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PROPAGATION OF WAVES AT IMPERFECT INTERFACE BETWEEN MICROPOLAR ELASTIC SOLID AND ELECTRO-MICROELASTIC SOLID HALF-SPACES

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Abstract

This paper deals with the propagation of waves at imperfect interface Micropolar Elastic Solid (MES) and Electro-microelastic Solid (EMS) half-spaces. A longitudinal (LD) wave is considered to be incident on the interface through micropolar elastic solid half space and this wave bumps obliquely at the interface. Amplitude ratios of various reflected and refracted waves are deduced using appropriate boundary conditions and results are represented graphically using the MATLAB software.

Introduction

The micropolar theory of elasticity constructed by Eringen and his co-workers intended to be applied on such materials and for problems where the ordinary theory of elasticity fails because of microstructure in the materials. Micropolar elastic materials, roughly speaking, are the classical elastic materials with extra independent degree of freedom for the local rotations. These materials respond to spin inertia, body and surface couples and as a

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consequence they exhibit certain new static and dynamic effects, e.g. new types of waves and couples stresses.

A micropolar elastic solid is distinguished from an elastic solid by the fact that it can support body and surface couples. These solids can undergo local deformations and micro-rotations such materials may be imagined as bodies which are made of rigid short cylinders or dumbbell type molecules.

From a continuum mechanical point of view, micropolar elastic solids may be characterized by a set of constitutive equations which define the elastic properties of such materials. A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi [3, 11]. Later, Eringen (1965) and (1966) recognized and extended this theory.

Eringen (1966a, 1990) developed the theories of 'micropolar continua' and 'microstructures continua' which are special cases of the theory of 'micromorphic continua' earlier developed by Eringen and his coworkers (1964). Thus, the Eringen's '3*M*' theories (Micromorphic, Microstretch, Micropolar) are the generalization the classical theory of elasticity. In classical continuum, each particle of a continuum is represented by a geometrical point and can have three degree of freedom of translation during the process of deformations.

Eringen's theory of micropolar elasticity keeps importance because of its applications in many physical substance for example material particles having rigid directors, chopped fibres composites, platelet composites, aluminium epoxy, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials.

Generalization of the theory of micropolar elasticity is linear theory of thermo-microstretch elastic solids developed by Eringen (1971, 1990) and extended this theory in (2004) including with electromagnetic interactions and known as electromagnetic theory of microstretch elasticity.

The present paper is concerned with wave propagation in micropolar elastic solid and Electro-microelastic Solid half-spaces. Computed the amplitude ratios of various reflected and refracted waves for a specific model and results are represented graphically according to angle of incidence of incident wave.

Fundamental equations and constitutive relations

For medium M_1 (Micropolar elastic solid half-space)

Eringen's (1968), equation of motion in micropolar elastic medium are as under:

$$(c_1^2 + c_3^2)\nabla^2 = \frac{\partial^2 \phi}{\partial t^2}, \tag{1}$$

$$(c_1^2 + c_3^2)\nabla^2 U + c_3^2 \nabla \times \Phi = \frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$(c_4^2 \nabla^2 - 2\omega_0^2)\Phi + \omega_0^2 \nabla \times U = \frac{\partial^2 \Phi}{\partial t^2}, \tag{3}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ c_3^2 = \frac{\kappa}{\rho},\ c_4^2 = \frac{\gamma}{\rho j},\ \omega_0^2 = \frac{\kappa}{\rho j}. \tag{4}$$

Equation (1) corresponding to LD wave moving with velocity V_1 and defined as $V_1^2 = c_1^2 + c_3^2$ given by Parfitt and Eringen (1969) and equations (2)-(3) are coupled equations in vector potentials U and Φ. These equations correspond to the waves named as coupled transverse and micro-rotations waves. If $\frac{\omega^2}{\omega_0^2} > 20$, there exists two set of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$

where

$$\lambda_1^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right],\ \lambda_2^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

and

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2^2 + c_3^2)} + \frac{1}{c_4^2},\ C = \left(\frac{1}{c_4^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2^2 + c_3^2)},$$

$$p = \frac{\kappa}{\mu + \kappa},\ q = \frac{\kappa}{\gamma}. \tag{6}$$

Considering the two dimensional problem by taking the following components of displacement and micro-rotation as

$$U = (u, 0, w), \Phi = (0, \Phi_2, 0), \tag{7}$$

where

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \; u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2 \phi}{\partial z^2} + \lambda \frac{\partial^2 \phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2 \psi}{\partial x \partial z}, \tag{9}$$

$$t_{zz} = (2\mu + \kappa)\frac{\partial^2 \phi}{\partial x \partial z} - (\mu + \kappa)\frac{\partial^2 \psi}{\partial z^2} + \mu \frac{\partial^2 \psi}{\partial x^2} - \kappa \Phi_2, \tag{10}$$

$$m_{zy} = \gamma \frac{\partial \Phi_2}{\partial z}, \tag{11}$$

For medium M_2 (Electro-microelastic solid half-space)

Electromagnetic fields in the continuum theory of microstretch elasticity introduced by Eringen (2004) and the absence of thermal effect, magnetic flux vector and microstretch continuum will be subjected only to electric field. So such type of continuum materials are known as eletro-microelastic solid medium are given by

$$\bar{t}_{kl} = (\bar{\lambda}_0 \bar{\psi} + \bar{\lambda} u_{r,r})\bar{\delta}_{kl} + \bar{\mu}(\bar{u}_{k,l} + \bar{u}_{l,k}) + \bar{\kappa}(\bar{u}_{l,k} - \in_{klr} \bar{\Phi}_r), \tag{12}$$

$$\overline{m}_{kl} = \alpha \overline{\Phi}_{r,r} \bar{\delta}_{kl} + \beta \overline{\Phi}_{k,l} + \gamma \overline{\Phi}_{l,k} + b_0 \in_{lkm} \overline{\Phi}_{,r}, \tag{13}$$

$$\overline{m}_k = \alpha_0 \overline{\psi}_{,k} + \lambda_2 E_k - b_0 \in_{klm} \overline{\Phi}_{l,m}, \tag{14}$$

