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PRIME EDGE MAGIC LABELING FOR SOME GRAPHS

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Abstract

In this paper we introduce new labeling called prime edge magic labeling and obtain the existence of this labeling for some graphs.

1. Introduction

All graphs in this paper are finite and simple. The graph G has the vertex set $V(G)$ and edge set $E(G)$. A labeling of a graph G is a mapping that carries graph elements to integers. The origin of this labeling is introduced by Rosa.

In this paper, we focus on one type of labeling called prime edge magic labeling. Recently in 2013, Neelam Kumari and Seema Mehra introduced n-edge magic labeling and shown the existence of this labeling for some class of graphs. Motivated by n-edge magic labeling we have introduced prime edge magic labeling for certain graphs as for p_t, c_t, $s_{m,t}$ (double star) etc. Now we shall drive the existence of prime edge magic labeling for some graphs.

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2. Preliminary

Definition 1. 0-Edge magic labeling

Let $G = (V, E)$ be a graph, where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-1, 1]$ and $f^* : E \to [0]$, such that for all $uv \in E$, $f^*(uv) = f(u) + f(v) = 0$ then the labeling is said to be 0-Edge magic labeling.

Definition 2. 1-Edge magic labeling

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-1, 2]$ and $f^* : E \to [1]$, such that for all $uv \in E$, $f^*(uv) = f(u) + f(v) = 1$ then the labeling is said to be 1-Edge magic labeling.

Definition 3. n-Edge magic labeling

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-1, n+1]$ and $f^* : E \to [n]$, such that for all $uv \in E$, $f^*(uv) = f(u) + f(v) = n$ then the labeling is said to be n-Edge magic labeling.

Definition 4. $G^+ = G \odot K_1$ is a graph obtained by joining exactly one pendant edge to every vertex of a graph G.

Definition 5. A sun S_t is a cycle on t vertices with an edge terminating in a vertex of degree 1 attached to each vertex on the cycle.

Definition 6. A complete binary tree T is a tree with a central vertex of degree 2, all other vertices that are not leaves of degree 3, and all leaves at the same distance from the central vertex.

2. Main Results

The concept of 0-Edge magic labeling, 1-Edge magic labeling and n-Edge magic labeling motivate us to define the following new definition of prime Edge magic labeling.

Prime Edge magic labeling:

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-p, 2p]$ and $f^* : E \to [p]$, if p is a prime number such that for all $uv \in E$, $f^*(uv) = f(u) + f(v) = p$ then the labeling is said to be prime Edge magic labeling.

Using this new definition, we prove some results as follows:

Theorem 1. *P_t admits prime Edge magic labeling for every t.*

Proof.

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-p, 2p]$ if p is a prime number.

Such that

$$f(v_i) = -p \text{ if } i \text{ is odd for } 1 \leq i \leq t$$

$$f(v_i) = 2p \text{ if } i \text{ is even for } 1 \leq i \leq t.$$

Edge weight calculated as follows

$$f^*(v_i v_{i+1}) = -p + 2p = p \text{ if } i \text{ is odd and } p \text{ is a prime number}$$

$$f^*(v_i v_{i+1}) = 2p - p \text{ if } i \text{ is even and } p \text{ is a prime number.}$$

Hence P_t admits prime Edge magic labeling.

Example. Prime edge magic labeling of P_5, P_6 is shown in the figure 1

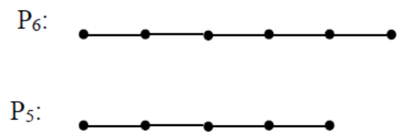

Figure 1

Theorem 2. *C_t admits prime Edge magic labeling when t is only for even.*

Proof.

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \leq i \leq t\} \text{ and } E = \{v_i v_{i+1}, 1 \leq i \leq t-1\}.$$

Let $f : V \to [-p, 2p]$ if p is a prime number.

Such that

$$f(v_i) = -p \text{ if } i \text{ is odd for } 1 \leq i \leq t$$

$$f(v_i) = 2p \text{ if } i \text{ is even for } 1 \leq i \leq t.$$

Edge weight calculated as follows

$$f^*(v_i, v_{i+1}) = -p + 2p = p \text{ if } i \text{ is odd and } p \text{ is a prime number}$$

$$f^*(v_i v_{i+1}) = 2p - p \text{ if } i \text{ is even and } p \text{ is a prime number.}$$

Hence C_t admits prime Edge magic labeling when t is only for even.

Example. Prime edge magic labeling of C_6 is shown in the figure 2

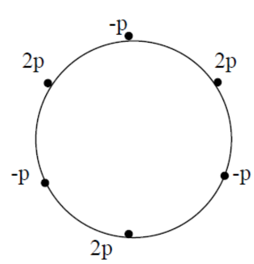

Figure 2.

