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A CLASS OF CONSTACYCLIC CODES OVER $Z_5 + vZ_5$

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Abstract

In the present paper, we study the structure of all cons acyclic codes over $Z_5 + vZ_5$, where $v^2 = v$ and establish relations to codes over Z_5 by defining the Gray Map that is from considered ring to Z_5^2 where, Z_5 is the field with 5 elements. Generators of such cons acyclic codes for an arbitrary length are also determined. Also, some examples of cons acyclic codes are cited in the paper.

1. Introduction

Finite ring codes were initiated by Blake in the early 1970s [3, 4]. The algebraic coding theory of linear codes has received notable interest in the last half of the century. Many articles have been written on cons acyclic codes over fields due to their applications. Moreover, cons acyclic codes consist of extraordinary generalization of cyclic codes. The focus on the construction of

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codes was mainly on fields, but received a lot of attention after investigating [1] over finite rings. Most of studies focus on codes over finite chain rings. Zhu and Wang studied a class of cons acyclic codes in [8]. Some consequence on cyclic codes over $F_2 + vF_2$ have been given by Zhu et al. in [7], who shows that cyclic codes over the ring are principally generated. Later, Ling and Blackford extended most of the results in [5, 6]. Since 2002, cons acyclic codes of different types of finite chain rings have been extensively examined [2, 9, 10].

This paper is structured as follows. Section 2 contains preliminaries that deal with some basic properties of the ring $Z_5 + vZ_5$ and some basic definitions. In section 3, Gray Map is defined over $Z_5 + vZ_5$. In section 4, theory for the construction of cons acyclic codes is given over the considered ring. Some examples are provided to illustrate the main result in section 5 of arbitrary length. Finally, paper is concluded in section 6.

2. Preliminaries

Let Z_5 is a finite filed having 5 elements that are $\{0, \mu, 1+\mu, 2+\mu, 3+\mu\}$ where $\mu = 1$. We first start with a general overview of the ring $\mathfrak{R} = Z_5 + vZ_5$ where $v^2 = v$, $\mathfrak{R}$ is a finite, commutative, non-chain, semi-local ring with $5^2 = 25$ elements. $\mathfrak{R}$ has total 16 units which are $\{\mu, 1+\mu, 2+\mu, 3+\mu, (\mu+v), (\mu+(1+\mu)v), (\mu+(2+\mu)v), ((1+\mu)+v), ((1+\mu)+(1+\mu)v), ((1+\mu)+(3+\mu)v), ((2+\mu)+v), ((2+\mu)+(2+\mu)v), ((2+\mu)+(3+\mu)v), ((3+\mu)+(1+\mu)v), ((3+\mu)+(2+\mu)v), ((3+\mu)+(3+\mu)v)\}$. The considered ring $\mathfrak{R}$ has two maximal ideals which are $\langle v\rangle$ and $\langle 1-v\rangle$. Since, it is clear that $\mathfrak{R}/\langle v\rangle$, $\mathfrak{R}/\langle 1-v\rangle$ both are isomorphic to Z_5.

Now by Chinese remainder theorem, the considered ring can be expressed as $\mathfrak{R} \cong \langle v\rangle \oplus \langle 1-v\rangle \cong Z_5 \oplus Z_5$. Therefore, an arbitrary element $\alpha + v\beta$ of the considered ring can be written as $(\alpha+\beta)(v) + (\alpha)(1-v)$ for all $\alpha, \beta \in Z_5$. Throughout the paper, we denote units of the ring $\mathfrak{R}$ as ϑ for sake of simplicity.

A nonempty subset $\mathcal{K}$ of $\mathfrak{R}^n$ is a linear code over $\mathfrak{R}$ of length n. If $\mathcal{K}$ is an $\mathfrak{R}$-sub module of $\mathfrak{R}^n$ and the elements of $\mathcal{K}$ are code words. Let $\mathcal{K}$ be a code over $\mathfrak{R}$ of length n and its polynomial representation be $T(\mathcal{K})$, that is,

$$T(\mathcal{K}) = \{\sum_{i=0}^{n-1} \chi_i t^i \mid (\chi_0, \chi_1, \ldots, \chi_{n-1}) \in \mathcal{K}\}$$

