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A NEW TECHNIQUE TO SOLVE AN UNBALANCED ASSIGNMENT PROBLEM USING PENTAGONAL FUZZY NUMBER

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Abstract

In this paper, an unbalanced assignment problem is solved using Ranking of Pentagonal Fuzzy Number (PFN). The First anticipated unbalanced assignment problem is articulated to a crisp assignment and solved by using Hungarian method and using Ranking of Pentagonal Fuzzy Number (PFN). Then Numerical examples are solved and proved. We further to resolve the fuzzy salesman problem via Hungarian method.

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1. Introduction

Fuzzy sets have been introduced by Lofti A. Zadeh [8] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0, 1]. The Fuzzy Assignment problem is a special type of fuzzy linear programming problem and it is a subclass of fuzzy transportation problem. The Fuzzy Assignment can be started as follows:

Let n number of jobs is performed by number of persons, where the costs depend on the specific assignments. Each job must be assigned to one and only one worker and each worker has to perform one and only one job. The problem is to find such an assignment so that the total cost is optimized. The fuzzy assignment problem can be applied to $n \times n$ fuzzy cost matrix (C_{ij}), where C_{ij} represents the fuzzy cost associated with worker $(i = 1, 2, \ldots, n)$ who has performed job $(j = 1, 2, \ldots, n)$. The fuzzy unbalanced assignment problems can be solved by the method proposed for unbalance assignment method. The unbalanced assignment problem can be changed to balance assignment problem and after solving the problem by assignment technique we use the method of pentagonal fuzzy number method.

The paper organized as follows, Firstly in section 2, we recall the definition of pentagonal fuzzy number and some operations on pentagonal fuzzy numbers (PFNs). In section 3, we have reviewed the definition of Unbalanced Assignment problem to change into Balanced Assignment problem. In section 4, we have the Numerical example are present and verified. Finally in section 5, conclusion is included.

2. Preliminaries

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1. fuzzy Set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs $A = \{(x, \mu_A(x)); x \in X\}$.

Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2. Membership Function

Let X denotes the universal set of the membership function.

$$\mu_A = \begin{cases} 1, & x \in A \\ 0, & x \notin A. \end{cases}$$

Definition 2.3. Fuzzy Number

A fuzzy set $\widetilde{A}$ defined on the set of real number R is said to be fuzzy number if its Membership function has the following characteristics

i $\widetilde{A}$ is normal

ii $\widetilde{A}$ is convex

iii The support of $\widetilde{A}$ is closed and bounded then $\widetilde{A}$ is called fuzzy number.

Definition 2.4. α of a fuzzy number

The α-cut of a fuzzy number $A(X)$ is defined as $A(\alpha) = \{X; \mu(x) \geq \alpha; \alpha \in [0, 1]\}$.

Definition 2.5. Pentagonal fuzzy number

A fuzzy number $\widetilde{A}^{PL} = (a_1, a_2, a_3, a_4, a_5)$ is said to be a pentagonal fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}^{PL}}(x) = 1;\ x = a_3 \begin{cases} 0; & x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}; & a_1 \leq x \leq a_2 \\ \dfrac{x - a_2}{a_3 - a_2}; & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}; & a_3 \leq x \leq a_4 \\ \dfrac{a_5 - x}{a_5 - a_{43}}; & a_4 \leq x \leq a_5 \\ 0, & x \geq a_5 \end{cases}$$

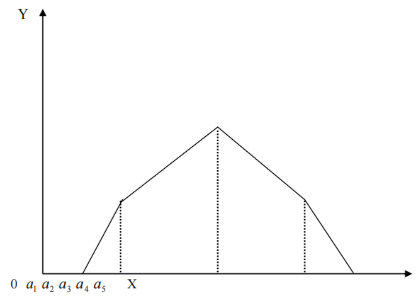

Figure 1. Pentagonal Fuzzy Number.

