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n- SPLIT DOMINATION IN VERTEX SQUARED INTERVAL-VALUED FUZZY GRAPHS

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Abstract

In this paper we study different concepts like vertex squared interval-valued fuzzy graph, vertex squared cardinality, vertex squared independent set, n-split dominating set, n-split domination number. We likewise, investigate a relationship between n-split dominating set and vertex squared independent set for vertex squared interval-valued fuzzy graphs.

1. Introduction

Fuzzy graphs differ from the classical ones in several ways, among them the most prominent one is connectivity. Distance and central concepts additionally assume important parts in applications related to fuzzy graphs. In 1965 Lotfi. A. Zadeh initiated fuzzy sets and later in 1983 Krassimir T. Bhattacharya [3] has discussed fuzzy graphs. Kalaiarasi and Mahalakshmi have also expressed fuzzy strong graphs [10].

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Generalized theory and fuzzy logic have been concentrated by Zadeh [21] [22] [23]. Hongmei and Lianhun have also expressed interval-valued sub semigroups and subgroups [7]. Akram et al. [1] gave the idea that fuzzy graphs. The concept of fuzzy sets has been concentrated by Turksen [19]. Pradip Debnath gave the characterization for a minimal dominating set [15]. Manjusha and Sunitha gave the notion of strong arcs [13].

In this paper, we build up the idea of n-split domination in VSIVFG and many fascinating outcomes including these ideas are researched. Additionally, we talk about n-split domination number and explored their many intriguing outcomes.

2. Vertex Squared Interval-Valued Fuzzy Graph

We consider,

G_{IV} - Vertex Squared Interval-Valued Fuzzy Graph

V_{IV} - Vertices

E_{IV} - Edges

Definition 2.1. An vertex squared interval-valued fuzzy set (VSIVFS) X_{IV} on a set V_{IV} is denoted by $X_{IV} = \{(i_{11}, \lfloor(\sigma^-_{X_{IV}}(i_{11}))^2, (\sigma^+_{X_{IV}}(i_{11}))^2\rfloor) : i_{11} \in V_{IV}\}$, where $(\sigma^-_{X_{IV}})^2$ and $(\sigma^+_{X_{IV}})^2$ are fuzzy subsets of V_{IV} such that $(\sigma^-_{X_{IV}}(i_{11}))^2 \leq (\sigma^+_{X_{IV}}(i_{11}))^2$ for all $i_{11} \in V_{IV}$. If $G^*_{IV} = (V_{IV}, E_{IV})$ is a crisp graph, then by an vertex squared interval-valued fuzzy relation Y_{IV} on V_{IV} we mean an VSIVFS on E_{IV} such that $\sigma^-_{Y_{IV}}(i_{11}i_{22}) \leq \min\{(\sigma^-_{X_{IV}}(i_{11}))^2, (\sigma^-_{X_{IV}}(i_{22}))^2\}$ and $\sigma^+_{Y_{IV}}(i_{11}i_{22}) \leq \max\{(\sigma^+_{X_{IV}}(i_{11}))^2, (\sigma^+_{X_{IV}}(i_{22}))^2\}$ for all $i_{11}i_{22} \in E_{IV}$ and we write $Y_{IV} = \{(i_{11}i_{22}, \lfloor(\sigma^-_{Y_{IV}}(i_{11}i_{22}))^2, (\sigma^+_{Y_{IV}}(i_{11}i_{22}))^2\rfloor) : i_{11}i_{22} \in E_{IV}\}$.

Definition 2.2. An VSIVFG of a graph $G^*_{IV} = (V_{IV}, E_{IV})$ is a pair

$G_{IV} = (X_{IV}, Y_{IV})$, where $X_{IV} = \lfloor(\sigma^{-}_{X_{IV}})^2, (\sigma^{+}_{X_{IV}})^2\rfloor$ is an VSIVFS on V_{IV} and $Y_{IV} = \lfloor\sigma^{-}_{Y_{IV}}, \sigma^{+}_{Y_{IV}}\rfloor$ is an vertex squared interval-valued fuzzy relation on V_{IV}.