Let Υ, Λ and $\mho$ are the maps from $\mathfrak{R}^n$ to $\mathfrak{R}^n$ defined as

$$\Upsilon(\chi_0, \chi_1, \ldots, \chi_{n-1}) = (\chi_{n-1}, \chi_0, \ldots, \chi_{n-2}),$$

$$\Lambda(\chi_0, \chi_1, \ldots, \chi_{n-1}) = (-\chi_{n-1}, \chi_0, \ldots, \chi_{n-2}),$$

$$\mho(\chi_0, \chi_1, \ldots, \chi_{n-1}) = (\vartheta\chi_{n-1}, \chi_0, \ldots, \chi_{n-2}),$$

respectively. Then $\mathcal{K}$ is a cyclic, negacyclic and ϑ-constacyclic if $\Upsilon(\mathcal{K}) = \mathcal{K}$, $\Lambda(\mathcal{K}) = \mathcal{K}$ and $\mho(\mathcal{K}) = \mathcal{K}$ respectively. A code $\mathcal{K}$ over $\mathfrak{R}$ of length n is cyclic, negacyclic and ϑ-constacyclic if and only if $T(\mathcal{K})$ is an ideal of $\mathfrak{R}[t]/\langle t^n - 1\rangle$, $\mathfrak{R}[t]/\langle t^n + 1\rangle$ and $\mathfrak{R}[t]/\langle t^n - \vartheta\rangle$ respectively.

For the arbitrary elements $\chi = (\chi_0, \chi_1, \ldots, \chi_{n-1})$ and $\psi = (\psi_0, \psi_1, \ldots, \psi_{n-1})$ of $\mathfrak{R}$, the inner product is defined as

$$\chi \cdot \psi = (\chi_0\psi_0 + \chi_1\psi_1 + \ldots + \chi_{n-1}\psi_{n-1}).$$

If $\chi \cdot \psi = 0$, then χ and ψ are orthogonal. If $\mathcal{K}$ is a linear code over $\mathfrak{R}$ of length n, then the dual code of $\mathcal{K}$ is defined as

$$\mathcal{K}^{\perp} = \{\chi \in \mathfrak{R}^n : \chi.\psi = 0 \text{ for all } \psi \in \mathcal{K}\}.$$

which is also a linear code over the ring $\mathfrak{R}$ of length n. A code $\mathcal{K}$ is said to be self orthogonal if $\mathcal{K} \subseteq \mathcal{K}^{\perp}$ and said to be self dual if $\mathcal{K} = \mathcal{K}^{\perp}$.

3. Gray Map over $\mathfrak{R}$

The hamming weight $w_H(\chi)$ for any codeword $\chi = (\chi_0, \chi_1, \ldots, \chi_{n-1}) \in \mathfrak{R}^n$

is defined as the number of all non-zero components in $\chi = (\chi_0, \chi_1, \ldots, \chi_{n-1})$. The minimum weight of a code $\mathcal{K}$, that is, $w_H(\mathcal{K})$ is the least weight among all of its non zero code words. The Hamming distance between two codes $\chi = (\chi_0, \chi_1, \ldots, \chi_{n-1})$ and $\hat{\chi} = (\hat{\chi}_0, \hat{\chi}_1, \ldots, \hat{\chi}_{n-1})$ of $\Re^n$, denoted by $d_H(\chi, \hat{\chi}) = w_H(\chi - \hat{\chi})$ and is defined as

$$d_H(\chi, \psi) = |\{i \mid \chi_i \neq \psi_i\}|.$$

Minimum distance of $\mathcal{K}$, denoted by d_H and is given by minimum distance between the different pairs of code words of the linear code $\mathcal{K}$. For any codeword $\chi = (\chi_0, \chi_1, \ldots, \chi_{n-1}) \in \Re^n$, the lee weight is defined as $w_L(\chi) = \sum_{i=0}^{n-1} w_L(\chi_i)$ and lee distance of $(\chi, \hat{\chi})$ is given by $d_L(\chi, \hat{\chi})$ $= w_L(\chi - \hat{\chi}) = \sum_{i=0}^{n-1} w_L(\chi_i - \hat{\chi}_i)$.