Definition 2.12. Ranking function

We defined a ranking function $\Re : F(R) \to R$ which maps each fuzzy numbers to real line $F(R)$ represent the set of all pentagonal fuzzy number. If R be any linear ranking function

$$\Re(\widetilde{A}^{PL}) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}\right).$$

Also we defined orders on $F(R)$ by

$\Re(\widetilde{A}^{PL}) \geq \Re(\widetilde{B}^{PL})$ if and only if $\widetilde{A}^{PL} \geq_R \widetilde{B}^{PL}$

$\Re(\widetilde{A}^{PL}) \leq \Re(\widetilde{B}^{PL})$ if and only if $\widetilde{A}^{PL} \leq_R \widetilde{B}^{PL}$

$\Re(\widetilde{A}^{PL}) = \Re(\widetilde{B}^{PL})$ if and only if $\widetilde{A}^{PL} =_R \widetilde{B}^{PL}$.

Definition 2.13. Robust's Ranking Method

Robust's ranking technique which satisfies compensation given a convex fuzzy number, the Robust's ranking index is defined as $R(C)\int_0^1 (C_\alpha^L, C_\alpha^L) d\alpha$ where (C_α^L, C_α^L) is the α level cut of the fuzzy number.

Definition 2.14. Arithmetic operations on pentagonal fuzzy numbers (PFNs)

Let $\widetilde{A}^{PL} = (a_1, a_2, a_3, a_4, a_5)$ and $\widetilde{B}^{PL} = (b_1, b_2, b_3, b_4, b_5)$ be pentagonal fuzzy numbers (PFNs) then we defined,

Addition

$$\widetilde{A}^{PL} + \widetilde{B}^{PL} = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4,\ a_5 + b_5)$$

Subtraction

$$\widetilde{A}^{PL} - \widetilde{B}^{PL} = (a_1 - b_5,\ a_2 - b_4,\ a_3 - b_3,\ a_4 - b_2,\ a_5 - b_1)$$

Multiplication

$$\widetilde{A}^{PL} \times \widetilde{B}^{PL} = (a_1\Re(B),\ a_2\Re(B),\ a_3\Re(B),\ a_4\Re(B),\ a_5\Re(B))$$

where $\Re(\widetilde{B}^{PL}) = \left(\dfrac{b_1 + b_2 + b_3 + b_4 + a_5}{5}\right)$ or $\Re(\widetilde{b}^{PL}) = \left(\dfrac{b_1 + b_2 + b_3 + b_4 + b_5}{5}\right)$.

Division

$$\widetilde{A}^{PL} / \widetilde{B}^{PL} = \left(\frac{a_1}{\Re(\widetilde{B}^{PL})}, \frac{a_2}{\Re(\widetilde{B}^{PL})}, \frac{a_3}{\Re(\widetilde{B}^{PL})}, \frac{a_4}{\Re(\widetilde{B}^{PL})}, \frac{a_5}{\Re(\widetilde{B}^{PL})}\right)$$

where $\Re(\widetilde{B}^{PL}) = \left(\dfrac{b_1 + b_2 + b_3 + b_4 + b_5}{5}\right)$ or $\Re(\widetilde{b}^{PL}) = \left(\dfrac{b_1 + b_2 + b_3 + b_4 + b_5}{5}\right)$.

Scalar multiplication

$$K\widetilde{A}^{PL} = \begin{cases} (ka_1,\ ka_2,\ ka_3,\ ka_4,\ ka_5) \text{ if } K \geq 0 \\ (ka_5,\ ka_4,\ ka_3,\ ka_2,\ ka_1) \text{ if } k < 0 \end{cases}.$$

Definition 2.15. Mathematical formulation of Fuzzy Assignment Problem

Mathematically, the fuzzy assignment problem is,

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}.$$

Subject to the constraints;

$$\sum_{i=1}^{n} x_{ij} = 1;\ i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1;\ j = 1, 2, \ldots, n$$

$$x_{ij} = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ person is assigned } j^{\text{th}} \text{ work} \\ 0, & \text{otherwise.} \end{cases}$$