$$\overline{D}_k = (1 + \chi^{\overline{E}})E_k + \lambda_3 \in_{lmk} \overline{\Phi}_{l,m} + \lambda_2 \overline{\psi}_k, \tag{15}$$

where $\bar{t}_{kl}, \overline{m}_{kl}, \overline{m}_k, \overline{D}_k; \bar{\lambda}, \bar{\mu}; \bar{\kappa}, \alpha, \beta, \gamma; b_0, \lambda_0, \alpha_0; \chi^{\overline{E}}, \lambda_2, \lambda_3; \bar{u}_k, \overline{\Phi}_k, \overline{\psi}$ and E_k are force stress tensor, couple stress, microstretch vector, dielectric displacement vector; Lame's constants; micropolar constants; microstretch constants; dielectric susceptibility, coupling constants; displacements, micropolar rotation vector, scalar microstretch and electric field vector respectively. The field equations under the section (7) of Eringen (2004) for an isotropic and homogeneous electro-microelastic solid medium are given by

$$(\bar{c}_1^2 + \bar{c}_3^2)\nabla\nabla \cdot \bar{u} - (\bar{c}_2^2 + \bar{c}_3^2)\nabla \times \nabla \times \bar{u} + c_3^2 \nabla \times \overline{\Phi} + \bar{\lambda}_0 \nabla \bar{\psi} = \ddot{\bar{u}} \tag{16}$$

$$(\bar{c}_4^2 + \bar{c}_5^2)\nabla\nabla \cdot \overline{\Phi} - \bar{c}_4^2 \nabla \times \nabla \times \overline{\Phi} + \overline{\omega}_0^2 \nabla \times \bar{u} - 2\overline{\omega}_0^2 \overline{\Phi} = \ddot{\overline{\Phi}} \tag{17}$$

$$\bar{c}_6^2 \nabla^2 \bar{\psi} - \bar{c}_7^2 \bar{\psi} - \bar{c}_8^2 \nabla \cdot \bar{u} + \bar{c}_9^2 \nabla . \overline{E} = \ddot{\bar{\psi}} \tag{18}$$

$$\nabla . \overline{D} = 0 \tag{19}$$

$$\nabla \times \overline{E} = 0 \tag{20}$$

where

$$\bar{c}_1^2 = \frac{\bar{\lambda} + 2\bar{\mu}}{\bar{\rho}}, \bar{c}_2^2 = \frac{\bar{\mu}}{\bar{\rho}}, \bar{c}_3^2 = \frac{\bar{\kappa}}{\bar{\rho}}, \bar{c}_4^2 = \frac{\bar{\gamma}}{\bar{\rho}\bar{j}}, \bar{c}_5^2 = \frac{\bar{\alpha} + \bar{\beta}}{\bar{\rho}\bar{j}}, \bar{c}_6^2 = \frac{2\bar{\alpha}_0}{\bar{\rho} j},$$

$$\bar{c}_7^2 = \frac{2\bar{\lambda}_1}{3\bar{\rho}\bar{j}}, \bar{c}_8^2 = \frac{2\bar{\lambda}_0}{3\bar{\rho}\bar{j}_0}, \bar{c}_9^2 = \frac{2\bar{\lambda}_2}{\bar{\rho}\bar{j}}, \overline{\omega}_0^2 = \frac{\bar{c}_3^2}{\bar{j}} = \frac{\bar{\kappa}}{\bar{\rho}\bar{j}}, \bar{\lambda}_0 = \frac{\lambda_0}{\bar{\rho}}. \tag{21}$$

Now, introducing the scalar potentials $\bar{q}$, ξ and $\in$; the vector potentials $\overline{U}, \overline{\Pi}$ as defined below:

$$\overline{U} = \nabla\bar{q} + \nabla \times \overline{U}, \overline{\Phi} = \nabla\xi + \nabla \times \overline{\Pi}, \overline{E} = -\nabla\varepsilon, \nabla \cdot \overline{U} = \nabla \cdot \overline{\Pi} = 0, \tag{22}$$

and using these into equations (16)-(20), obtained following equations

$$(\bar{c}_1^2 + \bar{c}_3^2)\nabla^2\bar{q} + \bar{\lambda}_0\bar{\psi} = \ddot{\bar{q}} \tag{23}$$

$$(\bar{c}_6^2 - \bar{c}_{10}^2)\nabla^2\bar{\psi} - \bar{c}_7^2\bar{\psi} - \bar{c}_8^2 \nabla^2\bar{q} = \ddot{\bar{\psi}} \tag{24}$$

$$(\bar{c}_2^2 + \bar{c}_3^2)\nabla^2\overline{U} + \bar{c}_3^2 \nabla \times \overline{\Pi} = \ddot{\overline{U}} \tag{25}$$

$$\bar{c}_4^2 \nabla^2 \overline{\Pi} - 2\overline{\omega}_0^2 \overline{\Pi} + \overline{\omega}_0^2 \nabla \times \overline{U} = \ddot{\overline{\Pi}} \tag{26}$$

$$(\bar{c}_4^2 + \bar{c}_5^2)\nabla^2\xi - 2\overline{\omega}_0^2 \xi = \ddot{\xi} \tag{27}$$

$$\nabla^2\varepsilon = \frac{\bar{\lambda}_2}{1 + \chi^{\overline{E}}} \nabla^2\bar{\psi} \tag{28}$$

where $\bar{c}_{10}^2 = \dfrac{2\bar{\lambda}_2^2}{\bar{\rho}\,\bar{j}_0(1+\chi^{\bar{E}})}$.

Here the equations (23) and (24) are coupled in scalar potentials $\bar{q}$ and $\bar{\psi}$, equation (28) also coupled in scalar potentials ε and $\bar{\psi}$. Equations (25) and (26) are coupled in vector potentials and Equation (27) is uncoupled in scalar potential $\bar{U}$ and $\bar{\Pi}$. Equation (27) is uncoupled in scalar potential ξ.