Minimum lee distance of $\mathcal{K}$ is denoted by d_L and is given by minimum lee distance of different pairs of code words of the linear code $\mathcal{K}$.

The Gray map φ from $\Re$ to Z_5^2, that is, $\varphi : \Re \to Z_5^2$ is defined as

$$\varphi(\eta = \eta_1 + \upsilon\eta_2) = (\eta_1, \eta_1 + \eta_2)$$

This map can be extended to $\Re^n$, that is $\varphi : \Re^n \to Z_5^{2n}$ as

$$\varphi(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}) = (\eta_1, \eta_2 + \eta_2, \eta_2 + \eta_2, \ldots, \eta_{n-1}, \eta_{n-1} + \eta_{n-1})$$

where $\alpha_i = \eta_i + \upsilon\eta_i^{\cdot}$ for all $0 \leq i \leq n-1$.

Proposition 3.1. *The Gray map φ is linear and distance preserving map from $(\Re^n, d_L)$ to (Z_5^{2n}, d_H).*

Proof. First we prove that φ is linear. Let $\chi, \hat{\chi} \in \Re^n$, and α_1, α_2 is $\in Z_5$ then from the definition of Gray map,

$$\varphi(\alpha_1\chi + \alpha_2\hat{\chi}) = \alpha_1\varphi(\chi) + \alpha_2\varphi(\hat{\chi})$$

which means that φ is linear map.

By the above definitions, it clear that $\varphi(\chi - \hat{\chi}) = \varphi(\chi) - \varphi(\hat{\chi})$ for $\chi, \hat{\chi} \in \Re^n$. Thus, $d_L(\chi, \hat{\chi}) = w_L(\chi - \hat{\chi}) = w_H(\varphi(\chi - \hat{\chi})) = w_H(\varphi(\chi) - \phi(\hat{\chi}))$ $= d_L(\varphi(\chi), \varphi(\hat{\chi}))$. Hence φ is distance preserving map. □

4. ϑ-Constacyclic Codes over $\Re$

Let S, D be two linear codes over $\Re$. The operations $\otimes, \oplus$ are defined as

$$S \otimes D = \{(s, d) \mid s \in S, d \in D\} \text{ and } S \oplus D = \{(s + d) \mid s \in S, d \in D\}$$

By the use of properties of Chinese Remainder theorem, any code $\mathcal{K}$ over $\Re$ is permutation equivalent to a code span by the below given matrix.

$$\begin{bmatrix} I_{k_1} & (1-v)D_1 & vS_1 & vS_2 + (1-v)D_2 & vS_3 + (1-v)D_3 \\ 0 & vI_{k_2} & 0 & vS_4 & 0 \\ 0 & 0 & (1-v)I_{k_3} & 0 & (1-v)D_4 \end{bmatrix}$$

where S_i, D_i are 5-ary matrices for all $1 \leq i, j \leq 4$.

For a linear code $\mathcal{K}$ of length n over $\Re$, we characterize

$$\mathcal{K}_\infty = \{a \in Z_5^2 \mid \text{for some } b \in Z_5^2 \text{ such that } (a + vb) \in \mathcal{K}\}$$

$$\mathcal{K}_\in = \{a + b \in Z_5^2 \mid \text{such that } (a + vb) \in \mathcal{K}\}$$

are 2, 5-ary codes such that

$$(1 - v) \in \mathcal{K}_\infty = \mathcal{K} \bmod v$$

and

$$v\mathcal{K}_\in = \mathcal{K} \bmod (1 - v).$$

Therefore, $\mathcal{K}_\infty$ and $\mathcal{K}_\in$ are linear codes over the ring Z_5 of length n. Moreover, the linear code $\mathcal{K}$ can be uniquely expressed as

$$\mathcal{K} = v\mathcal{K}_\infty \oplus (1 - v)\mathcal{K}_\in$$

and also $|\mathcal{K}| = |\mathcal{K}_\infty||\mathcal{K}_\in| = 25^{k_1}5^{k_2}5^{k_3} = 5^{2k_1+k_2+k_3}$.

The following proposition can be obtained directly by the above defined Gray map φ.

