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CORDIAL LABELING ON NEW CLASSES OF SUBDIVIDED SHELL GRAPHS

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Abstract

Cahit introduced cordial labeling [4] in the year 1987. Cordial labeling is defined as a function $\theta : V(K) \to \{0, 1\}$ when each edge ab is assigned the label $|\theta(a) - \theta(b)|$ with the condition $|v_\theta(0) - v_\theta(1)| \leq 1$ and $|e_\theta(0) - e_\theta(1)| \leq 1$ where $v_\theta(0)$ and $v_\theta(1)$ represent the number of vertices with 0's and 1's, similarly, $e_\theta(0)$ and $e_\theta(1)$ represent the number of edges with 0's and 1's. We intent to prove following graphs such as subdivided shell graph SSG(n) admits cordiality, subdivided shell graph SSG(n) with star graph attached to its apex admits cordiality, subdivided shell graph SSG(n) attached to each vertex of the path are cordial.

1. Introduction

Assigning numbers to the points or lines or to both with few constraints is termed as Graph labeling. Alexander Rosa [10] established the concept of graceful labeling in the research area of Graph Labeling. Cahit [4] introduced another labeling method called as cordial labeling. Cordial labeling is defined

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as a function $\theta : V(K) \rightarrow \{0, 1\}$ when each edge ab is assigned the label $|\theta(a) - \theta(b)|$ with the condition $|v_\theta(0) - v_\theta(1)| \leq 1$ and $|e_\theta(0) - e_\theta(1)| \leq 1$ where $v_\theta(0)$ and $v_\theta(1)$ denote the number of vertices with 0's and 1's, similarly $e_\theta(0)$ and $e_\theta(1)$ denote the number of edges with 0's and 1's. In [1], [2] Andar et al. proved that the multiple shells, helms, flowers and closed helms are cordial. Amit et al. [3] proved that the shadow graph of star, splitting graph of star, degree splitting graph of star, jewel graph and jelly fish graph are cordial. Cahit [5] proved the cordiality for the complete graph iff $n \leq 3$ and ladders, friendship graphs, paths, wheels, pinwheels are cordial. Deb and Limaye [6] introduced the shell graph and the subdivided shell graphs were introduced by Jeba Jesintha and Hilda [8]. Meena et al. [9] proved that shell graphs, multiple shell graph, star of shell graph, the cycle of shell graphs are cordial. For more results, one can refer dynamic survey by Gallian [7].

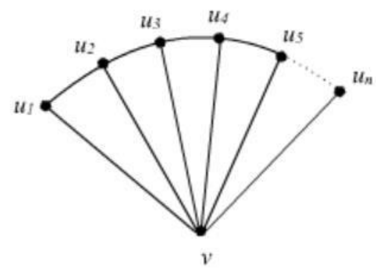
We intent to prove following graphs such as subdivided shell graph SSG (n) admits cordiality, subdivided shell graph SSG(n) with star graph attached to its apex admits cordiality, subdivided shell graph SSG(n) attached to each vertex of the path are cordial.

2. Definitions

In this section we give few definitions as in the literature [7].

Definition 2.1. A cycle C_n with $(n-3)$ chords and a common end point called as apex is defined as shell graph [6]. Shell graphs are also termed as fan graph. See Figure 1.

Definition 2. 2. In the shell graph [figure 1], if each path is subdivided, we get a subdivided shell graph [8]. See Figure 2.



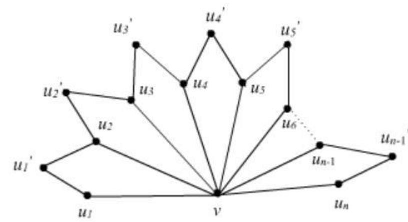


Figure 1. Shell graph. **Figure 2.** Subdivided shell graph.

Definition 2. 3. Path union of a graph H is defined as attaching a graph $H_i (i = 1, 2, \ldots, n)$ where $n \geq 2$ to each vertex of the given path P [11].

3. Main Result

Under this section, few theorems on subdivided shell graphs SSG are proved.

Theorem 3.1. *The graph* SSG(n) *is Cordial.*

Proof. The shell graph S with apex as v and the other vertices denoted by $u_1, u_2, u_3, \ldots, u_n$ as shown in Figure 1. We denote subdivided shell graph as SSG. The vertices of the SSG(n) be $u_1', u_2', u_3', \ldots, u_{n-1}'$ representing the vertices of the edges subdivided in S. The graph G is shown in Figure 2. Let $|V(G)| = p, |E(G)| = q$. Note that $p = 2n, q = 3n - 2$.