Formulation of the Problem

Assume the form of plane wave propagation in the positive direction of a unit vector $\bar{n}$ as given below:

$$\{\bar{q},\, \bar{\psi},\, \bar{U},\, \bar{\Pi}\} = \{a_1,\, b_1,\, A_0,\, B_0\}\exp\{ik(\bar{n}\cdot\bar{r} - \bar{V}t)\}. \tag{29}$$

Here a_1 and b_1 are using for complex constant; A_0 and B_0 are stand for complex constant vectors; $\bar{V}$, $\bar{r}$, k and ω having their usual meaning. Using the terms of $\bar{q}$ and $\bar{\psi}$ from (29) in equations (23) and (24), after that removing a_1 and b_2, consequently getting the equation

$$\bar{A}\bar{V}^4 - \bar{B}\bar{V}^2 + \bar{C} = 0 \tag{30}$$

where $\bar{A} = 1 - \dfrac{\bar{\lambda}_1\Omega}{3\bar{\kappa}}\left(\dfrac{j}{\bar{j}_0}\right)$, $\bar{B} = \left(\bar{c}_1^2 + \bar{c}_3^2 - \dfrac{\lambda_0\bar{\lambda}_0}{\lambda_1}\right)\bar{A} + \bar{c}_6^2 - c_{10}^2 + \dfrac{\lambda_0\bar{\lambda}_0}{\bar{\lambda}_1}$,

$\bar{C} = (\bar{c}_1^2 + \bar{c}_3^2)(\bar{c}_6^2 - \bar{c}_{10}^2)$ and $\Omega = \dfrac{2\bar{\omega}_0^2}{\omega}$. Equation (30) is quadratic in $\bar{V}^2$ and its roots are given by

$$\bar{V}_{1,2}^2 = \frac{1}{2\bar{A}}[\bar{B} \pm \sqrt{(\bar{B}^2 - 4\bar{A}\bar{C})}], \tag{31}$$

where '+' sign for the velocity $\bar{V}_1^2$ and '-' sign for the velocity $\bar{V}_2^2$.

It can be seen that from equations (23) and (29) the constants a_1 and b_1 are related to each other through the relation

$$b_1 = \zeta a_1 \tag{32}$$

where $\zeta = \frac{\omega^2}{\bar{\lambda}_0}\left[\frac{\bar{c}_1^2 + \bar{c}_3^2}{\bar{V}^2} - 1\right]$ is coupling parameter between $\bar{q}$ and $\bar{\psi}$.

With the help of the expression of $\bar{q}$ and $\bar{\psi}$ form the (23) into (16), the displacement vector $\bar{u}$ can be obtained as

$$\bar{u} = ika_1\bar{n}\exp\{ik(\bar{n}.\,\bar{r} - \bar{V}t)\}.$$

This shows that the vector $\bar{u}$ is parallel to the vector $\bar{n}$.

The equation (25) and (26) represent the two the set of coupled transverse waves propagating with these velocities $\bar{V}_3^2$ and $\bar{V}_4^2$ produced by Parfitt and Eringen (1969)

$$\bar{V}_{3,4}^2 = \frac{1}{2(1-\Omega)}\{\varepsilon \pm \sqrt{\varepsilon^2 - 4c_4^2}(1-\Omega)(\bar{c}_2^2 + \bar{c}_3^2)\}, \tag{34}$$

where $\varepsilon = \bar{c}_4^2 + \bar{c}_2^2(1-\Omega) + \bar{c}_3^2(1-\Omega/2)$, they have also produced the equation (27) represents a longitudinal microrotational wave propagating with the velocity

$$\bar{V}_5^2 = \bar{c}_4^2 + \bar{c}_5^2 + \frac{2\bar{\omega}_0^2}{\bar{\kappa}_2}.$$

Here, discussed the reflection and refraction phenomena of longitudinal wave at $(Z = 0)$ plane interface between micropolar elastic solid and electro-microelastic solid half-spaces. The problem is two dimensional *xz*-planes. So *x*-axis and *z*-axis are taken along the interface and along the directional vertically downward respectively. Taking the lower half-space as medium $M_1(Z > 0)$ for the micropolar elastic solid half-space and upper half-space as medium $M_2(Z < 0)$ for the electro-microelastic solid half-space.

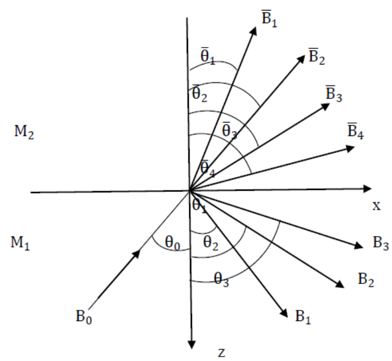


Figure 1. Geometry of the Problem.

In medium M_1

$$\phi = B_0 \exp\{ik_0(x \sin\theta_0 - z\cos\theta_0) + i\omega_1 t\}$$

$$+B_1 \exp\{ik_0(x\sin\theta_1 - z\cos\theta_1) + i\omega_1 t\}, \tag{35}$$

$$\psi = B_2 \exp\{i\delta_1(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}$$

$$+B_3 \exp\{i\delta_2(x\sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{36}$$

$$\Phi_2 = EB_2 \exp\{i\delta_1(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}$$

$$+FB_3 \exp\{i\delta_2(x\sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{37}$$

where

$$E \frac{\delta_1^2\left(\delta_1^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}},$$

$$F = \frac{\delta_2^2\left(\delta_2^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}},$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right),\ \delta_1^2 = \lambda_1^2\omega^2,\ \delta_2^2 = \lambda_2^2\omega^2,$$

where B_0, B_1, B_2, B_3 are amplitudes of incident longitudinal wave, reflected longitudinal wave, reflected coupled transverse and reflected micro-rotation waves and $\overline{B}_1, \overline{B}_2, \overline{B}_3, \overline{B}_4$ are amplitudes of refracted two coupled longitudinal waves, two sets of coupled transverse waves respectively.

