Advances and Applications in Mathematical Sciences
Volume 21, Issue 7, May 2022, Pages 3577-3594

DIVISOR CORDIAL LABELING FOR LADDERS AND TOTAL GRAPH OF SOME GRAPHS

C. M. BARASARA and Y. B. THAKKAR

Department of Mathematics
Hemchandracharya North Gujarat University
Patan - 384265, Gujarat, India
E-mail: chirag.barasara@gmail.com

Research Scholar
Department of Mathematics
Hemchandracharya North Gujarat University
Patan - 384265, Gujarat, India
E-mail: yogesh.b.thakkar21@gmail.com

Abstract

In 1967, Rosa [5] introduced β-valuation labeling of a graph. Golomb [20] subsequently called such labeling as a graceful labeling. In 1980, Graham and Sloane [15] introduced harmonious labeling. In 1987, Cahit [8] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling. In 2011, Varatharajan et al. [16] have introduced divisor cordial labeling as a variant of cordial labeling. In this paper, we investigate divisor cordial labeling for ladder, circular ladder, Möbius ladder, total graph of path and total graph of cycle.

1. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$. For all standard terminologies and notations we follow Harary [7]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the

2010 Mathematics Subject Classification: 05C78, 05C76.
Keywords: Graph labeling, Cordial labeling, Divisor cordial labeling, Graph operation.
Received June 6, 2021; Accepted August 17, 2021

mapping is the set of vertices (edges) then the labeling is called a vertex labeling (an edge labeling).

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [9].

In 1987, Cahit [8] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling, which is defined as follows.

Definition 1.2. For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as $f : V(G) \rightarrow \{0, 1\}$ and induced edge labeling function $f^* : E(G) \rightarrow \{0, 1\}$ such that for each edge uv, $f^*(uv) = |f(u) - f(v)|$. f is called cordial labeling of graph G if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Cordial Labeling is called a Cordial Graph.

Many researchers have explored variants of cordial labeling like prime cordial labeling, product cordial labeling, divisor cordial labeling etc. In 2011, Varatharajan et al. [16] have introduced divisor cordial labeling as a variant of cordial labeling, which is defined as follows.

Definition 1.3. For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as a bijection $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ such that induced edge labeling function $f^* : E(G) \rightarrow \{0, 1\}$ is given by

$$f^*(e = uv) = \begin{cases} 1, & \text{if } f(u)/f(v) \text{ or } f(v)/f(u); \\ 0, & \text{otherwise}. \end{cases}$$

Denote the number of edges labeled with 0 and 1 by $e_f(0)$ and $e_f(1)$ respectively. f is called divisor cordial labeling of graph G if $|e_f(0) - e_f(1)| \leq 1$. The graph that admits a divisor cordial labeling is called a divisor cordial graph.

Varatharajan et al. [16, 17] have investigated divisor cordial labeling for standard graph families. Vaidya and Shah [18, 19] have proved many results related to divisor cordial labeling for some star related graphs. Barasara and Thakkar [6] have derived results related to divisor cordial labeling for some cycle and wheel related graphs. Prajapati and Patel [21] discussed divisor cordial labeling in context of Friendship graph. Murugan and Nisha [4] have studied divisor cordial labeling of star attached paths and cycles.

Murugan and Devakiruba [3] as well as Rokad and Ghodasara [1] have obtained divisor cordial labeling for some cycle related graphs. Divisor cordial labeling for duplication of graph elements is studied by Thirusangu and Madhu [12]. Devaraj et al. [11] as well as Muthaiyan and Pugalenthi [2] obtained results related to divisor cordial labeling.

Definition 1.4. Let G and H be two graphs. The cartesian product of G and H, denoted by $G \square H$, has the vertex set $V(G) \times V(H)$ and (g, h) is adjacent to (g', h') if $g = g'$ and $hh' \in E(H)$, or $h = h'$ and $gg' \in E(G)$.

Definition 1.5. The Ladder graph L_n is defined as $P_2 \square P_n$.

Definition 1.6. The Circular ladder graph CL_n is defined as $P_2 \square C_n$.