Where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

Definition 2.16. Mathematical formulation of the Assignment Problem

Consider s problem of assignment of n resources (persons) to n-activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (C_{ij}) is given as follows

Activities

$$\begin{array}{ll} & \begin{array}{ccccc} A_1 & A_2 & A_3 & \ldots & A_n \end{array} \\ \begin{array}{l} \text{Available} \\ \text{Resource} \\ \text{Required} \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_4 \end{array} & \begin{bmatrix} C_{11} & C_{12} & C_{13} & \ldots & C_{1n} \\ C_{21} & C_{22} & C_{23} & \ldots & C_{2n} \\ C_{31} & C_{32} & C_{33} & \ldots & C_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{n1} & C_{n2} & C_{n3} & \ldots & C_{nn} \end{bmatrix} \begin{array}{c} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{array} \\ & \begin{array}{ccccc} 1 & 1 & 1 & \ldots & 1 \end{array} \end{array}$$

This cost matrix is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destination is unity (due to the fact that assignments are made on a one-to-one basis).

Let X_{ij} denotes the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is assigned to activity } j \\ 0, & \text{otherwise.} \end{cases}$$

Then the Mathematical formulation of the Assignment Problem is

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}.$$

Subject to the constraints;

$$\sum_{i=1}^{n} x_{ij} = 1;\ i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1;\ j = 1, 2, \ldots, n$$

3. Unbalanced Assignment Problem to Change into Balanced Assignment Problem

The number of rows is not equal to the number of columns, then the problem is termed as unbalanced assignment problem then this problem changed into balanced assignment problem as follows necessary number of dummy rows/columns are added such that the cost matrix is a square matrix, the values for the entries in the dummy rows/columns are assumed to be zero.

4. Numerical Example

Let us consider a fuzzy unbalanced assignment problem with rows representing four area A_1, A_2, A_3, A_4 and columns representing the salesman's B_1, B_2, B_3. The cost matrix $(\widetilde{C_{ij}}^{PL})$ is given whose elements are pentagonal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$(\widetilde{C_{ij}}^{PL}) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} (-2,-1,1,2,5) & (-2,1,2,3,6) & (-2,-1,1,3,4) \\ (-3,-2,2,3,5) & (-4,-1,2,3,5) & (-2,-1,1,2,5) \\ (-2,1,2,3,6) & (-2,-1,1,3,4) & (-3,-2,2,3,5) \\ (-5,-4,4,5,10) & (-2,1,2,3,6) & (-4,-1,2,3,5) \end{pmatrix} \end{array}.$$

Solution:

The given problem is a fuzzy unbalanced assignment problem. We have to change in to the fuzzy balanced problem as follows.

$$(\widetilde{C}_{ij}^{PL}) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{pmatrix} (-2, -1, 1, 2, 5) & (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) & (0, 0, 0, 0. 0) \\ (-3, -2, 2, 3, 5) & (-4, -1, 2, 3, 5) & (-2, -1, 1, 2, 5) & (0, 0, 0, 0, 0) \\ (-2, 1, 2, 3, 6) & (-2, -1, 1, 3, 4) & (-3, -2, 2, 3, 5) & (0, 0, 0, 0, 0) \\ (-5, -4, 4, 5, 10) & (-2, 1, 2, 3, 6) & (-4, -1, 2, 3, 5) & (0, 0, 0, 0, 0) \end{pmatrix} \end{array}.$$