In medium M_2

For the two dimensional plane using

$$\overline{u} = (\overline{u}_1, 0, \overline{u}_3), \overline{\Phi} = (0, \overline{\Phi}_2, 0), \frac{\partial}{\partial y} \equiv 0. \tag{41}$$

Putting these into (22), obtained following expressions

$$\overline{u}_1 = \frac{\partial \overline{q}}{\partial x} - \frac{\partial \overline{U}_2}{\partial z}, \overline{u}_3 = \frac{\partial \overline{q}}{\partial z} + \frac{\partial \overline{U}_2}{\partial x}, \overline{\Phi}_2 = \frac{\partial \overline{\Pi}_3}{\partial x} - \frac{\partial \overline{\Pi}_1}{\partial z}$$

$\overline{U}_2$ Stands for y-component of $\overline{U}$, $\overline{\Pi}_1$ and $\overline{\Pi}_3$ are correspondingly the x and z-components of $\overline{\Pi}$. Potentials of various reflected and refracted waves in medium M_1 and medium M_2 respectively are given by

$$\overline{q} = \sum_{p=1, 2} \overline{B}_p \exp\{i\overline{k}_p(\sin \overline{\theta}_p x - \cos \overline{\theta}_p z) - \overline{\omega}_p t\}, \tag{42}$$

$$\overline{\psi} = \sum_{p=1,2} \zeta_p \overline{B}_p \exp\{i\overline{k}_p(\sin \overline{\theta}_p x - \cos \overline{\theta}_p z) - \overline{\omega}_p t\}, \tag{43}$$

$$\overline{U}_2 = \sum_{p=3, 4} \overline{B}_p \exp\{i\overline{k}_p(\sin \overline{\theta}_p x - \cos \overline{\theta}_p z) - \overline{\omega}_p t\}, \tag{44}$$

$$\overline{\Phi}_2 = \sum_{p=3,4} \eta_p \overline{B}_p \exp\{i\overline{k}_p(\sin \overline{\theta}_p x - \cos \overline{\theta}_p z) - \overline{\omega}_p t\}, \tag{45}$$

where $i = \sqrt{-1}$ and $\overline{\omega}_p = \overline{k}_p \overline{V}_p$, $\zeta_{1, 2}$ are coupling parameters between $\overline{q}$ and $\overline{\psi}$, and $\eta_{3, 4}$ are coupling parameters between $\overline{U}_2$ and $\overline{\Phi}_2$. The expressions of ζ_i are computed earlier through equation (32) can be define as

$$\zeta_{1, 2} = \frac{\omega^2}{\overline{\lambda}_0}\left[\frac{\overline{c}_1^2 + \overline{c}_3^2}{\overline{V}_{1,2}^2} - 1\right]$$

and the expressions of η_i are computed by applying curl operator in equation

(26) and then using the equations (44) and (45). These expressions are defined as

$$\eta_{3,4} = \overline{\omega}_0^2 \left[\overline{V}_3^2, - \frac{2\overline{\omega}_0^2}{k_{3,4}^2} - \overline{c}_4^2 \right]^{-1}$$

using the equations from (22) into equations (12)-(15), the required components of stress, microrotation, microstretch and displacements are given by

$$\overline{t}_{zz} = (\overline{\lambda} + 2\overline{\mu} + \overline{\kappa})\overline{q}_{zz} + (2\overline{\mu} + \overline{\kappa})\overline{U}_{2,xz} + \overline{\lambda}\overline{q}_{,xx} + \lambda_0 \overline{\psi},$$

$$\overline{t}_{zx} = (2\overline{\mu} + \overline{\kappa})\overline{q}_{,xz} + (\overline{\mu} + \overline{\kappa})\overline{U}_{2,zz} + \overline{\mu}\overline{U}_{2,xx} + \overline{\kappa}\overline{\Phi}_2,$$

$$\overline{m}_{zy} = \overline{\gamma}\overline{\Phi}_{2,z}, \ \overline{m}_z = \left(\alpha_0 - \frac{\overline{\lambda}_2^2}{1 + \chi^{\overline{E}}} \right) \overline{\psi}_{,z}$$

$$\overline{u}_1 = \overline{q}_{,x} - \overline{U}_{2,z}, \ \overline{u}_3 = \overline{q}_{,z} + \overline{U}_{2,x}. \tag{46}$$

Boundary Conditions

The suitable boundary conditions at the interface between micropolar elastic solid halfspace and electromicroelastic solid half-space are: continuity of stresses, couple stress, microstretch, displacements and microrotations.

At the interface $z = 0$

$$t_{zz} = \overline{t}_{zz}, \ t_{zx} = \overline{t}_{zx}, \ m_{zy} = \overline{m}_{zy}, \ m_z = \overline{m}_z,$$

$$\overline{t}_{zx} = k_t(u_1 - \overline{u}_1), \ \overline{t}_{zz} = k_n(u_3 - \overline{u}_3), \ \Phi_2 = \overline{\Phi}_2, \ \psi = \overline{\psi}. \tag{47}$$

Using the equations (9)-(11) and (42)-(45), the boundary conditions given above in equation (47) are identically satisfied iff $k_i \sin\theta_i = \overline{k}_i \sin\overline{\theta}_i$ and $\omega_i = \overline{\omega}_i$, obtained the required result:

$$a_{11} = -\{\lambda + (2\mu + \kappa)\cos^2\theta_1\}, \ a_{12} = -(2\mu + \kappa)\sin\theta_2 \cos\theta_2 \frac{\delta_1^2}{k_0^2},$$

$$a_{13} = -(2\mu + \kappa)\sin\theta_3 \cos\theta_3 \frac{\delta_2^2}{k_0^2}, \ a_{14} = \left\{ \overline{\lambda} + (2\overline{\mu} + \overline{\kappa})\cos^2\overline{\theta}_1 - \frac{\overline{\lambda}_0\overline{\xi}_1}{\overline{k}_1^2} \right\} \frac{\overline{k}_1^2}{k_0^2},$$

$$a_{15} = \left\{\bar{\lambda} + (2\bar{\mu} + \kappa)\cos^2 \bar{\theta}_2 - \frac{\bar{\lambda}_0 \bar{\xi}_2}{\bar{k}_1^2}\right\}\frac{\bar{k}_1^2}{k_0^2},\ a_{16} = -(2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_3 \cos\bar{\theta}_3 \frac{\bar{k}_3^2}{k_0^2},$$

$$a_{17} = -(2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_4 \cos\bar{\theta}_4 \frac{\bar{k}_4^2}{k_0^2},\ Y_1 = -a_{11}.$$

$$a_{21} = \sin\theta_1 \cos\theta_1,\ a_{22} = -\left\{\mu(1 - 2\sin^2 \theta_2) + \kappa\cos^2 \theta_2 - \frac{\kappa E}{\delta_1^2}\right\}\frac{\delta_1^2}{k_0^1},$$