Proposition 4.1. *Let $\mathcal{K}$ be a linear code over the ring $\Re$ of length n. If $\mathcal{K}$ is self orthogonal, then $\varphi(\mathcal{K})$ is also self orthogonal.*

Proof. Let $\mathcal{K}$ be a self orthogonal code and $\eta_1, \eta_2 \in \mathcal{K}$ such that $\eta_1 = \xi_1 + v\varpi_1$ and $\eta_2 = \xi_2 + v\varpi_2$ where $\xi_1, \xi_2, \varpi_1, \varpi_2 \in Z_5$. From the definition of self orthogonality, $\eta_1 \cdot \eta_2 = 0$, that is, $\xi_1\xi_2 + v(\xi_1\varpi_2 + \xi_2\varpi_1 + \varpi_1\varpi_2) = 0$, it follow that $\xi_1\xi_2 = \xi_1\varpi_2 + \xi_2\varpi_1 + \varpi_1\varpi_2 = 0$. Now, applying φ on η_1, η_2 we have $\varphi(\eta_1) = (\xi_1, \xi_1 + \varpi_1)$ and $\varphi(\eta_2) = (\xi_2, \xi_2 + \varpi_2)$ and hence $\varphi(\eta_1) \cdot \varphi(\eta_2) = 2\xi_1\xi_2 + \xi_1\varpi_2 + \xi_2\varpi_1 + \varpi_1\varpi_2 = 0$ this implies $\varphi(\mathcal{K})$ is self orthogonal.

Proposition 4.2. *Let $\mathcal{K} = v\mathcal{K}_\infty \oplus (1-v)\mathcal{K}_\in$ be a linear code over the ring $\mathfrak{R}$ of length n such that $\mathcal{K}_1$ be a linear code having parameters $[n, k_1, d_1]$ and $\mathcal{K}_\in$ be a linear code having parameters $[n, k_2, d_2]$. Then $\varphi(\mathcal{K})$ is a 5-ary linear code having parameters $[2n, k_1 + k_2, \min(d_1, d_2)]$.*

Theorem 4.3. *Let $\mathcal{K} = v\mathcal{K}_\infty \oplus (1-v)\mathcal{K}_\in$ be linear code over the ring $\Re$ of length n where $\mathcal{K}_\infty, \mathcal{K}_\in$ are linear codes over the ring Z_5. Then $\mathcal{K}$ is a ϑ-constacyclic codes over the ring $\mathfrak{R}$ of length n if and only if $\mathcal{K}_\infty$ is Cyclic or negacyclic or constacyclic code and $\mathcal{K}_\in$ is Cyclic or negacyclic or constacyclic code over the ring Z_5 of length n.*

Note. To prove the theorem, we consider $\vartheta = ((3+\mu) + (1+\mu)v)$.

Proof. Let $\dot{a} = (\dot{a}_0, \dot{a}_1, \ldots, \dot{a}_{n-1}) \in \mathcal{K}_\infty$ and $\dot{b} = (\dot{b}_0, \dot{b}_1, \ldots, \dot{b}_{n-1}) \in \mathcal{K}_\in$. For an arbitrary element $\zeta_i = v\dot{a}_i + (1-v)\dot{b}_i$ where $\dot{a}_i, \dot{b}_i \in Z_5$ for $i = 0, 1, \ldots, n-1$.

Let $\zeta = (\zeta_0, \zeta_1, \ldots, \zeta_{n-1}) \in \mathcal{K}$.

First we assume that $\mathcal{K}$ is a $((3+\mu)+(1+\mu)v)$-constacyclic code over the ring $\mathfrak{R}$ of length n then,

$$\mho(\zeta) = (((3+\mu)+(1+\mu)v)\zeta_{n-1}, \zeta_0, \ldots, \zeta_{n-2})$$

$$= (v\dot{a}_{n-1} - (1-v)\dot{b}_{n-1}, v\dot{a}_0 + (1-v)\dot{b}_0, \ldots, v\dot{a}_{n-2} + (1-v)\dot{b}_{n-2})$$

$$= v\Upsilon(\dot{a}) + (1-v)\Lambda(\dot{b})$$

which is an element of the linear code $\mathcal{K}$. Therefore, $\mathcal{K}_\infty$ is a cyclic and $\mathcal{K}_\in$ is a negacyclic codes over the ring Z_5 of length n.