Example 2.1. G_{IV} :

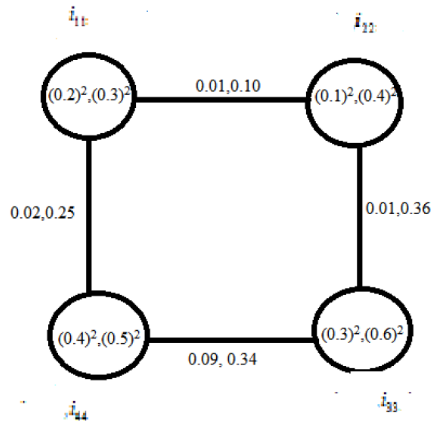

Figure 1. VSIVFG (G_{IV}).

In the above figure,

$V_{IV} = \{i_{11}, i_{22}, i_{33}, i_{44}\}$

$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{44}, i_{44}i_{11}\}$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ defined by

$$X_{IV} = \left\langle\left(\frac{i_{11}}{(0.2)^2}, \frac{i_{22}}{(0.1)^2}, \frac{i_{33}}{(0.3)^2}, \frac{i_{44}}{(0.4)^2}\right)\right\rangle\left\langle\left(\frac{i_{11}}{(0.3)^2}, \frac{i_{22}}{(0.4)^2}, \frac{i_{33}}{(0.6)^2}, \frac{i_{44}}{(0.5)^2}\right)\right\rangle$$

$$Y_{IV} = \left\langle\left(\frac{i_{11}i_{22}}{0.01}, \frac{i_{22}i_{33}}{0.01}, \frac{i_{33}i_{44}}{0.09}, \frac{i_{44}i_{11}}{0.02}\right)\right\rangle\left\langle\left(\frac{i_{11}i_{22}}{0.10}, \frac{i_{22}i_{33}}{0.36}, \frac{i_{33}i_{44}}{0.34}, \frac{i_{44}i_{11}}{0.25}\right)\right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

Definition 2.3. The order p_{IV} and size q_{IV} of an VSIVFG $G_{IV} = (X_{IV}, Y_{IV})$ of a graph $G^{*}_{IV} = (V_{IV}, E_{IV})$ are denoted by

$$p_{IV} = \sum_{i_{11} \in V_{IV}} \frac{1 + (\sigma^{+}_{X_{IV}}(i_{11}))^2 - (\sigma^{-}_{X_{IV}}(i_{11}))^2}{2} \text{ and}$$

$$q_{IV} = \sum_{i_{11}i_{22} \in V_{IV}} \frac{1 + \sigma^{+}_{X_{IV}}(i_{11}i_{22}) - \sigma^{-}_{X_{IV}}(i_{11}i_{22})}{2}.$$

Definition 2.4. Let $G_{IV} = (X_{IV}, Y_{IV})$ be an VSIVFG on $G^{*}_{IV} = (V_{IV}, E_{IV})$ and $S_{IV} \subseteq V_{IV}$. Then the vertex squared cardinality of S_{IV} is defined to be $\sum_{i_{11} \in V_{IV}} \frac{1 + (\sigma^{+}_{X_{IV}}(i_{11}))^2 - (\sigma^{-}_{X_{IV}}(i_{11}))^2}{2}$.

Definition 2.5. An arc $e_{IV} = i_{11}i_{22}$ of the VSIVFG is called a vertex squared effective edge if $\sigma^{-}_{Y_{IV}}(i_{11}i_{22}) = \min\{(\sigma^{-}_{X_{IV}}(i_{11}))^2, (\sigma^{-}_{X_{IV}}(i_{22}))^2\}$ and $\sigma^{+}_{Y_{IV}}(i_{11}i_{22}) = \max\{(\sigma^{+}_{X_{IV}}(i_{11}))^2, (\sigma^{+}_{X_{IV}}(i_{22}))^2\}$.