Definition 1.7. The Möbius ladder M_n is a graph obtained from the ladder $P_2 \square P_n$ by joining the opposite end vertices of two copies of P_n.

Definition 1.8. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent whenever they are either adjacent or incident in G.

In this paper, we investigate divisor cordial labeling for ladder, circular ladder, Möbius ladder, total graph of path and total graph of cycle.

2. Main Results

Theorem 2.1. *The ladder graph L_n is a divisor cordial graph.*

Proof. Let $v_1, v_4, v_5, \ldots, v_{4k}, v_{4k+1}, \ldots$ be the vertices of first path and $v_2, v_3, v_6, \ldots, v_{4k+2}, v_{4k+3}, \ldots$ be the vertices of second path of ladder L_n. Then $|V(L_n)| = 2n$ and $|E(L_n)| = 3n - 2$.

We define a bijective vertex labeling $f : V(L_n) \to \{1, 2, \ldots, 2n\}$ as follows.

$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \leq i \leq a_1$,

such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2$;

where p_1 is largest integer such that $2^{p_1} \leq 2n$,

$$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i};\text{ for } 1 \leq i \leq a_2,$$

such that $3 \times 1 \times 2^{p_2+1-i} \geq 3 \times 1 \times 1$;

where p_2 is largest integer such that $3 \times 2^{p_2} \leq 2n$,

$$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i};\text{ for } 1 \leq i \leq a_3,$$

such that $3 \times 3 \times 2^{p_3+1-i} \geq 3 \times 3 \times 1$;

where p_3 is largest integer such that $3 \times 3 \times 2^{p_3} \leq 2n$,

$$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i};\text{ for } 1 \leq i \leq a_4,$$

such that $3 \times 3^2 \times 2^{p_4+1-i} \geq 3 \times 3^2 \times 1$;

where p_4 is largest integer such that $3 \times 3^2 \times 2^{p_4} \leq 2n$,

$$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i};\text{ for } 1 \leq i \leq a_5,$$

such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1$;

where p_5 is largest integer such that $3 \times 3^3 \times 2^{p_5} \leq 2n$,

Continuing in this way up to $3^{m_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}}) = 5 \times 1 \times 2^{p_1+1-i};\text{ for } 1 \leq i \leq b_1,$$

such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1$;

where q_1 is largest integer such that $5 \times 1 \times 2^{q_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i};\text{ for } 1 \leq i \leq b_2,$$

such that $5 \times 3 \times 2^{q_2+1-i} \geq 5 \times 3 \times 1$;

where q_2 is largest integer such that $5 \times 3 \times 2^{q_2} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i};\ \text{for } 1 \leq i \leq b_3,$$

such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1$;

where q_3 is largest integer such that $5 \times 3^2 \times 2^{q_3} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_4+1-i};\ \text{for } 1 \leq i \leq b_4,$$

such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1$;

where q_4 is largest integer such that $5 \times 3^3 \times 2^{q_4} \leq 2n$.

Continuing in this way till we get at least $\left\lfloor \frac{3n-2}{2} \right\rfloor - 3$ edges with label 1.

Sub case 1. If we get $\left\lfloor \frac{3n-2}{2} \right\rfloor - 3$ edges with label 1, take $f(v_{2n-2}) = 1$.

Sub case 2. If we get $\left\lfloor \frac{3n-2}{2} \right\rfloor - 2$ edges with label 1, take $f(v_{2n}) = 1$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices, (except when one of the vertex label is 1).

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{3n-2}{2} \right\rceil$, and $e_f(1) = \left\lfloor \frac{3n-2}{2} \right\rfloor$. Thus, $|\, e_f(0) - e_f(1) \,| \leq 1$.

Hence, the ladder graph L_n is a divisor cordial graph. □

Illustration 2.2. The graph L_6 and its divisor cordial labeling is shown in Figure 1.

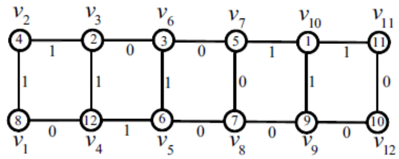

Figure 1. The graph L_6 and its divisor cordial labeling.

