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SPHERICAL SYMMETRIC DISTRIBUTION OF WET DARK FLUID ADMITTING CONFORMAL MOTIONS

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Abstract

In this article, we have studied the role of wet dark fluid (WDF) in static spherical symmetric space-time admitting one parameter group of conformal motions in the presence of cosmological constant Λ. The various properties of the solutions are also discussed.

1. Introduction

In modern astrophysics and cosmology, understanding the nature of dark energy is one of the most challenging problems. Observational data like la Supernovae suggest that the universe is dominated by two dark components containing dark energy (DE) and dark matter (DM). Dark energy with negative pressure is used to explain the present cosmic accelerating expansion while dark matter is used to explain galactic curves and large-scale structure formation [Chirde and Rahate [5]].

Recent cosmological observations, such as type I supernovae (SNeIa) [Riess et al. [21]], Sloan Digital Sky Survey (SDSS) [Tegmark et al. [27]], Wilkinson Microwave Anisotropy Probe (WMAP) [Nolta et al. [20]; Hinshaw et al. [14]], contradict the matter dominated universe with decelerating

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expansion indicating that our universe experiences accelerated expansion. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on [Mishra and Sahoo [18]].

There is a new matter for dark energy called Wet Dark Fluid (WDF). This model is in the spirit of generalized Chaplygin gas (GCG), where a physically motivated equation of state is offered with properties relevant for the dark energy problem [Mishra and Sahoo [18], Chaubey [4]]. The cosmological models with wet dark fluid have been discussed by many researchers [Mishra and Sahoo [19], Samanta et al. [22-23], Mete et al. [17], Shobhane and Deo [18], Katore et al. [16], Deo and Singh [9], Chirde and Kadam [6], Deo et al. [10], Mahanta and Sheikh (2017), Dagwal [8]].

General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein's equations. Symmetries of geometrical/Physical relevant quantities of this theory are known as collineations and the most useful collineation is conformal killing vector defined by

$$\mathcal{L}_{\xi} g_{ij} = \xi_{i;j} + \xi_{j;i} = \psi g_{ij}, \psi = \psi(x^i),$$

where $\mathcal{L}_{\xi}$ signifies the Lie derivative along ξ^i and $\psi = \psi(x^i)$ is the conformal factor. In particular, ξ is a special conformal killing vector, if $\psi_{;ij} = 0$ and $\psi_{,i} \neq 0$. Here (;) and (,) denote covariant and ordinary derivatives, respectively.

Conformal killing vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations and hence many of the authors [Herreraetal. [12], Herrera and Leon [13], Coley and Tupper [7], Sharif [24], Yavuz et al. [29], Yilmaz et al. [28], Aktas and Yilmaz [1], Kandalkar et al. [15], Shobhane and Deo [26]] have been studied conformal collineations.

Motivated with the work of these authors, we have examined the wet dark fluid matter in the static spherical symmetric space-time admitting one parameter group of conformal motions.

The paper is outlined as follows:

In section 2, we have obtained Einstein field equations for static spherical symmetric distribution of wet dark fluid admitting one-parameter group of conformal motions.

In Section 3, the solutions of the Einstein field equations are obtained for wet dark fluid. At the end, the properties of the solutions obtained are discussed in concluding section 4.

2. Field Equations

The most general static spherically symmetric line element in isotropic form [Banerjee and Santos [2] and Hajj-Boutros [11]] is given by

$$ds^2 = e^{\nu(r)}dt^2 - e^{\omega(r)}(dr^2 + r^2 d\Omega^2), \tag{1}$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2, \; x^{1, 2, 3, 4} = r, \theta, \phi, t.$$

The energy momentum tensor for wet dark fluid (WDF) is given by

$$T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j - p_{WDF}\, g_{ij} \tag{2}$$

together with

$$g_{ij}u^i u^j = 1, \tag{3}$$

where u^i is the four-velocity vector of the fluid, p_{WDF} and ρ_{WDF} are the pressure and the energy density of wet dark fluid respectively.

Einstein field equations can be expressed as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \wedge g_{ij} = -8\pi G T_{ij}, \tag{4}$$

where Λ is the cosmological constant.

Here, we shall use geometrized units so that $8\pi G = c = 1$.

Then using equations (1), (2) and (4), we get

$$e^{-\omega}\left[\frac{\omega'+\nu'}{r}+\frac{\omega'}{2}\left(\upsilon'+\frac{\omega'}{2}\right)\right]+\Lambda=p_{WDF} \tag{5}$$

$$\frac{1}{2}e^{-\omega}\left[(\omega''+\nu'')+\left(\frac{\omega'+\nu'}{r}\right)+\frac{\nu'^2}{4}\right]+\Lambda=p_{WDF} \tag{6}$$

and

$$-e^{-\omega}\left[\omega''+\frac{2\omega'}{r}+\frac{\omega'^2}{4}\right]-\Lambda=\rho_{WDF}, \tag{7}$$

where primes denote differentiation w.r.t. r.