Conversely, for any $\zeta = (\zeta_0, \zeta_1, \ldots, \zeta_{n-1}) \in \mathcal{K}$, where $\zeta_i = v\dot{a}_i + (1-v)\dot{b}_i$ and $\dot{a}_i\dot{b}_i \in Z_5$ for $i = 0, 1, \ldots, n-1$. If $\mathcal{K}_\infty$ is a cyclic codes and $\mathcal{K}_\in$ is a negacyclic codes over the ring Z_5 of length n, then $\Upsilon(\dot{\alpha}) \in \mathcal{K}_\infty$ and $\Lambda(\dot{b}) \in \mathcal{K}_\in$. Hence, we have $v\Upsilon(\dot{a}) + (1-v)\Lambda(\dot{b}) \in \mathcal{K}$ where $\mho(\zeta) = v\Upsilon(\dot{a}) + (1-v)\Lambda(\dot{b})$, which implies that $\mho(\zeta) \in \mathcal{K}$.

Therefore, $\mathcal{K}$ is a $((3+\mu)+(1+\mu)v)$-constacyclic codes over the ring $\mathfrak{R}$ of length n. □

Now, we discuss all other cases for ϑ over $\mathfrak{R}$ that is, relation between ϑ-constacyclic codes $\mathcal{K}$ and the codes $\mathcal{K}_\infty, \mathcal{K}_\in$.

Note.

ϑ		$\mathcal{K}$	$\mathcal{K}_\infty$	$\mathcal{K}_\in$
1	$\vartheta = (\mu + v)$	$(\mu + v)$ -constacylic	$(1+\mu)$ -constacyclic	Cyclic
2	$\vartheta = (\mu + (1+\mu)v)$	$(\mu + (1+\mu)v)$ -constacyclic	$(2+\mu)$ -constacyclic	Cyclic
3	$\vartheta = (\mu + (2+\mu)v)$	$(\mu + (2+\mu)v)$ -constacyclic	Negacyclic	Cyclic
4	$\vartheta = ((1+\mu)+v)$	$((1+\mu)+v)$ -constacyclic	$(2+\mu)$ -constacyclic	$(1+\mu)$ -constacyclic

5	$\vartheta = ((1+\mu)+(1+\mu)v)$	$((1+\mu)+(1+\mu)v)$ -constacyclic	Negacyclic	$(1+\mu)$ -constacyclic
6	$\vartheta = ((1+\mu)+(3+\mu)v)$	$((1+\mu)+(3+\mu)v)$ -constacyclic	Cyclic	$(1+\mu)$ -constacyclic
7	$\vartheta = ((2+\mu)+v)$	$((2+\mu)+v)$ -constacyclic	Negacyclic	$(2+\mu)$ -constacyclic
8	$\vartheta = ((2+\mu)+(2+\mu)v)$	$((2+\mu)+(2+\mu)v)$ -constacyclic	Cyclic	$(2+\mu)$ -constacyclic
9	$\vartheta = ((2+\mu)+(3+\mu)v)$	$((2+\mu)+(3+\mu)v)$ -constacyclic	$(1+\mu)$ -constacyclic	$(2+\mu)$ -constacyclic
10	$\vartheta = ((3+\mu)+(1+\mu)v)$	$((3+\mu)+(1+\mu)v)$ -constacyclic	Cyclic	Negacyclic
11	$\vartheta = ((3+\mu)+(2+\mu)v)$	$((3+\mu)+(2+\mu)v)$ -constacyclic	$(1+\mu)$ -constacyclic	Negacyclic
12	$\vartheta = ((3+\mu)+(3+\mu)v)$	$((3+\mu)+(3+\mu)v)$ -constacyclic	$(2+\mu)$ -constacyclic	Negacyclic

Remaining ϑ of $\mathfrak{R}$ are trivially hold.