Theorem 2.3. *The circular ladder CL_n is a divisor cordial graph.*

Proof. Let $v_1, v_4, v_5, \ldots, v_{4k}, v_{4k+1}, \ldots$ be the vertices of an outer cycle and $v_2, v_3, v_6, \ldots, v_{4k+2}, v_{4k+3}, \ldots$ be the vertices of an inner cycle of circular ladder CL_n. Then $|V(CL_n)| = 2n$ and $|E(CL_n)| = 3n$.

We define a bijective vertex labeling $f : V(CL_n) \to \{1, 2, \ldots, 2n\}$ as follows.

$f(v_{2n}) = 1,$

$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \le i \le a_1$,

such that $1 \times 1 \times 2^{p_1+1-i} \ge 1 \times 1 \times 2$;

where p_1 is largest integer such that $2^{p_1} \le 2n$,

$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i}$; for $1 \le i \le a_2$,

such that $3 \times 1 \times 2^{p_2+1-i} \ge 3 \times 1 \times 1$;

where p_2 is largest integer such that $3 \times 2^{p_2} \le 2n$,

$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i}$; for $1 \le i \le a_3$,

such that $3 \times 3 \times 2^{p_3+1-i} \ge 3 \times 3 \times 1$;

where p_3 is largest integer such that $3 \times 3 \times 2^{p_3} \le 2n$,

$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i}$; for $1 \le i \le a_4$,

such that $3 \times 3^2 \times 2^{p_4+1-i} \ge 3 \times 3^2 \times 1$;

where p_4 is largest integer such that $3 \times 3^2 \times 2^{p_4} \leq 2n$,

$$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i}; \text{ for } 1 \leq i \leq a_5,$$

such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1$;

where p_5 is largest integer such that $3 \times 3^3 \times 2^{p_5} \leq 2n$,

Continuing in this way up to $3^{m_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i}; \text{ for } 1 \leq i \leq b_1,$$

such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1$;

where q_1 is largest integer such that $5 \times 1 \times 2^{q_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i}; \text{ for } 1 \leq i \leq b_2,$$

such that $5 \times 3 \times 2^{q_2+1-i} \geq 5 \times 3 \times 1$;

where q_2 is largest integer such that $5 \times 3 \times 2^{q_2} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i}; \text{ for } 1 \leq i \leq b_3,$$

such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1$;

where q_3 is largest integer such that $5 \times 3^2 \times 2^{q_3} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_4+1-i}; \text{ for } 1 \leq i \leq b_4,$$

such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1$;

where q_4 is largest integer such that $5 \times 3^3 \times 2^{q_4} \leq 2n$.

Continuing in this way till we get at least $\left\lfloor \frac{3n}{2} \right\rfloor$ edges with label 1.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices, (except when one of the vertex label is 1).

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{3n}{2} \right\rceil$, and $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the circular ladder CL_n is a divisor cordial graph. □

Illustration 2.4. The graph CL_6 and its divisor cordial labeling is shown in Figure 2.

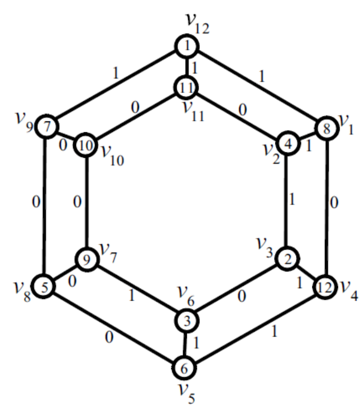

Figure 2. The graph CL_6 and its divisor cordial labeling.

Theorem 2.5. *The Möbius ladder M_n is a divisor cordial graph.*

Proof. Let $v_1, v_4, v_5, \ldots, v_{4k}, v_{4k+1}, \ldots$ be the vertices of a first cycle and $v_2, v_3, v_6, \ldots, v_{4k+2}, v_{4k+3}, \ldots$ be the vertices of a second cycle of graph M_n. Then $|V(M_n)| = 2n$ and $|E(M_n)| = 3n$.

