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A NOVEL NARRATION FOR PROPAGATION OF WAVE IN RELAXING MEDIUM BY EXPANSION SCHEME

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Abstract

In this task, the Jacobi elliptic function expansion scheme {JEFES}, which is more general than the tanh function method, is derived to assemble the new wave solutions of the modal of wave's propagation of in a relaxing media. It is proposing a new method to assemble more general exact solutions of nonlinear Vakhnenko-Parkes system and its craving to look up the works which are finished. It is shown that more new solutions can be got on their limit circumstance.

Introduction

The physical phenomenon and process that take place in nature generally have complicated nonlinear features. This leads to nonlinear mathematical models for the real process. There is much interest in the practical issues involved, as well as the development of methods to investigate the associated nonlinear wave propagation. The wave phenomena are pragmatic in fluid dynamics, plasma, elastic media, optical fibers, etc. So, looking for exact travelling wave solutions particularly exact solitary wave solutions has long been a major role in the study of physical phenomena.

In recent times, an interesting and important innovation has been made by Vakhnenko and Parkes (Vakhnenko and Parkes [22]) who have verified that the reduced Ostrovsky equation (L. A. Ostrovsky, [11]).

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$$(E_t + \alpha E_x + KEE_{xx})_x = \lambda E \quad (1)$$

can be transformed to the new integrable equation

$$EE_{xxt} - E_x E_{xt} + E^2 E_t = 0 \quad (2)$$

Where, $E(x; t)$ is a function of the spatial variable x and the temporal variable t. This is the Vakhnenko-Parkes (VP) equation with power law nonlinearity and is describing propagation of waves in a relaxing medium [(V. O. Vakhnenko [23]), Xiao Hua Liu and Caixia He, [24])] this system are investigated by the improved tanh function method introduced [(E. Yasar, [5]), E. Yusufoğlu, A. Bekir [6]], auxiliary equation method (F. Kangalgil and F. Ayaz [8]), and G/G- expansion method (R. Abazari [17]). We are interested to improve the works which was done to relate of this system.

1. Jacobi Elliptic Function Expansion Method

We now present in brief the main steps of the Jacobi elliptic function expansion strategy that will be applied. Consider a given nonlinear wave equation

$$P(u, u_t, u_x, u_{tt}, u_{xx} \ldots) = 0 \quad (3)$$

can be converted to an ODE

$$P(u, u', u'', u''' \ldots) = 0 \quad (4)$$

upon using a wave variable $\xi = \alpha(x - ct)$ where α and c are wave number and wave speed, respectively. The equation (4) is after that integrated as long as all terms contain derivatives where integration constants are considered zeros. Introducing a new independent variable

$$Y = Y(\xi).$$

By the Jacobi elliptic function expansion method, $Y(\xi)$ can be expressed as a finite series in the form of Jacobi elliptic functions (R. D. Pankaj, B. Singh, A. Kumar [15]),

$$Y(\xi) = \sum_{i=0}^{n} a_i (sn(\xi))^i \quad (5)$$

is prepared and its highest degree is $O\{Y(\xi)\} = n$. $sn(\xi) = sn(\xi, m)$, $dn(\xi) = dn(\xi, m)$ and $cn(\xi) = cn(\xi, m)$ are the Jacobi elliptic function with modulus m, where $0 < m < 1$, and a_i is constant. These functions are convincing the following formulas:

$$sn^2(\xi) + cn^2(\xi) = 1, \quad dn^2(\xi) + m^2 sn^2(\xi) = 1$$

$$sn'(\xi) = \frac{d(sn(\xi))}{d\xi} = cn(\xi)dn(\xi),\ dn'(\xi) = \frac{d(dn(\xi))}{d\xi} - m^2 cn(\xi)sn(\xi),$$

$$cn'(\xi) = \frac{d(cn(\xi))}{d\xi} - dn(\xi)sn(\xi).$$

These functions degenerate into hyperbolic functions when $m \to 1$ as follows:

$$sn(\xi) \to \tanh(\xi)cn(\xi) \to \sec h(\xi)dn(\xi) \to \sec h(\xi).$$

We can select n in (5) to balance the derivative term of the highest order and the nonlinear term. So, the Jacobi elliptic function expansion method is more general than the hyperbolic tangent function expansion method.

2. Application Jacobi Elliptic Function Expansion Method

We introduce a transformation for equation (2)

$$E(x, t) = U(\xi)$$

$$\xi = (x - \omega t)$$

$$3UU'' - 3(U')^2 + U^3 = 0 \text{ (6)}$$

into (6), integrating once with respect to ξ, and setting the integration constant equal to zero yield Balancing U^3 with UU'' gives the leading order $n = 2$, so the equation (3) convert

$$U((\xi)) = a_0 + a_1 sn(\xi) + a_2 sn^2(\xi). \tag{7}$$

Substituting Equation (7) into Equation (6), we get

$$3(a_0 + a_1 sn(\xi) + a_2 sn^2(\xi))[2a_2\{1-(1+k^2)sn^2(\xi) + k^2 sn^4(\xi)\}]$$

$$+ 3(a_0 + a_1 sn(\xi) + a_2 sn^2(\xi))[\{a_1 + 2a_2 sn(\xi)\}\{k^2 sn^2(\xi) + k^2 sn^3(\xi)\}]$$

$$-3(1+k^2)(a_0 + a_1 sn(\xi) + a_2 sn^2(\xi))\{a_1 sn(\xi) + 2a_2 sn^2(\xi)\}$$

$$-3 dn^2(\xi) cn^2(\xi)(a_1 + 2a_2 sn(\xi))^2 + \{a_0 + a_1 sn(\xi) + a_2 sn^2(\xi)\}^3 = 0.$$