Theorem 4.4 [8]**.** *Let $\mathcal{K}$ be a ϑ-constacyclic codes over the ring $\mathfrak{R}$ of length n. Then*

$$\mathcal{K} = \langle vg_1(t),\ (1-v)g_2(t)\rangle = \langle vg_1(t) + (1-v)g_2(t)\rangle$$

with $|\mathcal{K}| = 5^{2n-\deg(g_1(t))-\deg(g_2(t))}$

where $g_i(t)$ for $i = 1, 2$ are the monic generator polynomials of $\mathcal{K}_\infty$ and $\mathcal{K}_\in$ respectively.

Theorem 4.5. [8] *Let $\mathcal{K}$ be a ϑ-constacyclic codes over the ring $\mathfrak{R}$ of length n with*

$$\mathcal{K} = \langle vg_1(t),\ (1-v)g_2(t)\rangle = \langle vg_1(t) + (1-v)g_2(t)\rangle,$$

where $g_i(t)$ for $i = 1, 2$ are the monic generator polynomials of $\mathcal{K}_\infty$ and $\mathcal{K}_\in$ respectively then

$$\varphi(\mathcal{K}) = \langle g_1(t)g_2(t)\rangle$$

Theorem 4.6 [8]. *Let $\mathcal{K}$ be a φ-constacyclic code over $\mathfrak{R}$ of length n. Then dual code $\mathcal{K}^{\perp}$ is also ϑ-constacyclic code over $\mathfrak{R}$.*

Corollary 4.7 [8]. *Let $\mathcal{K} = \langle vg_1(t), (1-v)g_2(t)\rangle$ be a ϑ-constacyclic codes over the ring $\mathfrak{R}$ of length n with $g_i(t)$ for $i = 1, 2$ are the monic generator polynomials of $\mathcal{K}_\infty$ and $\mathcal{K}_\in$ respectively then $\mathcal{K}^{\perp} = v\mathcal{K}_\infty^{\perp} \oplus (1-v)\mathcal{K}_\in^{\perp}$ is also ϑ-constacyclic codes over the ring $\mathfrak{R}$ of length n and*

$$\mathcal{K}^{\perp} = \langle vg_1^*(t), (1-v)g_2^*(t)\rangle = \langle vg_1^*(t) + (1-v)g_2^*(t)\rangle$$

with $|\mathcal{K}^{\perp}| = p^{\deg(g_1(t))+\deg(g_2(t))}$, $\varphi(\mathcal{K}^{\perp}) = \langle g_1^*(t)g_2^*(t)\rangle$ and $\varphi(\mathcal{K}^{\perp}) = \varphi(\mathcal{K})^{\perp}$ where $g_i^*(t)$ for $i = 1, 2$ are reciprocal polynomials of $\frac{t^n-1}{g_1(t)}$ and $\frac{t^n+1}{g_2(t)}$ respectively.

5. Examples

In this section, some examples are provided to illustrate the main result. Here, the quantum codes through ϑ-constacyclic codes over the ring $\mathfrak{R} = Z_5 + vZ_5$ where $v^2 = v$ are also obtained.

Example 5.1. In $Z_5(t)$, $t^3 - 1 = (t-1)(t^2+t+1)$ and $t^3 + 1 = (t+1)(t^2 - t + 1)$. Now, let $\mathcal{K}$ be a $((3+\mu) + (1+\mu)v)$-constacyclic codes over the ring $\mathfrak{R} = Z_5 + vZ_5$ where $v^2 = v$ of length 3. Let $g_1(t) = t - 1$ and $g_2(t) = t + 1$ then $g(t) = v(t-1) + (1-v)(t+1)$ be the generator polynomial of $\mathcal{K}$. Then by the use of Theorem 4.5, we get $\varphi(\mathcal{K})$ is a linear code having parameters [6, 4, 2] with generator polynomial $\langle g_1(t)g_2(t)\rangle = (t-1)(t+1)$.

Example 5.2. In $Z_5(t)$, $t^6 - 1 = (t-1)(t+1)(t^2+t+1)(t^2+4t+1)$ and $t^6 + 1 = (t+2)(t+3)(t^2+2t+4)(t^2+3t+4)$. Now, let $\mathcal{K}$ be a $((3+\mu)+(1+\mu)v)$-constacyclic codes over the ring $\mathfrak{R} = Z_5 + vZ_5$ where $v^2 = v$ of length 6. Let $g_1(t) = (t-1)(t+1)$ and $g_2(t) = t + 2$ then

$g(t) = v(t-1)(t+1) + (1-v)(t+2)$ be the generator polynomial of $\mathcal{K}$. Then by using Theorem 4.5, we get $\varphi(\mathcal{K})$ is a linear code having parameters [12, 9, 3] with generator polynomial $\langle g_1(t)g_2(t)\rangle = (t-1)(t+1)(t+2)$.