We define a bijective vertex labeling $f : V(M_n) \to \{1, 2, \ldots, 2n\}$ as follows:

$f(v_{2n}) = 1$,

$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \leq i \leq a_1$,

such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2$;

where p_1 is largest integer such that $2^{p_1} \leq 2n$,

$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i}$; for $1 \le i \le a_2$,

such that $3 \times 1 \times 2^{p_2+1-i} \ge 3 \times 1 \times 1$;

where p_2 is largest integer such that $3 \times 2^{p_2} \le 2n$,

$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i}$; for $1 \le i \le a_3$,

such that $3 \times 3 \times 2^{p_3+1-i} \ge 3 \times 3 \times 1$;

where p_3 is largest integer such that $3 \times 3 \times 2^{p_3} \le 2n$,

$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i}$; for $1 \le i \le a_4$,

such that $3 \times 3^2 \times 2^{p_4+1-i} \ge 3 \times 3^2 \times 1$;

where p_4 is largest integer such that $3 \times 3^2 \times 2^{p_4} \le 2n$,

$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i}$; for $1 \le i \le a_5$,

such that $3 \times 3^3 \times 2^{p_5+1-i} \ge 3 \times 3^3 \times 1$;

where p_5 is largest integer such that $3 \times 3^3 \times 2^{p_5} \le 2n$,

Continuing in this way up to $3^{m_1} \le 2n$,

$f(v_{i+a_1+a_2+\ldots+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i}$; for $1 \le i \le b_1$,

such that $5 \times 1 \times 2^{q_1+1-i} \ge 5 \times 1 \times 1$;

where q_1 is largest integer such that $5 \times 1 \times 2^{q_1} \le 2n$,

$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i}$; for $1 \le i \le b_2$,

such that $5 \times 3 \times 2^{q_2+1-i} \ge 5 \times 3 \times 1$;

where q_2 is largest integer such that $5 \times 3 \times 2^{q_2} \le 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i};\ \text{for } 1 \le i \le b_3,$$

such that $5 \times 3^2 \times 2^{q_3+1-i} \ge 5 \times 3^2 \times 1$;

where q_3 is largest integer such that $5 \times 3^2 \times 2^{q_3} \le 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_3+1-i};\ \text{for } 1 \le i \le b_4,$$

such that $5 \times 3^3 \times 2^{q_4+1-i} \ge 5 \times 3^3 \times 1$;

where q_4 is largest integer such that $5 \times 3^3 \times 2^{q_4} \le 2n$.

Continuing in this way till we get at least $\left\lfloor \frac{3n}{2} \right\rfloor$ edges with label 1.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices, (except when one of the vertex label is 1).

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{3n}{2} \right\rceil$, and $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$. Thus, $|\, e_f(0) - e_f(1) \,| \le 1$.

Hence, the Möbius ladder M_n is a divisor cordial graph. □

Illustration 2.6. The graph M_6 and its divisor cordial labeling is shown in Figure 3.

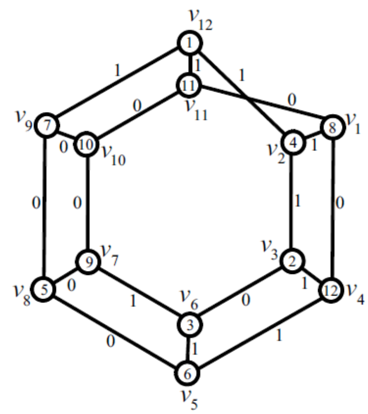

Figure 3. The graph M_6 and its divisor cordial labeling.

Theorem 2.7. *The total graph of path P_n is a divisor cordial graph.*

Proof. Let P_n be the path with vertices $v_1, v_3, \ldots, v_{2n-1}$ and edges $e_1, e_2, \ldots, e_{n-1}$. To construct the total graph $T(P_n)$, let the added vertex corresponding to edge e_i is v_{2i}, for $1 \leq i \leq n-1$. Then $|V(T(P_n))| = 2n-1$ and $|E(T(P_n))| = 4n-5$.