Corollary. *Similarly, we can prove that the graph* mC_t *is prime Edge Magic for all m and t is only for even t.*

Theorem 3. *A sun graph* S_t *is prime Edge magic labeling when t is only for even.*

Proof. Let $v_1, v_2, v_3, \ldots, v_t$ be the vertices of cycle S_t and $u_1, u_2, u_3, \ldots, u_t$ be the end vertices of each edge attached to $v_1, v_2, v_3, \ldots, v_t$.

$$f(v_i) = -p \text{ if } i \text{ is odd for } 1 \leq i \leq t$$

$$f(v_i) = 2p \text{ if } i \text{ is even for } 1 \leq i \leq t$$

and

$$f(u_i) = -p \text{ if } f(v_i) = 2p \text{ for } 1 \leq i \leq t$$

$$f(u_i) = 2p \text{ if } f(v_i) = -p \text{ for } 1 \leq i \leq t$$

Edge weight calculated as follows

$$f^*(v_i v_{i+1}) = -p + 2p = p \text{ if } i \text{ is odd and } p \text{ is a prime number}$$

$$f^*(v_i v_{i+1}) = 2p - p = p \text{ if } i \text{ is even and } p \text{ is a prime number.}$$

and

$$f^*(v_i u_{i+1}) = -p + 2p = p \text{ if } i \text{ is odd and } p \text{ is a prime number}$$

$$f^*(v_i u_{i+1}) = 2p - p = p \text{ if } i \text{ is even and } p \text{ is a prime number.}$$

Hence *A* sun graph S_t is prime Edge magic labeling when t is only for even.

Example. Prime edge magic labeling of sun graph S_t is shown in the figure 3.

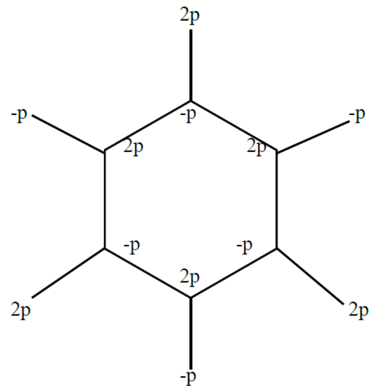

Figure 3

Theorem 4. *If G admits prime Edge magic labeling, then* $G \odot K_1$ *admits prime Edge magic labeling.*

Proof.

Let $G = (V, E)$ be a graph where

$$V = \{v_i, 1 \le i \le t\} \text{ and } E = \{v_i v_{i+1}, 1 \le i \le t-1\}.$$

Let $f : V \to [-p, 2p]$ and $f^* : E \to [p]$, such that for all $uv \in E$ $f^*(v_i v_{i+1}) = f(v_i) = p$.

Let $G \odot K_1 = (V, E) U \{v_j : 1 \le j \le t\} U \{v_i v_j : 1 \le i, j \le t\}$, then let $g : V \to [-p, 2p]$ and $g^* : E \to [p]$, then for all $v_i v_{i+1} \in E$

$v_j = -p$ if $v_i = 2p$ or $v_j = 2p$ if $v_i = -p$ then $g^*(v_i v_j) = g^*(v_i) + g^*(v_j) = p$. Hence the theorem.

Example. Prime edge magic labeling of $P_4 \odot K_1$ is shown in the figure 4

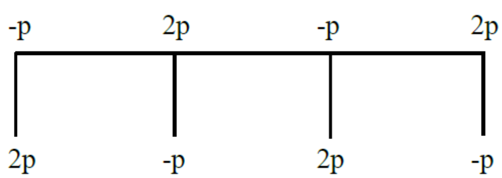

Figure 4

Theorem 5. *Let* $G = S_{m,t}$ *be a double star graph then* G *admits prime Edge magic labeling*.

Proof. Let $G = (V, E)$ be a double star graph denoted by $S_{m,t}$ and v_1 and v_2 are two vertices in $S_{m,t}$ which are not pendent. Let u_i's are m pendent vertices to v_1 and u_j's are t pendent vertices to v_2.

Let $f : V \to [-p, 2p]$ such that $f(v_1) = -p$ and $f(v_2) = 2p$ and $f(u_i) = 2p$ for $1 \le i \le m$, and $f(u_j) = -p$ for $1 \le j \le t$, $f^*(v_1 u_i) = -p + 2p = p$ if $1 \le i \le m$.

Also $f^*(v_2 u_j) = 2p - p = p$ if $1 \le j \le t$. Hence star graph admits prime edge magic labeling.

Example. Prime Edge magic labeling of star graph is shown in the figure 5.

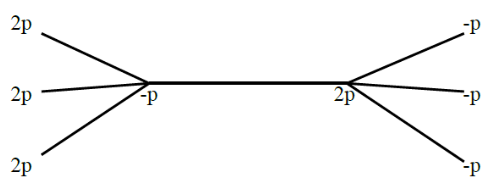

Figure 5.

Conclusion

In this paper we have investigated some class of graphs which admits prime edge magic labeling. Further investigation can be done to obtain the condition at which some graphs admit prime edge magic labeling.

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