$$a_{23} = -\left\{\mu(1 - 2\sin^2 \theta_3) + \kappa\cos^2 \theta_3 - \frac{\kappa F}{\delta_2^2}\right\}\frac{\delta_2^2}{k_0^2},\ a_{24} = (2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_1 \cos\bar{\theta}_1 \frac{\bar{k}_1^2}{k_0^2},$$

$$a_{25} = (2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_2 \cos\bar{\theta}_2 \frac{\bar{k}_2^2}{k_0^2},\ a_{26} = \left\{\frac{\bar{\mu}}{\bar{k}}(\cos^2 \bar{\theta}_3 - \sin^2 \bar{\theta}_3) - \cos^2 \bar{\theta}_3 - \frac{\eta_3}{\bar{k}_3^2}\right\}\frac{\bar{\kappa}\bar{k}_3^2}{k_0^2},$$

$$a_{27} = \left\{\frac{\bar{\mu}}{\bar{\kappa}}(\cos^2 \bar{\theta}_4 - \sin^2 \theta_4) - \cos^2 \bar{\theta}_4 - \frac{\eta_4}{\bar{k}_4^2}\right\}\frac{\bar{\kappa}\bar{k}_4^2}{k_0^2},\ Y_2 = a_{21}.$$

$$a_{31} = a_{34} = a_{35} = 0,\ a_{32} = \gamma\delta_1 E\cos\theta_2,\ a_{33} = \gamma\delta_2 F\cos\theta_3,\ a_{36} = \bar{\gamma}\eta_3\bar{k}_3\cos\bar{\theta}_3,$$

$$a_{37} = \bar{\gamma}\eta_4\bar{k}_4\cos\bar{\theta}_4,\ Y_3 = a_{31}.$$

$$a_{41} = -i\sin\theta_1,\ a_{42} = i\cos\theta_2 \frac{\delta_1}{k_0},\ a_{43} = i\cos\theta_3 \frac{\delta_2}{k_0,}$$

$$a_{44} = \left[\frac{1}{k_t}\{(2\bar{\mu} + \bar{\kappa})\bar{\kappa}\sin\bar{\theta}_1 \cos\bar{\theta}_1\} + i\sin\bar{\theta}_1\right]\frac{\bar{k}_1}{k_0},$$

$$a_{45} = \left[\frac{1}{k_t}\{(2\bar{\mu} + \bar{\kappa})\bar{\kappa}\sin\bar{\theta}_2 \cos\bar{\theta}_2\} + i\sin\bar{\theta}_2\right]\frac{\bar{k}_2}{k_0},$$

$$a_{46} = \left[\frac{1}{k_t}\left\{\bar{\mu}(\cos^2 \bar{\theta}_3 - \sin^2 \bar{\theta}_3)\bar{k}_3 + \cos^2 \bar{\theta}_3\bar{k}_3 - \frac{\bar{\kappa}\eta_3}{\bar{k}_3}\right\} + i\cos\bar{\theta}_3\right]\frac{\bar{k}_3}{k_0},$$

$$a_{47} = \left[\frac{1}{k_t}\left\{\bar{\mu}(\cos^2 \bar{\theta}_4 - \sin^2 \bar{\theta}_4)\bar{k}_4 + \cos^2 \bar{\theta}_4\bar{k}_4 - \frac{\bar{\kappa}\eta_4}{\bar{k}_4}\right\} + i\cos\bar{\theta}_4\right]\frac{\bar{k}_4}{k_0},\ Y_4 = -a_{41}.$$

$$a_{51} = i\cos\theta_1,\ a_{52} = i\sin\theta_2 \frac{\delta_1}{k_0},\ a_{53} = i\sin\theta_3 \frac{\delta_2}{k_0},$$

$$a_{54} = \left[\frac{1}{k_n}\left\{\left(\bar{\lambda} + (2\bar{\mu} + \bar{\kappa})\bar{k}_1 \cos^2 \bar{\theta}_1 + \bar{\lambda} \sin^2 \bar{\theta}_1 - \frac{\bar{\lambda}_0 \bar{\xi}_1}{k_0 \bar{k}_1}\right)\right\} + i \cos \bar{\theta}_1\right] \frac{\bar{k}_1}{k_0},$$

$$a_{55} = \left[\frac{1}{k_n}\left\{\left(\bar{\lambda} + (2\bar{\mu} + \bar{\kappa})\bar{k}_2 \cos^2 \bar{\theta}_2 + \bar{\lambda} \sin^2 \bar{\theta}_2 - \frac{\bar{\lambda}_0 \bar{\xi}_2}{k_0 \bar{k}_2}\right)\right\} + i \cos \bar{\theta}_2\right] \frac{\bar{k}_2}{k_0},$$

$$a_{56} = -\left[\frac{1}{k_n}\{(2\bar{\mu} + \bar{\kappa})\bar{\kappa} \sin \bar{\theta}_3 \cos \bar{\theta}_3\} + i \sin \bar{\theta}_3\right] \frac{\bar{k}_3}{k_0},$$

$$a_{57} = -\left[\frac{1}{k_n}\{(2\bar{\mu} + \bar{\kappa})\bar{\kappa} \sin \bar{\theta}_4 \cos \bar{\theta}_4\} + i \sin \bar{\theta}_4\right] \frac{\bar{k}_4}{k_0}, \; Y_5 = a_{51}.$$

$$a_{61} = a_{64} = a_{65} = 0, \; a_{62} = E, \; a_{63} = F, \; a_{66} = -\eta_3, \; a_{67} = -\eta_4, \; Y_6 = a_{61}.$$

$$a_{7r} = 0, \; (r = 2, 3, 6, 7); \; a_{74} = \bar{\xi}_1 \cos \bar{\theta}_1 \bar{k}_1, \; a_{75} = \bar{\xi}_2 \cos \bar{\theta}_2 \bar{k}_2, \; Y_7 = a_{71}. \quad (48)$$

Prticular cases.

Case I: Transverse force stiffness $(K_n \to \infty, K_t \neq 0)$.