Definition 2.6. A set S_{IV} of vertices of the VSIVFG is called the vertex squared independent set (VSIS) if $\sigma^{-}_{Y_{IV}}(i_{11}i_{22}) < \min\{(\sigma^{-}_{X_{IV}}(i_{11}))^2, (\sigma^{-}_{X_{IV}}(i_{22}))^2\}$ and $\sigma^{+}_{Y_{IV}}(i_{11}i_{22}) < \max\{(\sigma^{+}_{X_{IV}}(i_{11}))^2, (\sigma^{+}_{X_{IV}}(i_{22}))^2\}$ for all $i_{11}, i_{22} \in S_{IV}$.

3. *n*-Split Domination in Vertex Squared Interval-Valued Fuzzy Graph

Definition 3.1. Let $G_{IV} = (X_{IV}, Y_{IV})$ be an VSIVFG on V_{IV} and $i_{11}, i_{22} \in V_{IV}$ We say 'i_{11}' n-split dominates 'i_{11}' if

$$\sigma^{-}_{Y_{IV}}(i_{11}i_{22}) = \min\left\{\frac{(\sigma^{-}_{X_{IV}}(i_{11}))^2}{n}, \frac{(\sigma^{-}_{X_{IV}}(i_{22}))^2}{n}\right\} \text{ and}$$

$$\sigma^{+}_{Y_{IV}}(i_{11}i_{22}) = \max\left\{\frac{(\sigma^{+}_{X_{IV}}(i_{11}))^2}{n}, \frac{(\sigma^{+}_{X_{IV}}(i_{22}))^2}{n}\right\}.$$

Example 3.1. G_{IV} :

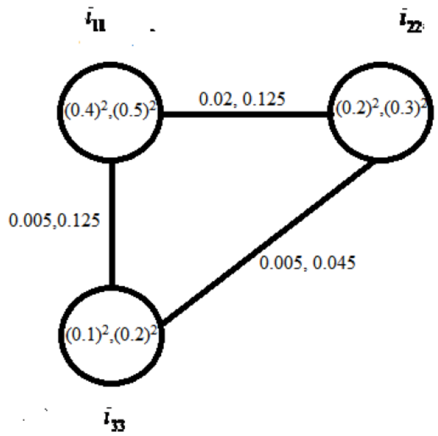

Figure 2. VSIVFG (G_{IV}) with 2-Split Dominates.

In the above figure,

$V_{IV} = \{i_{11}, i_{22}, i_{33}\}$

$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{11}\}$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ denoted by

$$X_{IV} = \left\langle \left(\frac{i_{11}}{(0.4)^2}, \frac{i_{22}}{(0.2)^2}, \frac{i_{33}}{(0.1)^2} \right) \right\rangle \left\langle \left(\frac{i_{11}}{(0.5)^2}, \frac{i_{22}}{(0.3)^2}, \frac{i_{33}}{(0.2)^2} \right) \right\rangle$$

$$Y_{IV} = \left\langle \left(\frac{i_{11}i_{22}}{0.02}, \frac{i_{22}i_{33}}{0.005}, \frac{i_{33}i_{11}}{0.005} \right) \right\rangle \left\langle \left(\frac{i_{11}i_{22}}{0.125}, \frac{i_{22}i_{33}}{0.045}, \frac{i_{33}i_{11}}{0.125} \right) \right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

Definition 3.2. A subset S_{IV} of V_{IV} is called a n-split dominating set (n-SDS) in VSIVFG if for every $i_{22} \notin S_{IV}$, there exist $i_{22} \in S_{IV}$ such that i_{22} n-split dominates i_{22}. A n-SDS IV R of a VSIVFG is called the minimal n-split dominating set if no proper subset of R_{IV} is a n-SDS of VSIVFG.

Definition 3.3. The minimal vertex squared cardinality of a n-SDS in

VSIVFG is said to be n-split domination number of VSIVFG and is denoted by $\gamma_{nSPD}(G_{IV})$.

Example 3.2.