We define a bijective vertex labeling $f : V(T(P_n)) \to \{1, 2, \ldots, 2n-1\}$ by following two cases,

Case 1. For $n = 2$.

The graph $T(P_2)$ is isomorphic to cycle C_3 and Varatharajan et al. [16] proved that cycles are divisor cordial graph. Hence, $T(P_2)$ is divisor cordial graph.

Case 2. For $n \geq 3$.

$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \leq i \leq a_1$,

such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2$;

where p_1 is largest integer such that $2^{p_1} \leq 2n-1$,

$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i}$; for $1 \leq i \leq a_2$,

such that $3 \times 1 \times 2^{p_2+1-i} \geq 3 \times 1 \times 2$;

where p_2 is largest integer such that $3 \times 2^{p_2} \leq 2n - 1$,

$$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i}; \text{ for } 1 \leq i \leq a_3,$$

such that $3 \times 3 \times 2^{p_3+1-i} \geq 3 \times 3 \times 2$;

where p_3 is largest integer such that $3 \times 3 \times 2^{p_3} \leq 2n - 1$,

$$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i}; \text{ for } 1 \leq i \leq a_4,$$

such that $3 \times 3^2 \times 2^{p_4+1-i} \geq 3 \times 3^2 \times 1$;

where p_4 is largest integer such that $3 \times 3^2 \times 2^{p_4} \leq 2n - 1$,

$$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i}; \text{ for } 1 \leq i \leq a_5,$$

such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1$;

where p_5 is largest integer such that $3 \times 3^3 \times 2^{p_5} \leq 2n - 1$,

Continuing in this way up to $3^{m_1} \leq 2n - 1$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i}; \text{ for } 1 \leq i \leq b_1,$$

such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1$;

where q_1 is largest integer such that $5 \times 1 \times 2^{q_1} \leq 2n - 1$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i}; \text{ for } 1 \leq i \leq b_2,$$

such that $5 \times 3 \times 2^{q_3+1-i} \geq 5 \times 3 \times 1$;

where q_2 is largest integer such that $5 \times 3 \times 2^{q_2} \leq 2n - 1$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i}; \text{ for } 1 \leq i \leq b_3,$$

such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1$;

where q_3 is largest integer such that $5 \times 3^2 \times 2^{q_3} \leq 2n-1$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_3+1-i}; \text{ for } 1 \leq i \leq b_4,$$

such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1$;

where q_4 is largest integer such that $5 \times 3^3 \times 2^{q_4} \leq 2n-1$.

Continuing in this way till we get at least $\left\lfloor \frac{4n-5}{2} \right\rfloor - 4$ edges with label 1.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices, (except when one of the vertex label is 1).

Sub case 1. If we get $\left\lfloor \frac{4n-5}{2} \right\rfloor - 4$ edges with label 1, take $f(v_{2n-3}) = 1$.

Sub case 2. If we get $\left\lfloor \frac{4n-5}{2} \right\rfloor - 3$ edges with label 1, take $f(v_{2n-2}) = 1$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{4n-5}{2} \right\rceil$, and $e_f(1) = \left\lfloor \frac{4n-5}{2} \right\rfloor$. Thus $|e_f(0) - e_f(1)| \leq 1$.

Hence, the total graph of path P_n is a divisor cordial graph. □

Illustration 2.8. The graph $T(P_5)$ and its divisor cordial labeling is shown in Figure 4.

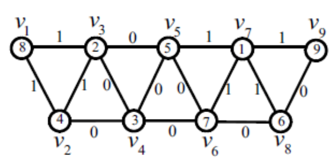

Figure 4. The graph $T(P_5)$ and its divisor cordial labeling.

Theorem 2.9. *The total graph of cycle C_n is a divisor cordial graph.*

Proof. Let the C_n be the cycle with vertices $v_2, v_4, \ldots, v_{2n}$ and edge set $e_2, e_2, \ldots, e_n$. To construct the total graph $T(C_n)$, let the added vertices corresponding to edge e_1 is v_{2i-1} for $1 \leq i \leq n$. Then $|V(T(C_n))| = 2n$ and $|E(T(C_n))| = 4n$.