3. Solutions of Field Equations

Now we shall assume that space-time admits one-parameter group of conformal motions [Aktas and Yilmaz [1]], that is

$$\mathcal{L}_\xi g_{ij}=\xi_{i;j}+\xi_{j;i}=\psi g_{ij}, \tag{8}$$

where $\mathcal{L}_\xi$ signifies the Lie derivative along ξ^i and Ψ is an arbitrary function of r. Using equations (1) and (8) by virtue of spherical symmetry, we get the following expressions:

$$\xi^1\nu'=\psi, \tag{9}$$

$$\xi^1=\frac{\psi r}{r\omega'+2}, \tag{10}$$

$$\omega'\xi^1+2\xi^1_{,1}=\psi, \tag{11}$$

and

$$\xi^2=\xi^3=0,\ \xi^4=\alpha=\text{constant}, \tag{12}$$

where comma denotes partial derivative.

Using equations (9), (10) and (11), we obtain

$$\omega'=\nu'-\frac{2}{r} \tag{13}$$

and

$$\xi^1 = kr, \tag{14}$$

where $k(>0)$ is an arbitrary constant.

Using equations (9), (13) and (14), we get

$$\nu' = \frac{\psi}{kr} \tag{15}$$

and

$$\omega' = \frac{\psi}{kr} - \frac{2}{r}. \tag{16}$$

Further, on integration equation (13) gives

$$e^{\omega} = \frac{e^{\nu}}{a^2 r^2}, \tag{17}$$

Where $a(>0)$ is an arbitrary constant and $r > 0$.

Using the equations (5) and (6), we get

$$\frac{2\psi'}{kr} - \frac{\psi^2}{k^2 r^2} + \frac{2}{r^2} = 0 \tag{18}$$

The general solution of differential equation (18) is given by

$$\psi = \sqrt{2}\, k \left[\frac{1 + (br)^{\sqrt{2}}}{1 - (br)^{\sqrt{2}}} \right], \tag{19}$$

Where $b(>0)$ is an arbitrary constant and $r \neq \frac{1}{b}$.

Then using equation (15)-(17) and (19), we get

$$e^{\nu} = \frac{(cr)^{\sqrt{2}}}{[1 - (br)^{\sqrt{2}}]^2} \tag{20}$$

and

$$e^{\omega} = \frac{(cr)^{\sqrt{2}}}{a^2 r^2 [1 - (br)^{\sqrt{2}}]^2}, \tag{21}$$

where $c(>0)$ is an arbitrary constant.

For simplicity, we denote

$$B = (br)^{\sqrt{2}} \text{ and } C = (cr)^{\sqrt{2}}. \tag{22}$$

Then (19), (20) and (21) take the form:

$$\psi = \sqrt{2}k\left(\frac{1+B}{1-B}\right), \tag{23}$$

$$e^{\nu} = \frac{C}{(1-B)^2} \tag{24}$$

and

$$e^{\omega} = \frac{C}{a^2 r^2 (1-B)^2}. \tag{25}$$

Using equations (5) and (7), we obtain

$$\rho_{WDF} = \frac{e^{-\omega}}{4k^2 r^2}(8k^2 - 3\psi^2) - \Lambda \tag{26}$$

and

$$p_{WDF} = \frac{e^{-\omega}}{4k^2 r^2}(3k^2 - 4k^2) + \Lambda. \tag{27}$$

We have

$$p_{WDF} + \rho_{WDF} = \frac{e^{-\omega}}{r^2} \geq 0. \tag{28}$$

This implies that wet dark fluid will not violate the strong energy condition. Further,

$$\rho_{WDF} - p_{WDF} = \frac{3e^{-\omega}}{2k^2 r^2}(2k^2 - \psi^2) - 2\Lambda \geq 0. \tag{29}$$

The equation of state for wet dark fluid is given by

$$p_{WDF} = \gamma(\rho_{WDF} - \rho^*), \tag{30}$$

Where the parameters γ and ρ^* are taken to be positive and $0 < \gamma < 1$, and it is good approximation for many fluids including water, where the internal attraction of the molecules make negative pressure possibly.

Using equations (23)-(27) and (30), we get

$$\Lambda = -\frac{a^2[(4-n)(1+B^2)+12(4+n)B]}{2(4+n)C} \leq 0 \tag{31}$$

and

$$\rho^* = \frac{a^2(4-n)}{n} \geq 0, \tag{32}$$

Where $\gamma = \frac{n}{4}$, $0 < n < 4$ and B, C are given by (22).

Using (1), the space-time geometry, that is, the line element is given by

$$ds^2 = \frac{(cr)^{\sqrt{2}}}{[1-(br)^{\sqrt{2}}]^2}\left[dt^2 - \frac{1}{a^2r^2}(dr^2 + r^2d\Omega^2)\right]. \tag{33}$$

4. Conclusion

From space-time (33), it is clear that there are two singularities; one at $r = 0$ and another at $r = \frac{1}{b}$. Since $B - 1 < B^2$, both $p_{WDF} > 0$ and $\rho_{WDF} > 0$. At $r = \frac{(4+n) \pm \sqrt{(4+3n)(4-n)}}{2nb^{\sqrt{2}}}$, $p_{WDF} = 0$, but for real positive value of r, $\rho_{WDF} \neq 0$. Further, wet dark fluid does not violate the strong energy condition as $p_{WDF} + \rho_{WDF} \geq 0$. As $r \to 0$, $p_{WDF} \to \infty$ and $\rho_{WDF} \to \infty$ and $r \to \infty$, $p_{WDF} \to \infty$ and $\rho_{WDF} \to \infty$.

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