In the figure,

$V_{IV} = \{i_{11}, i_{22}, i_{33}\}$

$E_{IV} = \{i_{11}i_{22}, i_{22}i_{33}, i_{33}i_{11}\}$

Here we take X_{IV} be an VSIVFS on V_{IV} and Y_{IV} be an VSIVFS on $E_{IV} \subseteq V_{IV} \times V_{IV}$ denoted by

$$X_{IV} = \left\langle \left(\frac{i_{11}}{(0.4)^2}, \frac{i_{22}}{(0.3)^2}, \frac{i_{33}}{(0.2)^2} \right) \right\rangle \left\langle \left(\frac{i_{11}}{(0.5)^2}, \frac{i_{22}}{(0.4)^2}, \frac{i_{33}}{(0.4)^2} \right) \right\rangle$$

$$Y_{IV} = \left\langle \left(\frac{i_{11}i_{22}}{0.045}, \frac{i_{22}i_{33}}{0.02}, \frac{i_{33}i_{11}}{0.02} \right) \right\rangle \left\langle \left(\frac{i_{11}i_{22}}{0.125}, \frac{i_{22}i_{33}}{0.08}, \frac{i_{33}i_{11}}{0.125} \right) \right\rangle$$

Then $G_{IV} = (X_{IV}, Y_{IV})$ is an VSIVFG.

G_{IV} :

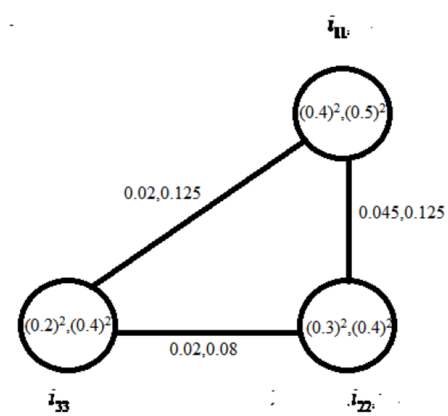

Figure 3. VSIVFG (G_{IV}) with 2-Split domination number.

In the above figure 3 having 2 -split dominating sets are

$D_1 = \{i_{11}\}, D_2 = \{i_{22}\}, D_3 = \{i_{33}\}, D_4 = \{i_{11}, i_{22}\}, D_5 = \{i_{22}, i_{33}\}$ and

$$D_6 = \{i_{33}, i_{11}\}.$$

Then the minimal vertex squared cardinality of a 2-split dominating set is $\{i_{22}\}$ and $\gamma_{2SPD}(G_{IV}) = 0.54$.

Theorem 3.1. *A vertex squared independent set is a maximal vertex squared independent set of a VSIVFG iff it is a vertex squared independent set and n-SDS.*

Proof. Let S_{IV} is a maximal vertex squared independent set of a VSIVFG. Thus for each $x \in V_{IV} - S_{IV}$, the set $S_{IV} \cup \{x\}$ is not vertex squared independent set. In this way, for each vertex $x \in V_{IV} - S_{IV}$, there is a vertex $y \in S_{IV}$ to such an extent that y is n-split dominated by x. Consequently S_{IV} is a n-SDS. Hence S_{IV} is an vertex squared independent and n-SDS.

Conversely, let S_{IV} be vertex squared independent set and n-SDS. If conceivable, assume S_{IV} is not a maximal vertex squared independent set. Then there exists $x \in V_{IV} - S_{IV}$ to such an extent that the set $S_{IV} \cup \{x\}$ is vertex squared independent set. Then no vertex in S_{IV} is n-split dominated by x. Hence S_{IV} cannot be a n-SDS, which is a contradiction. Hence S_{IV} should be a maximal vertex squared independent set.

Example 3.3. G_{IV} :

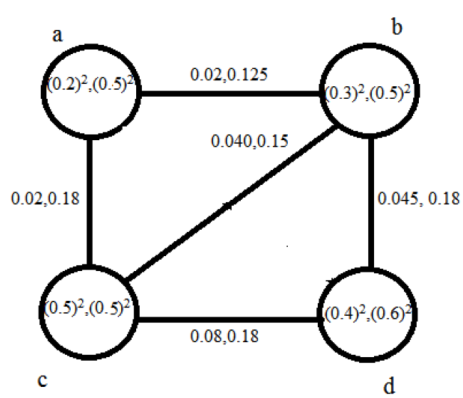

Figure 4. VSIVFG (G_{IV}) with 2-Split Dominating Set.