We define a bijective vertex labeling $f : V(T(C_n)) \to \{1, 2, \ldots, 2n\}$ by following two cases,

Case 1. For $n = 5$.

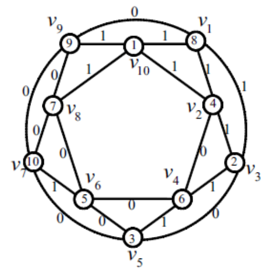

Figure 5. The graph $T(C_5)$ and its divisor cordial labeling.

Case 2. For $n \neq 5$.

$f(v_{2n}) = 1,$

$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \leq i \leq a_1$,

such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2;$

where p_1 is largest integer such that $2^{p_1} \leq 2n,$

$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i};$ for $1 \leq i \leq a_2,$

such that $3 \times 1 \times 2^{p_2+1-i} \geq 3 \times 1 \times 1;$

where p_2 is largest integer such that $3 \times 2^{p_2} \leq 2n,$

$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i};$ for $1 \leq i \leq a_3,$

such that $3 \times 3 \times 2^{p_3+1-i} \geq 3 \times 3 \times 1$;

where p_3 is largest integer such that $3 \times 3 \times 2^{p_3} \leq 2n$,

$$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i};\ \text{for } 1 \leq i \leq a_4,$$

such that $3 \times 3^2 \times 2^{p_4+1-i} \geq 3 \times 3^2 \times 1$;

where p_4 is largest integer such that $3 \times 3^2 \times 2^{p_4} \leq 2n$,

$$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i};\ \text{for } 1 \leq i \leq a_5,$$

such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1$;

where p_5 is largest integer such that $3 \times 3^3 \times 2^{p_5} \leq 2n$,

Continuing in this way up to $3^{m_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i};\ \text{for } 1 \leq i \leq b_1,$$

such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1$;

where q_1 is largest integer such that $5 \times 1 \times 2^{q_1} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i};\ \text{for } 1 \leq i \leq b_2,$$

such that $5 \times 3 \times 2^{q_2+1-i} \geq 5 \times 3 \times 1$;

where q_2 is largest integer such that $5 \times 3 \times 2^{q_2} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i};\ \text{for } 1 \leq i \leq b_3,$$

such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1$;

where q_3 is largest integer such that $5 \times 3^2 \times 2^{q_3} \leq 2n$,

$$f(v_{i+a_1+a_2+\ldots+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_4+1-i};\ \text{for } 1 \leq i \leq b_4,$$

such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1;$

where q_4 is largest integer such that $5 \times 3^3 \times 2^{q_4} \leq 2n.$

Continuing in this way till we get at least $2n-1$ edges with label 1.

If we get $2n-1$ edges with label 1, take $f(v_{2n-1}) = 2p'$ and $f(v_{2n-2}) = p'$, where p' is largest prime number such that $p' \leq n.$

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices, (except when one of the vertex label is 1).

In view of above defined labeling pattern, we have $e_f(0) = 2n$, and $e_f(1) = 2n.$ Thus, $|e_f(0) - e_f(1)| = 0.$

Hence, the total graph of cycle C_n is a divisor cordial graph. □

Illustration 2.10. The graph $T(C_6)$ and its divisor cordial labeling is shown in Figure 6.

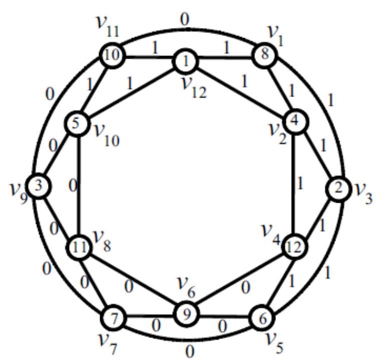

Figure 6. The graph $T(C_6)$ and its divisor cordial labeling.

