + Y_2 + Y_3$ then recruitment is done and in this case $T = B_1 = D_1$. However, if the non-recruitment period will continue till the arrival of next exit epoch. If the cumulative sum S_2 of wastage in the first two exit epochs exceeds

$Y_1 + Y_2 + Y_3$, then recruitment is done and $T = B_1 + B_2 = D_2$. If $S_2 = Y_1 + Y_2 + Y_3$ then the non-recruitment period will continue till the arrival of next exit epoch. Depending on $S_3 = Y_1 + Y_2 + Y_3$ or $S_3 = Y_1 + Y_2 + Y_3$ recruitment is done or the non-recruitment period continues and so on.

In terms of indicator function of an event, we can write T as

$$T = \sum_{i=0}^{\infty} D_{i+1}\chi(S_i \leq Y_1 + Y_2, Y_3 < S_{i+1}) \tag{1}$$

First we shall express $E(T)$ in terms of $E(B)$ which is the mean of the inter-exit times.

Taking expectation on both sides of (1) and using the result (for any event L)

$$E(T) = \sum_{i=0}^{\infty} E(D_{i+1})P(S_i \leq Y_1 + Y_2, Y_3 < S_{i+1}) \tag{2}$$

From (2) and from the definition of D_{i+1}, we get

$$E(T) = \sum_{i=0}^{\infty} E\left(\sum_{i=0}^{i+1} B_k\right)P(S_i \leq Y_1 + Y_2, Y_3 \leq S_{i+1}) \tag{3}$$

$$E(T) = \sum_{i=0}^{\infty} (i+1)E(B)P(S_i \leq Y_1 + Y_2 + Y_3 \leq S_{i+1}) \tag{4}$$

By using law of total probability, we get

$$P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1}) = P(0 \leq (Y_1, Y_2, Y_3) - S_i < X_{i+1})$$

$$= \int_0^{\infty}\int_0^{t} \widetilde{M}(t-x)m_i(x)h(t)dxdt \tag{5}$$

By hypothesis,

$\widetilde{M}(t-x) = e^{-\alpha d^i(t-x)}$, $m_i(x) = d^{i-1}m(d^{i-1}x)$ and

$$h_1(t) = e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_2)t} - e^{-(\theta_2+\theta_3)t}$$

Therefore

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})$$

$$= d^{i-1}\int_0^{\infty}\int_0^{t} e^{-\alpha d^i (t-x)}[m(d^{i-1}x)]\frac{\theta_1\theta_2}{\theta_1 - \theta_2}[e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_2)t}$$

$$- e^{-(\theta_1+\theta_3)t}]dxdt$$

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}) = \frac{\alpha\theta_1\theta_2}{\theta_1 - \theta_2}\int_0^{\infty}[1 - e^{-\alpha d^{i-1}\bar{d}t}]$$

$$[e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_2)t} - e^{-(\theta_2+\theta_3)t}]e^{-\alpha d^i}dt \qquad (6)$$

Consider,

$$\int_0^{\infty}[1 - e^{-\alpha d^{i-1}}\bar{d}t][e^{-\theta_2 t}]e^{-\alpha d^i t}dt$$

$$= \int_0^{\infty} e^{-(\alpha d^i + \theta_2)t} - e^{-\alpha d^{i-1}}\bar{d}t[e^{-\theta_2 t}]e^{-\alpha d^i t}dt$$

$$\int_0^{\infty}[1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-\theta_2 t}]e^{-\alpha d^i}dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_2)(\alpha d^{i-1} + \theta_2)} \qquad (7)$$

Similarly

$$\int_0^{\infty}[1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-\theta_2 t}]e^{-\alpha d^i}dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_1)(\alpha d^{i-1} + \theta_1)} \qquad (8)$$

$$\int_0^{\infty}[1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-(\theta_1+\theta_2)t}]e^{-\alpha d^i t}dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)}$$

$$(9)$$

$$\int_0^{\infty}[1-e^{-\alpha d^{i-1}\overline{d}t}][e^{-(\theta_1+\theta_3)t}]e^{-\alpha d^i t}dt = \frac{\alpha d^{i-1}\overline{d}}{(\alpha d^i+\theta_2+\theta_3)(\alpha d^{i-1}+\theta_2+\theta_3)} \quad (10)$$

Sub's (7), (8), (9) and (10) in (6) we get,

$$P(S_i \le Y_1+Y_2+Y_3 < S_{i+1}) = \frac{\alpha\theta_1\theta_2}{\theta_1-\theta_2}\alpha d^{i-1}$$

$$+\left[\frac{(d^i+d^{i-1})(\alpha\theta_1-\alpha\theta_2)+(\theta_1^2-\theta_2^2)}{(\alpha d^i+\theta_2)(\alpha d^{i-1}+\theta_2)(\alpha d^i+\theta_1)(\alpha d^{i-1}+\theta_1)}\right.$$

$$\left.+\frac{(d^i+d^{i-1})(\alpha\theta_1-\alpha\theta_3)+(\theta_1^2-\theta_2^2)+(\theta_1^2-\theta_3^2)+2\theta_1\theta_2-2\theta_2\theta_3}{(\alpha d^i+\theta_1+\theta_2)(\alpha d^{i-1}+\theta_1+\theta_2)(\alpha d^i+\theta_1+\theta_2)(\alpha d^{i-1}+\theta_1+\theta_2)}\right] \quad (11)$$