Theorem 3.2. *In a VSIVFG, every maximal vertex squared independent set is a minimal n-split dominating set.*

Proof. Let S_{IV} be a maximal vertex squared independent set in VSIVFG. By the theorem 3.1, S_{IV} is a n-SDS. Assume S_{IV} be not a minimal n-split dominating set. Then there exists somewhere around one vertex $x \in S_{IV}$ for which $S_{IV} - \{x\}$ is a n-SDS. Yet, if $S_{IV} - \{x\}$ n-split dominates $V_{IV} - (S_{IV} - \{x\})$ then at least one vertex in $S_{IV} - \{x\}$ must n-split dominate x. This contradicts the way that S_{IV} is a VSIS of VSIVFG. Hence S_{IV} should be a minimal n-split dominating set.

Example 3.4. G_{IV} :

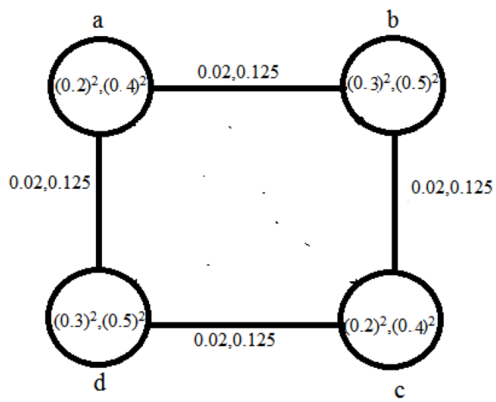

Figure 5. VSIVFG (G_{IV}) with 2-Split dominating set.

Theorem 3.3. *Let G_{IV} be a VSIVFG with n-split dominate edges. If S_{IV} is a minimal n-split dominating set, then $V_{IV} - S_{IV}$ is a n-SDS.*

Proof. Let S_{IV} be a minimal n-split dominating set of VSIVFG. Assume $V_{IV} - S_{IV}$ is not n-SDS. Then there exist a vertex to $x \in S_{IV}$ such an extent that x is not n-split dominated by anyone vertex in $V_{IV} - S_{IV}$. Since G_{IV} has n-split dominate edges, x is a n-split dominate of somewhere around one vertex in $S_{IV} - \{x\}$. Then $S_{IV} - \{x\}$ is a n-SDS, which contradicts the minimality of S_{IV}. Subsequently, every vertex in S_{IV} is a n-split dominate of no less than one vertex in $V_{IV} - S_{IV}$. Hence $V_{IV} - S_{IV}$ is a n-SDS.

Example 3.5. G_{IV} :

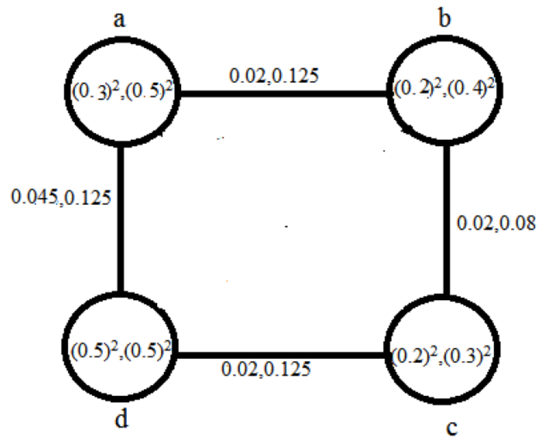

Figure 6. VSIVFG (G_{IV}) with 2-Split Dominating Set.

Conclusion

The new thought has been explained in this paper for vertex squared cardinality, vertex squared effective edge, n-split dominating set, and n-split domination number. Theorems identified with this concept are inferred and the relation between n-split domination set and vertex squared independent set are set up.

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