3. Concluding Remarks

Maheo [13] has proved that ladder is a graceful graph and Frucht and Gallian [14] have investigated graceful labeling for circular ladder while Gallian [10] has obtained that Möbius ladder is graceful graph. In this paper, we have shown that ladder, circular ladder and Möbius ladder are divisor cordial graphs. Moreover, we have obtained divisor cordial labeling of total graph of path and total graph of cycle.

Acknowledgement

The authors are highly thankful to the anonymous referees for the kind comments and fruitful suggestions on the first draft of this paper. The second author is supported by Knowledge Consortium of Gujarat, Government of Gujarat, Ahmedabad through SHODH Scholarship-2021-23 with Ref. No.: 202001400029.

References

[1] A. H. Rokad and G. V. Ghodasara, Divisor cordial labeling of cycle related graphs, Int. J. for Research in Appl. Sci. and Eng. Tech. 3(X) (2015), 341-346.

[2] A. Muthaiyan and P. Pugalenthi, Some new divisor cordial graphs, Int. J. Math. Trends Tech. 12(2) (2014), 81-88.

[3] A. N. Murugan and G. Devakiruba, Cycle related divisor cordial graphs, Int. J. Math. Trends Tech. 12(1) (2014), 34-43.

[4] A. N. Murugan and M. T. Nisha, A study on divisor cordial labeling of star attached paths and cycles, Paripex-Indian Journal of Research 3(3) (2014), 12-17.

[5] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris, (1967), 349-355.

[6] C. M. Barasara and Y. B. Thakkar, Divisor cordial labeling for some cycle and wheel related graphs, Malaya Journal of Matematik 8(3) (2020), 966-972.

[7] F. Harary, Graph Theory, Narosa Publication House Reading, New Delhi, (1998).

[8] I. Cahit, Cordial Graphs: A weaker version of graceful and harmonious Graphs, Ars Combinatoria 23 (1987), 201-207.

[9] J. A. Gallian, A Dynamic Survey of Graph Labeling, The Electronics Journal of Combinatorics, 23, (2020), #DS6.

[10] J. A. Gallian, Labeling Prisms and prism related graphs, Congr. Numer., 59 (1989), 89-100.

[11] J. Devaraj, C. Sunitha and S. P. Reshma, On Divisor Cordial Graph, Bulletin of Pure and Appl. Sci., 37E (Math. and Stat.) (2) (2018), 290-302.

[12] K. Thirusangu and M. Madhu, Divisor Cordial Labeling in Extended duplicate Graph of Star, Bistar and Double Star, J. Appl. Sci. and Compu., VI(I) (2019), 583-594.

[13] M. Maheo, Strongly graceful graphs, Discrete. Math. 29 (1980), 39-46.

[14] R. Frucht and J. A. Gallian, Labeling Prisms, Ars Combin. 26 (1988), 69-82.

[15] R. L. Graham and N. J. A. Sloane, On additive bases and harmonious graphs, SIAM J. Alg. Discrete Methods 1 (1980), 382-404.

[16] R. Varatharajan, S. Navanaeethakrishnan and K. Nagarajan, Divisor Cordial Graphs, Int. J. of Mathematics and Combinatorics, 4 (2011), 15-25.

[17] R. Varatharajan, S. Navanaeethakrishnan and K. Nagarajan, Special Classes of Divisor Cordial Graphs, Int. Mathematical Forum, 7(35) (2012), 1737-1749.

[18] S. K. Vaidya and N. H. Shah, Some star and bistar related divisor cordial graphs, Annals Pure Appl. Math. 3(1) (2013), 67-77.

[19] S. K. Vaidya and N. H. Shah, Further results on divisor cordial labeling, Annals Pure Appl. Math., 4(2) (2013), 150-159.

[20] S. W. Golomb, How to number a graph, in Graph Theory and Computing, R. C. Read, ed., Academic Press, New York, (1972), 23-37.

[21] U. M. Prajapati and P. A. Patel, Divisor Cordial Labeling in the Context of Friendship Graph, Journal of Xidian University 14(5) (2020), 167-177.