Define the vertex labeling for G as $\gamma : V(G) \to \{0, 1\}$

$$\gamma(v) = 0$$

$$\gamma(u_\alpha) = \begin{cases} 1; & \alpha \equiv 1 (\text{mod } 2) \\ 0; & \alpha \equiv 0 (\text{mod } 2) \end{cases} \text{ for } 1 \leq \alpha \leq n$$

$$\gamma(u_\alpha') = \begin{cases} 0; & \alpha \equiv 1 (\text{mod } 2) \\ 1; & \alpha \equiv 0 (\text{mod } 2) \end{cases} \text{ for } 1 \leq \alpha \leq n - 1$$

Number of vertices with labels 0's and 1's be denoted as for $e_\gamma(0)$ and $v_\gamma(1)$ respectively.

From the above defined vertex labeling we get $v_\gamma(0) = n = v_\gamma(1)$.

Number of edges with labels 0's and 1's be denoted as $e_\gamma(0)$ and $e_\gamma(1)$ respectively. $e_\gamma(0)$ and $e_\gamma(1)$ are computed as follows.

Case 1. Where n is even

$$e_\gamma(0) = \frac{3n}{2} - 1 = e_\gamma(1)$$

Case 2. Where n is odd

$$e_\gamma(0) = \left\lfloor \frac{3n}{2} \right\rfloor$$

$$e_\gamma(1) = \left\lfloor \frac{3n}{2} \right\rfloor + 1$$

Thus, the above labeling pattern satisfies the conditions $|v_\gamma(0) - v_\gamma(1)| \le 1$ and $|e_\gamma(0) - e_\gamma(1)| \le 1$. Hence subdivided shell graph admits cordial labeling.

Theorem 3 2. *The subdivided shell graph SSG(n) with star attached at its apex admits cordiality.*

Proof. Let H be the SSG(n) with star attached at its apex. Let v be the apex of the subdivided shell graph H. We denote the pendant vertices from v in the graph H be $w_1, w_2, w_3, \ldots, w_r$. Let the other vertices in H adjacent to v be denoted as $u_1, u_2, u_3, \ldots, u_n$. The vertices in H not adjacent to v be denoted as $u'_1, u'_2, u'_3, \ldots, u_{n-1}$. The graph H is shown in Figure 3. Let $|V(H)| = p$, $|E(H)| = q$. The graph H has $p = 2n + r$, $q = 3n + r - 2$.

Define the vertex labeling of H as $\delta : V(H) \to \{0, 1\}$.

$$\delta(v) = 0$$

$$\delta(u_\beta) = \begin{cases} 1; & \beta \equiv 1 \pmod 2 \\ 0; & \beta \equiv 0 \pmod 2 \end{cases} \text{ for } 1 \le \beta \le n$$

$$\delta(u'_\beta) = \begin{cases} 0; & \beta \equiv 1 \pmod 2 \\ 1; & \beta \equiv 0 \pmod 2 \end{cases} \text{ for } 1 \le \beta \le n-1$$

$$\delta(w_\beta) = \begin{cases} 1; & \beta \equiv 1 (\text{mod } 2) \\ 0; & \beta \equiv 0 (\text{mod } 2) \end{cases} \text{ for } 1 \le \beta \le r$$

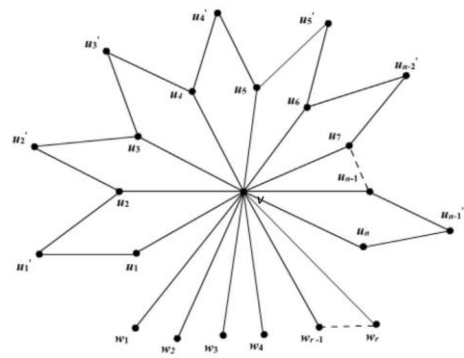


Figure 3. The graph H.

Number of vertices and edges with labels 0 and 1 are denoted as $v_\delta(0)$, $e_\delta(0)$ and $v_\delta(1)$, $e_\delta(1)$ respectively and they are computed as follows.