In this case, a system of seven non homogeneous equations as those given by equation (48) is gained with changed a_{ij} as given below

$$a_{54} = \frac{i\bar{k}_1}{k_0} \cos \bar{\theta}_1, \; a_{55} = \frac{i\bar{k}_2}{k_0} \cos \bar{\theta}_2,$$

$$a_{56} = -\frac{i\bar{k}_3}{k_0} \sin \bar{\theta}_3, \; a_{57} = -\frac{i\bar{k}_4}{k_0} \sin \bar{\theta}_4. \quad (49)$$

Case II: Welded contact $(K_n \to \infty, K_t \to 0)$. In this case, we obtained with changed a_{ij} in equation (48) as

$$a_{44} = \frac{i\bar{k}_1}{k_0} \sin \bar{\theta}_1, \; a_{45} = \frac{i\bar{k}_2}{k_0} \sin \bar{\theta}_2, \; a_{46} = \frac{i\bar{k}_3}{k_0} \cos \bar{\theta}_3, \; a_{47} = \frac{i\bar{k}_4}{k_0} \cos \bar{\theta}_4,$$

$$a_{54} = \frac{i\bar{k}_1}{k_0} \cos \bar{\theta}_1, \; a_{55} = \frac{i\bar{k}_2}{k_0} \cos \bar{\theta}_2,$$

$$a_{56} = -\frac{i\bar{k}_3}{k_0} \sin \bar{\theta}_3, \; a_{57} = -\frac{i\bar{k}_4}{k_0} \sin \bar{\theta}_4. \quad (50)$$

Discussion and Numerical Results

To solve the equations of stresses, microstretch, displacements and microrotations with the help of equations of displacements, potentials of various reflected and refracted waves, Snell's Law and boundary conditions. After that, write these equations in the matrix form such that $[a_{ij}][Z_j] = [Y_j]$, where $[a_{ij}]_{7\times 7}$, $[Z_i]_{7\times 1}$ and $[Y_i]_{7\times 1}$ are matrices of respective order. Making a program using the coefficients $[a_{ij}]$ in the computer software MATLAB and execute.

Consequently, obtain the various graphs with respects to amplitude ratios $Z_i(i = 1, 2, 3, 4, 5, 6, 7)$. Following Gauthier (1982), the physical values of constants for MES halfspace are

$$\lambda = 7.85\times 10^{11}\,\text{dyne/cm}^2,\ \mu = 6.46\times 10^{11}\,\text{dyne/cm}^2,\ \kappa = 0.0139\times 10^{11}\,\text{dyne/cm}^2,$$

$$\rho = 1.19\text{gm/cm}^3,\ \gamma = 0.0365\times 10^{11}\text{dyne},\ j = 0.0212\text{cm}^2,\ \frac{\omega^2}{{\omega_0}^2} = 20. \tag{49}$$

the physical constants for EMS half-space are given as

$$\bar{\lambda} = 7.59\times 10^{11}\,\text{dyne/cm}^2,\ \bar{\mu} = 1.89\times 10^{11}\,\text{dyne/cm}^2,\ \bar{\kappa} = 0.0149\times 10^{11}\,\text{dyne/cm}^2,$$

$$\bar{\rho} = 2.2\text{gm/cm}^3,\ \alpha_0 = 0.095\times 10^{11}\text{dyne},\ \bar{\lambda}_0 = 0.032\times 10^{11}\text{dyne/cm}^2,$$

$$\bar{\lambda}_1 = 0.030\times 10^{11}\text{dyne/cm}^2,\ \bar{\lambda}_2 = 0.3364\times 10^{11}\text{dyne},\ \bar{j}_0 = 0.0196\text{cm}^2,$$

$$\bar{\gamma} = 0.0345\times 10^{11}\text{dyne},\ \chi^{\bar{E}} = 298,\ \bar{\omega}/\bar{\omega}_0 = 10. \tag{50}$$

Figures (2)-(17) show the variations of amplitude ratios Z_i, $(i = 1, 2, 3)$ for reflected and Z_j, $(j = 1, 2, 3, 4)$ for refracted waves, when the incident wave is (LD) wave.

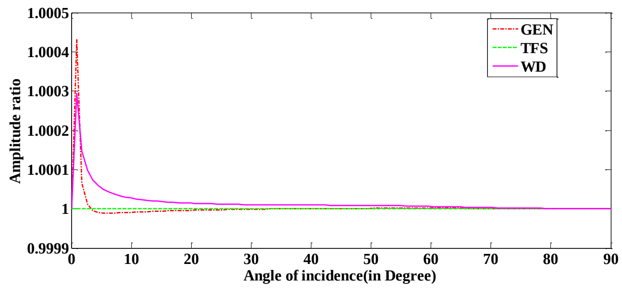


Figure (2) shows that the minimum values of amplitude ratio Z_1 are approximately at angle 4° and 90° and maximum values are attained approximately at angle 1° for the GEN and WD cases, while values are unaltered in the case of TFS from the corresponding angles. The values of Z_1 are speedily decreasing from the angles 1° to 10° and after that from the angles 10° to 90° values have minor changing.

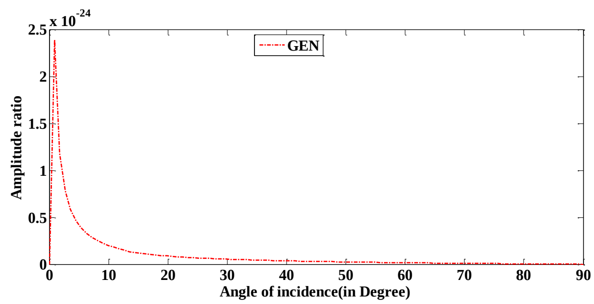


Figure (3), shows that the minimum values of amplitude ratio Z_2 are approximately at angle 0° and 90° and maximum values are attained approximately at angle 1° for the GEN case but the values of Z_1 are speedily decreasing from the angles 1° to 20° and after that from the angles 20° to 90° values are slowly decreasing.