The fuzzy balanced assignment problem can be formulated in the following mathematical programming form

$$Min\{R(-2, -1, 1, 2, 5)x_{11} + R(-2, 1, 2, 3, 6)x_{12} + R(-2, -1, 1, 3, 4)x_{13} + R(0, 0, 0, 0, 0)$$

$$x_{14} + R(-, -2, 2, 3, 5)x_{21} + R(-4, -1, 2, 3, 5)x_{22} + R(-2, -1, 1, 2, 5)x_{23}$$

$$+R(0, 0, 0, 0, 0)x_{24} + R(-2, 1, 2, 3, 6)x_{31} + R(-2, -1, 1, 3, 4)x_{32} + R(-3, -, 2, 3, 5)$$

$$x_{33} + R(0, 0, 0, 0, 0)x_{34} + R(-5, -, 4, 5, 10)x_{41} + R(-2, 1, 2, 3, 6)x_{42} +$$

$$R(-4, -, 2, 3, 5)x_{43} + R(0, 0, 0, 0, 0)x_{44}\}.$$

Subject to the constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$

$$x_{ij} \in [0, 1].$$

Now we conclude $R(-2, -1, 1, 2, 5)$ by applying Robust's ranking method.

The membership function of the pentagonal fuzzy number $(-2, -1, 1, 2, 5)$ is

$$\mu_{\widetilde{A}^{PL}}(x) = \begin{cases} 0; \ x \le -2 \\ \dfrac{x+2}{3}; \ -2 \le x \le -1 \\ \dfrac{x+1}{2}; \ -1 \le x \le 1 \\ 1; \ x = 1 \\ \dfrac{2-x}{1}; \ 1 \le x \le 2 \\ \dfrac{5-x}{3}; \ 2 \le x \le 5 \\ 0; \ x \ge 5 \end{cases}$$

Ranking of pentagonal fuzzy number

$$\mathfrak{R}(\widetilde{A}^{PL}) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}\right)$$

$$R(C_{11}) = \frac{-2 -1 + 1 + 2 + 5}{5} = \frac{5}{5} = 1.$$

Similarly, the ranking of pentagonal fuzzy number for the fuzzy costs $(\widetilde{C_{ij}}^{PL})$ are calculated as follows:

$$(\widetilde{C_{ij}}^{PL}) = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

Row reduction:

$$(\widetilde{C_{ij}}^{PL}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

Column reduction:

$$(\widetilde{C_{ij}}^{PL}) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

By using Hungarian Assignment Method:

$$(\widetilde{C_{ij}}^{PL}) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

$$(\widetilde{C_{ij}}^{PL}) = \begin{pmatrix} (0) & 2 & 1 & 0 \\ 1 & (0) & 0 & 0 \\ 2 & 1 & (0) & 0 \\ 2 & 2 & 1 & (0) \end{pmatrix}.$$

The optimal Assignment $A_1 \to B_1,\ A_2 \to B_2,\ A_3 \to B_3,\ A_4 \to B_4$.

The optimal total minimum cost $= Rs. 1 + 1 + 1 + 0 = Rs. 3$.

The fuzzy optimal Assignment $A_1 \to B_1$, $A_2 \to B_2$, $A_3 \to B_3$, $A_4 \to B_4$.

The fuzzy optimal total minimum cost $= \widetilde{C_{11}}^{PL} + \widetilde{C_{22}}^{PL} + \widetilde{C_{33}}^{PL} + \widetilde{C_{44}}^{PL}$

$$= R(-2, -1, 1, 2, 5) + R(-4, -1, 2, 3, 5) + R(-3, -2, 2, 3, 5) + R(0, 0, 0, 0, 0)$$

$$= R(-9, -4, 5, 8, 15) = Rs. 3.$$

5. Conclusion

In this article, the fuzzy unbalanced assignment problem has been transformed into crisp assignment problem using Ranking of Pentagonal fuzzy number. Numerical example shows that by using this method we can have the optimal assignment as well as the fuzzy optimal total cost. By using Ranking of PFN method, we have shown that the total cost obtained is optimal moreover; one can conclude that the solution of fuzzy problem can be obtained by Pentagonal Fuzzy Number method effectively.

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