Case 1. Where n, r is odd

$$v_\delta(0) = n + \frac{r}{2} + 1$$

$$v_\delta(1) = \frac{r}{2} + n$$

$$e_\delta(0) = \frac{3n + r}{2} - 1 = e_\delta(1)$$

Case 2. Where n is ever, r odd

$$v_\delta(0) = n\frac{r}{2} + 1$$

$$v_\delta(1) = n + \frac{r}{2}$$

$$e_\delta(0) = \left\lfloor \frac{3n + r}{2} \right\rfloor - 1$$

$$e_\delta(1) = \left\lfloor \frac{3n + r}{2} \right\rfloor$$

Case 3. Where n, r is even

$$v_\delta(0) = n + \frac{r}{2} = v_\delta(1)$$

$$e_\delta(0) = \frac{3n+r}{2} - 1 = e_\delta(1)$$

Case 4. Where n is odd r is even

$$v_\delta(0) = n + \frac{r}{2} = v_\delta(1)$$

$$e_\delta(0) = \left\lfloor \frac{3n+r}{2} \right\rfloor - 1$$

$$e_\delta(1) = \left\lfloor \frac{3n+r}{2} \right\rfloor$$

From the above labeling it is observed that $|v_\delta(0) - v_\delta(1)| \leq 1$ and $|e_\delta(0) - e_\delta(1)| \leq 1$. Hence, the subdivided shell graph with star attached at its apex admits cordiality.

Theorem 3.3. SSG(n) *attached to each vertex of the path admits cordiality.*

Proof. SSG(n) attached to each vertex of the path obtained by attaching r copies of subdivided shell graphs namely $S_1, S_2, S_3, \ldots, S_r$ to the vertices $p_1, p_2, p_3, \ldots, p_r$ of the path P be denoted as G. Graph G is described in the following way. The vertices of S_1 which are adjacent to p_1 are denoted by $a_1^1, a_2^1, a_3^1, \ldots, a_n^1$. The vertices of S_1 which are not adjacent to p_1 are denoted by $b_1^1, b_2^1, b_3^1, \ldots, b_{n-1}^1$. Similarly, the vertices of S_2 which are adjacent to p_2 are denoted by $a_1^2, a_2^2, a_3^2, \ldots, a_n^2$. The vertices of S_2 which are not adjacent to p_2 are denoted by $b_1^2, b_2^2, b_3^2, \ldots, b_{n-1}^2$. In general, the vertices of S_r which are adjacent to p_r are denoted by $a_1^m, a_2^m, a_3^m, \ldots, a_n^m$. The vertices of S_r which are not adjacent to p_r are denoted by $b_1^m, b_2^m, b_3^m, \ldots, b_n^m$. Figure 4 shows the graph for G. Labeling for the vertex be defined as $\mu : V(G) \to \{0, 1\}$. The vertices and edges of the graph G be

$|V(G)| = 2nr, |E(G)| = 3nr - r - 1.$

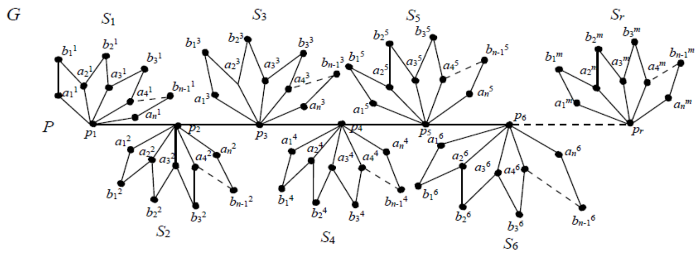


Figure 4. SSG(n) attached to each vertex of the path.

Define the vertex labeling of G as:

Case 1. Where n is ever

$$\mu(p_\sigma) = \begin{cases} 1; & \sigma \equiv 1, 2 (\text{mod } 3) \\ 0; & \sigma \equiv 0 (\text{mod } 3) \end{cases} \text{ for } 1 \leq \sigma \leq r$$

$$\mu(a_\sigma^t) = \begin{cases} 1; & \sigma \equiv 1, 2 (\text{mod } 2) \\ 0; & \sigma \equiv 0 (\text{mod } 2) \end{cases}, \text{ for } 1 \leq t \leq m, 1 \leq \sigma \leq n,$$

Where $t \equiv 1, 2 (\text{mod } 3)$

$$\mu(b_\sigma^t) = \begin{cases} 0; & \sigma \equiv 1 (\text{mod } 2) \\ 1; & \sigma \equiv 0 (\text{mod } 2) \end{cases}, \text{ for } 1 \leq t \leq m, 1 \leq \sigma \leq n - 1$$