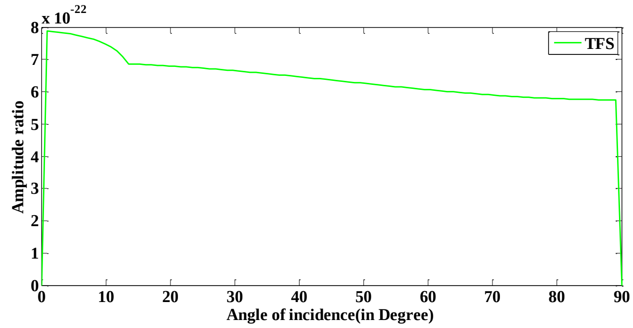


Figure (4) shown that the minimum values of amplitude ratio Z_2 are same as the figure (3) at angles 0° and 90°. The maximum value attains at angle 1° for the TFS case and after that the values are decreasing from the angles 1° to 89°, while values are suddenly down from 89° to 90°.

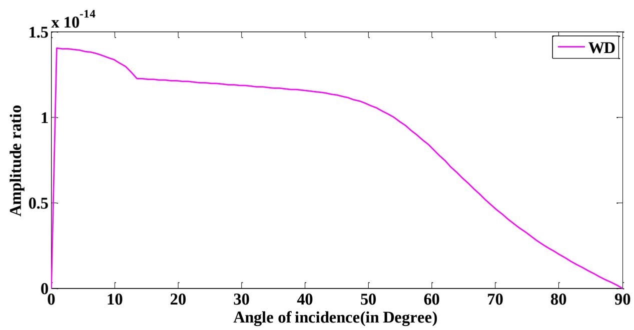
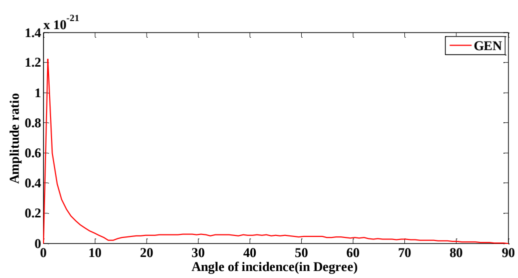


Figure (5) shows that maximum value amplitude ratio Z_2 attained at angle 1° for the TFS case and after that the values are decreasing from the angles 1° to 90° and the minimum values are same as the figure (3) at angles 0° and 90°.



In the figure (6), behavior the values of amplitude ratio Z_3 for the GEN case are just like the figure (3).

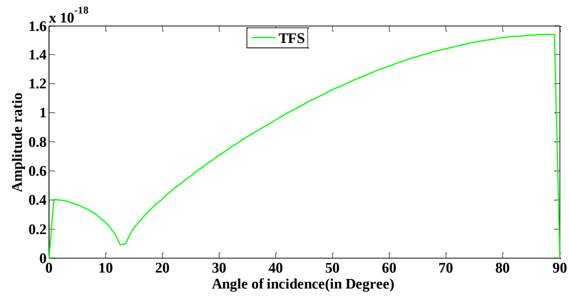


Figure (7) shows that the values of amplitude ratio Z_3 for the TFS case, are increasing from the angles 0° to 1° and then decreasing from the angles 1° to 14° and again increasing from the angles 14° to 89° and to reach their maximum value at the angle 89°. The minimum values are same as the figure (3) at angles 0° and 90°.

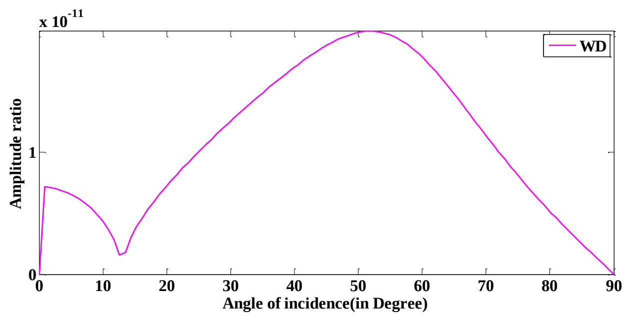


Figure (8) shows that the behavior of values of amplitude ratio Z_3 for the WD case are just like the figure (7) from the angles 0° to 14° and after that the values are increasing from the angles 14° to 52° and to get maximum value at the angle 52°. But the values are again decreasing form the angles 52° and 90°.

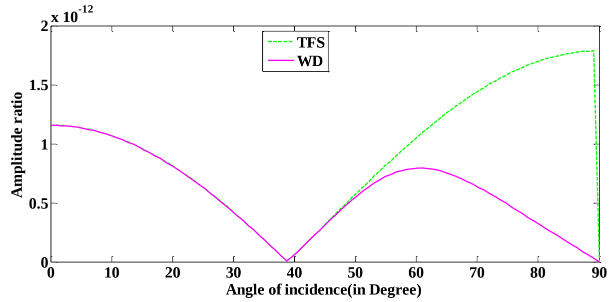


Figure (9) shows that the values of amplitude ratio Z_4 for the TFS and WD cases are decreasing from the angles 0° to 39° and then increasing from the angles 39° to 61° and decreases and approaches to its minimum value at 90° for WD case, whereas in the case of TFS values are increasing from the angles 39° to 89° and decreases and approaches to its minimum value.

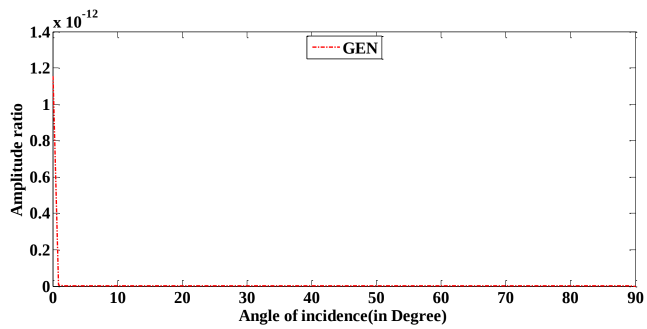
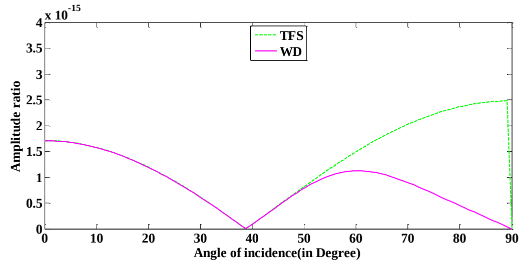
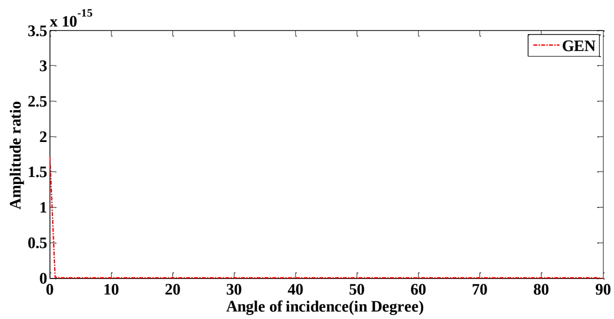


Figure (10) depicts that the maximum values of amplitude ratio Z_4 are just near about the angle 0° and after that values are speedily decreasing from the angles 0° to 1° for the GEN case while in the other values of Z_4 are negligible changes from the angles 1° to 90°.