Computing the coefficients of power $sn(\xi)$, constant setting each of the obtained coefficients for functions to be zero, yields the set of algebraic equations in $a_0, a_1, a_2, k = m$. On solving the algebraic equations, we have the some solutions.

Solution 1. $a_2 = 0,\ a_1 = \dfrac{-2k^2}{k^2 + a_2} a_0 = 2\sqrt{3}$

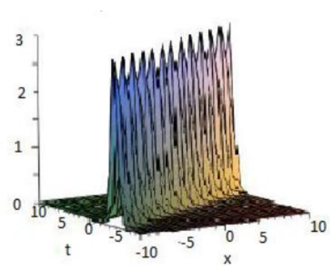
$$E_1(x, t) = 2\sqrt{3} - \left(\frac{2k^2}{k^2 + a_2}\right) sn((x - \omega t)). \tag{8}$$

Solution 2.

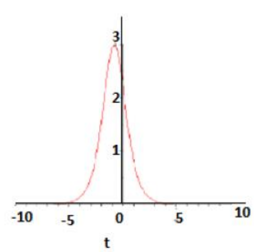
$$a_2 = -24k^2,\ a_1 = \frac{-2k^2}{k^2 + a_2},\ a_0 = \frac{-(1+k^2) \pm \sqrt{(1+k^2)^2 + 8a_2}}{2}$$

$$E_2(x, t) = \left(\frac{-(1+k^2) \pm \sqrt{(1+k^2)^2 + 8a_2}}{2}\right) - \left(\frac{2k^2}{k^2 + a_2}\right)$$

$$sn(x - \omega t) - a_2 sn^2(x - \omega t) \tag{9}$$



(a) Shape of in 3D



(b) Shape of in 2D

For $E_2(x, t) - 10 \leq x \leq 10$; the computation is done for $-10 \leq t \leq 10$, with $k \to 1$, $\omega = 0.05$.

Solution 3. $a_1 = 0\, a_3 = -3k^2 a_0 = -6a_2$

$$E_3(x, t) = -6a_2 - a_2 sn^2(x - \omega t). \tag{10}$$

Solution 4. $a_0 = 0\, a_1 \neq 0\, a_2 = \dfrac{k^2 - a_1 k^2}{3k^2 + a_1}$

$$E_4(x, t) = a_1 sn(x - \omega t) + \left(\frac{k^2 - a_1 k^2}{3k^2 + a_1}\right) sn^2(x - \omega t). \tag{11}$$

The limit condition on $m = k \to 1$

- Then solution 1

$$E_1(x, t) = 2\sqrt{3} - \left(\frac{2k^2}{k^2 + a_2}\right) sn(\xi)$$

is convert in

$$E_1(x, t) = 2\sqrt{3} - 2\tanh(x - \omega t). \tag{12}$$

- When $k \to 1$ then solution-1 convert in the form of

$$E_1(x, t) = 2\sqrt{3} - 2\tan(x - \omega t) \tag{13}$$

- When $k \to 1$ Solution 3 convert in the form of

$$E_3(x, t) = -6a_2 - a_2 \tanh^2(x - \omega t). \tag{14}$$

This is closed to as (V. O. Vakhnenko [21])

- When $k \to 1$ Solution 4 convert in the form of

$$E_4(x, t) = \left[a_1 \tanh(x - \omega t) + \left(\frac{k^2 - a_1 k^2}{3k^2 + a_1}\right)\tanh^2(x - \omega t)\right]. \tag{15}$$

This is closed to as (F. Kangalgil and F. Ayaz [8]).

3. Conclusion

The Jacobi elliptic function expansion method is proposed and applied to Vakhnenko-Parkes Equation. Using this method, we found some new solutions of Jacobi elliptic function type. In the limiting case of the Jacobi elliptic function [namely, modulus setting 0 or 1], the solutions are completely new and have not found in earlier. But the solutions found in the (Xiao Hua Liu and Caixia He, [24]), (F. Kangalgil and F. Ayaz [8]), (V. O. Vakhnenko [21]) are the closed to our obtain solutions 1, 3 and 4. By means of this scheme, we found a new solution-2 of the above-mentioned equation (9). Therefore, this scheme may be easily applied to solve the coupled NLPDEs and provides some new solutions. Actually, this method may be applied to obtain the solutions and classify modulational instability to more nonlinear wave equations, as long as the odd- and even-order derivative terms do not coexist in the nonlinear wave equations. The solutions obtained in this article have been checked by putting them back into the original equation and found correct.

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