From (4) and (11)

$$E(T) = E(B)\sum_{i=0}^{\infty}(i+1)\frac{\alpha\theta_1\theta_2}{\theta_1-\theta_2}\alpha d^{i-1}$$

$$\left[\frac{(d^i+d^{i-1})(\alpha\theta_1-\alpha\theta_2)+(\theta_1^2-\theta_2^2)}{(\alpha d^i+\theta_2)(\alpha d^{i-1}+\theta_2)(\alpha d^i+\theta_1)(\alpha d^{i-1}+\theta_1)}\right.$$

$$\left.+\frac{(d^i+d^{i-1})(\alpha\theta_1-\alpha\theta_3)+(\theta_1^2-\theta_2^2)+(\theta_2^2-\theta_3^2)+2\theta_1\theta_2-2\theta_2\theta_3}{(\alpha d^i+\theta_1+\theta_2)(\alpha d^{i-1}+\theta_1+\theta_2)(\alpha d^i+\theta_1+\theta_2)(\alpha d^{i-1}+\theta_1+\theta_2)}\right]$$

$$E(T) = K_1 E(B) \quad (12)$$

We now find $E(B)$ for the present model.

$$E(B) = \int_0^{\infty} x\, dG(x) \quad (13)$$

$$E(B) = \frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2 q} \quad (14)$$

Using (14) in (12), we get

$$E(T) = K_1\left(\frac{a_1\lambda_2 + a_2\lambda_1}{\lambda_1\lambda_2 q}\right) \quad (15)$$

We now evaluate $E(T^2)$ for the present model.

Squaring both sides on (2), we get

$$E(T^2) = \sum_{i=0}^{\infty} E(D_{i+1}^2) P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}). \tag{16}$$

Now

$$E(D_{i+1}^2) = (i+1)V(B) + [(i+1)E(B)]^2 \tag{17}$$

Using (17) in (16), we get

$$E(T^2) = \sum_{i=0}^{\infty} (i+1)V(B)[P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})]$$

$$+ \sum_{i=0}^{\infty} (i+1)^2 E^2(B)[P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})] \tag{18}$$

We now evaluate $E(B^2)$ for the present model

From (13) we get

$$E(B^2) = \int_0^{\infty} x^2 dG(x) = \int_0^{\infty} x^2 \left[q \sum\nolimits_{n=1}^{\infty} (1-q)^{n-1} f_n(x) \right] dx$$

$$E(B^2) = q \sum_{n=1}^{\infty} (1-q)^{n-1} \int_0^{\infty} x^2 f_n(x) dx. \tag{19}$$

We now evaluate $\int_0^{\infty} x^2 f_n(x) dx$ in (19).

$$\int_0^{\infty} x^2 f_n \, dx = V(C_n) + [E(C_n)]^2$$

i.e.

$$\int_0^\infty x^2 f_n(x)dx = \frac{2n[a_1\lambda_2^2 + a_2\lambda_1^2}{(\lambda_1\lambda_2)} - n[E(A)]^2 + n^2[E(A)]^2. \tag{20}$$

From (19) and (20), we get

$$E(B^2) = \left[\frac{2q\,[a_1\lambda_2^2 + a_2\lambda_1^2] + 2(1-q)\,[a_1\lambda_2 + a_2\lambda_1]^2}{(\lambda_1\lambda_2 q)^2}\right] \tag{21}$$

From (21) and (18) we get

$$E(T^2) = K_1 E(B^2) + E^2(B)[K_1 - K_2] \tag{22}$$

$$V(T) = K_1\left[\frac{2q\,[a_1\lambda_2^2 + a_2\lambda_1^2] + 2(1-q)\,[a_1\lambda_2 + a_2\lambda_1]^2}{(\lambda_1\lambda_2 q)^2}\right]$$

$$-[K_1^2 + K_1 - K_2]\left[\frac{[a_1\lambda_2 + a_2\lambda_1]}{\lambda_1\lambda_2 q}\right]^2$$

3. Conclusion

The models discussed in this paper improve the earlier relevant research work in the context of admitting the realistic assumption of non-instantaneous exits in the system and taking into account wastage due to policy decisions, frequent breaks and backup sources for this system. They will be useful in the process of planning recruitment when the system has the above cited provision. The suitability of distributions assumed in the present work can be tested by data analysis.

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