In the figure (11), behavior the values of amplitude ratio Z_5 are same as the figure (9).



Discussion about the figure (12) for the values of amplitude ratio Z_5 is just like the figure (10).

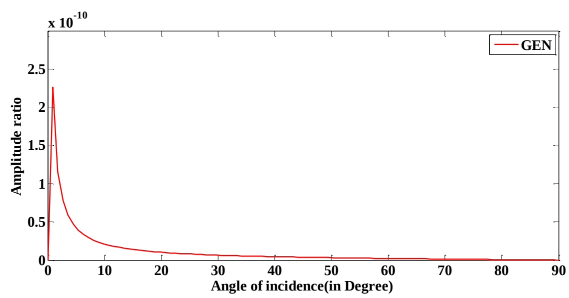
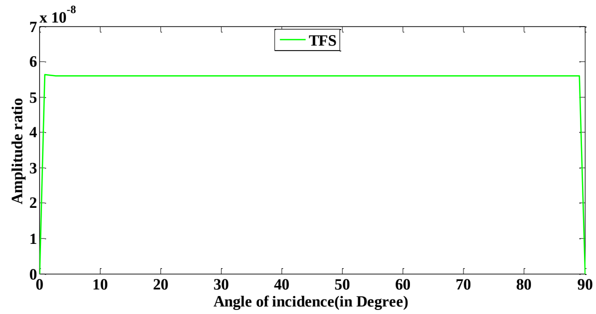


Figure (13) shows that the minimum values of amplitude ratio Z_6 are lies at angle 0° and 90° and maximum values are attained approximately at angle 1° for the GEN case but the values of Z_1 are speedily decreasing from the angles 1° to 10° and after that from the angles 10° to 90° values are slowly decreasing.



In the figure (14), the minimum values of amplitude ratio Z_6 are same the value of the figure (3) and maximum values are attained at angle 1° to 89°.

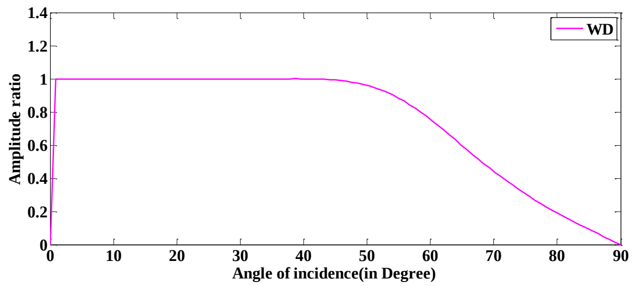
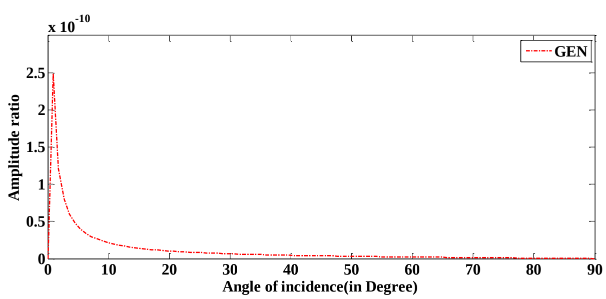
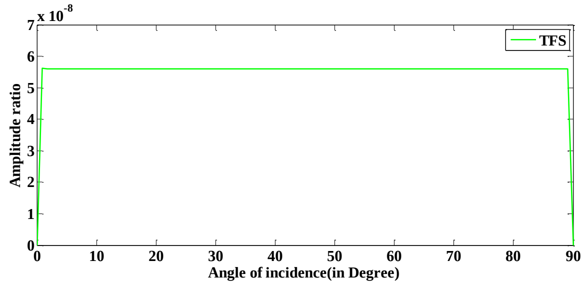
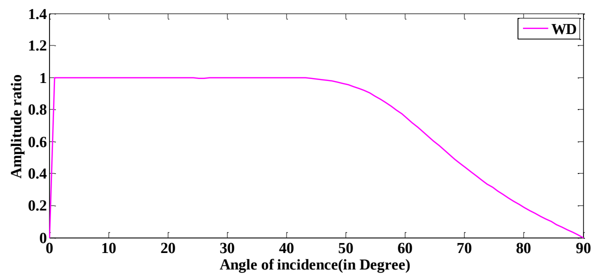


Figure (15) depicts that maximum values of amplitude ratio Z_6 are from the angle 1° to 45° and after that the values are decreasing from the angles 45° to 90° and the minimum values are same as the figure (3) at angles 0° and $90.^\circ$







Discussions about the figures (16-18) for the values of amplitude ratio Z_7 are just like the figures (13-15) respectively.

Conclusion

In the present paper, we have discussed about the plane wave propagation obliquely at an imperfect interface between MES and EMS half-spaces. Reflection and refraction coefficients of various reflected wave and refracted waves has been derived for transverse force stiffness (TFS) and welded contact (WD). The results are considered to be useful in further theoretical and observational studies of propagation of waves in more models of MES. Making the use of appropriate set of boundary conditions, the system of simultaneous equations giving the amplitudes of various reflected and refracted waves are obtained.

a. The amplitudes of various reflected and refracted waves are found to be complex valued.

b. The modulus of amplitudes of various reflected and refracted waves depend upon angle of incidence, stiffness of forces and elastic properties of materials of the medium.

c. The minimum values of almost amplitude ratios lies at the angles 0° and 90°.

d. All the amplitude ratios have different values but some of them have same behave.

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