Where $t \equiv 0 (\text{mod } 3)$

$$\mu(b_\sigma^t) = \begin{cases} 1; & \sigma \equiv 1 (\text{mod } 2) \\ 0; & \sigma \equiv 0 (\text{mod } 2) \end{cases}, \text{ for } 1 \leq t \leq m, 1 \leq \sigma \leq n - 1$$

Case 2. When n is odd

$\mu(p_\sigma) = 0$, for $1 \leq \sigma \leq r$

$$\mu(a_\sigma^t) = \begin{cases} 1; & \sigma \equiv 1 (\text{mod } 2) \\ 0; & \sigma \equiv 0 (\text{mod } 2) \end{cases}, \text{ for } 1 \leq t \leq m, 1 \leq \sigma \leq n,$$

$$\mu(b_\sigma^t) = \begin{cases} 0; & \sigma \equiv 1 (\text{mod } 2) \\ 1; & \sigma \equiv 0 (\text{mod } 2) \end{cases}, \text{ for } 1 \leq t \leq m, 1 \leq \sigma \leq n - 1$$

Let $v_\mu(0)$, $e_\mu(0)$ and $v_\mu(1)$, $e_\mu(1)$ be the number of vertices and edges with 1 respectively in G.

Now $v_\mu(0)$ and $v_\mu(1)$ are computed as follows:

From the above defined vertex labeling we get $v_\mu(0) = nr = v_\mu(1)$.

Now $v_\mu(0)$ and $v_\mu(1)$ computed in the following cases.

Case a. Where r is even and for any n

$$e_\mu(0) = \left\lfloor \frac{3nr-1}{2} \right\rfloor - 2$$

$$e_\mu(1) = \left\lfloor \frac{3nr-1}{2} \right\rfloor - 1$$

Case b. Where r is odd and n is odd

$$e_\mu(0) = \left\lfloor \frac{3nr}{2} \right\rfloor - 3$$

$$e_\mu(1) = \left\lfloor \frac{3nr}{2} \right\rfloor - 2$$

Case c. Where r is odd and n is even

$$e_\mu(0) = \left\lfloor \frac{3nr-5}{2} \right\rfloor = e_\mu(1)$$

From the above labeling it follows that $| v_\mu(0) - v_\mu(1) | \leq 1$ and $| e_\mu(0) - e_\mu(1) | \leq 1$. Hence path union of subdivided shell graph admits cordial labeling.

4. Conclusion

This paper proved the following graphs such as subdivided shell graph SSG(n) admits cordiality, subdivided shell graph SSG(n) with star graph attached to its apex admits cordiality, subdivided shell graph SSG(n) attached to each vertex of the path are cordial.

References

[1] M. Andar, S. Boxwala and N. Limaye, Cordial labeling of some wheel related graphs, J. Combin. Math. Combin. Comput., 41 (2002), 203-208.

[2] M. Andar, S. Boxwala and N. Limaye, A note on cordial lableing of multiple shells, Trends Math. (2002), 77-80.

[3] Amit H. Rokad and Kalpesh M. Patadiya, Cordial labeling of some graphs, Aryabhata Journal of Mathematics and Informatics (Impact factor-5.856) ISSN:2394-9309(E)/0975-7139(P) 09(01) (Jan- June, 2017).

[4] I. Cahit, Cordial Graphs: A weaker Version of Graceful and Harmonious Graphs, Ars Combin. 23 (1987), 201-207.

[5] I. Cahit, On cordial and 3-equitable labelings of graphs, Util. Math., 37(1990), 189-198.

[6] P. Deb and N. B. Limaye, On Harmonious Labeling of some cycle related graphs, Ars Combinatoria 65 (2002), 177-97.

[7] J. A. Gallian, A dynamic survey of Graph labeling, The Electronic Journal of Combinatorics (2020), pp. 77-81.

[8] J. Jeba Jesintha and K. Ezhilarasi Hilda, Sub divided uniform shell bow graphs and subdivided shell graphs are odd graceful, Eduventure 7(13) (2014), 103-105.

[9] S. Meena, M. Renugha and M. Sivasakthi, Cordial labeling for different types of shell graph, International Journal of Scientific and Engineering Research ISSN 2229 - 5518 6(9) (2015), 1282-1288.

[10] A. Rosa, On certain valuation of graph, Theory of graphs, Gordon and Breach. N.Y and Paris (1967), 349-355.

[11] S. C. Shee and Y. S. Ho, The cordiality of the path-union of n copies of a graph, Discrete Math. 151 (1996), 221-229.