Example 5.3. In $Z_5(t), t^{10}+1 = (t+2)^5(t+3)^5$ and $t^{10}-1 = (t+1)^5(t+4)^5$. Now, let $\mathcal{K}$ be a $(\mu + (2+\mu)v)$-constacyclic codes over the ring $\Re = Z_5 + vZ_5$ where $v^2 = v$ of length 10. Let $g_1(t) = t+2$, $g_2(t) = t+1$, then $g(t) = v(t+2) + (1-v)(t+1)$ be the generator polynomial of $\mathcal{K}$. Then by the use of Theorem 4.5, we get $\varphi(\mathcal{K})$ is a linear code having parameters [20, 18, 2] with generator polynomial $\langle g_1(t)g_2(t)\rangle = (t+2)(t+1)$.

Example 5.4. In $Z_5(t), t^{20}+1 = ((t^2+2)^5)((t^3+3)^5)$ and $t^{20}-1 = (t-1)^5(t-2)^5(t-3)^5(t-4)^5$. Now, let $\mathcal{K}$ be a $(\mu + (2+\mu)v)$-constacyclic codes over the ring $\Re = Z_5 + vZ_5$ where $v^2 = v$ of length 20. Let $g_1(t) = t^2-3$, $g_2(t) = t^2-1$, then $g(t) = v(t^2-3) + (1-v)(t-1)$ be the generator polynomial of $\mathcal{K}$. Then by using Theorem 4.5, we get $\varphi(\mathcal{K})$ is a linear code having parameters [40, 37, 3] with generator polynomial $\langle g_1(t)g_2(t)\rangle = (t^2-3)(t-1)$.

6. Conclusion

In this work, we derived the complete structure of ϑ-constacyclic codes over the finite non-chain ring $\Re = Z_5 + vZ_5$ where $v^2 = v$ by defining Gray map from consider ring $\Re$ to Z_5^2 and proved that map is linear and distance preserving. We discuss about the generator polynomial of constacyclic codes over consider ring. Also, we discuss about the dual codes of constacyclic coder over the ring. It may be interesting to consider the structure of other class of constacyclic codes over $Z_p + vZ_p$.

References

[1] A. R. Hammons, Jr. P. V. Kumar, J. A. Calderbank, N. J. A. Sloane and P. Sole, The Z4-linearity of Kerdock, Preparata, Goethals and related codes, IEEE Trans. Inform. Theory 40(2) (1994), 301-319.

[2] H. Q. Dinh and S. R. Lpez-Permouth, Cyclic and negacyclic codes over finite chain rings, IEEE Trans. Inform. Theory 50(8) (2004), 1728-1744.

[3] I. F. Blake, Codes over certain rings, Inform. Control 20(1972), 396-404.

[4] I. F. Blake, Codes over integer residue rings, Inform. Control 29 (1975), 295-300.

[5] J. Wolfmann, Negacyclic and cyclic codes over Z_4, IEEE Trans. Inform. Theory 45(7) (1999), 2527-2532.

[6] J. Wolfmann, Binary image of cyclic codes over Z_4, IEEE Trans. Inform. Theory 47(5) (2001), 1773-1779.

[7] S. Zhu, Y. Wang and M. Shi, Some results on cyclic codes over $F_2 + vF_2$, IEEE Trans. Inform. Theory 56(4) (2010), 1680-1684.

[8] S. Zhu and L. Wang, A class of constacyclic codes over $F_p + vF_p$ and its Gray image, Discrete Math 311 (2011), 2677-2682.

[9] T. Abualrub and I. Siap, Constacyclic codes over $F_2 + uF_2$, J. Franklin Inst. 346 (2009), 520-529.

[10] T. Blackford, Negacyclic codes over Z_4 of even length, IEEE Trans. Inform. Theory 49 (2003), 1417-1424.