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INTUITIONISTIC FUZZY TRANSLATION OF INTUITIONISTIC FUZZY SUBFIELD

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Abstract

In this paper, we introduced the concept of intuitionistic fuzzy translation of intuitionistic fuzzy subfield. Also we investigate some of their properties.

1. Introduction

The concept of fuzzy subset was introduced by L. A. Zadeh [8] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: K. T. Atanassov [2, 3] introduced intuitionistic fuzzy subset, as a generalization of the notion of fuzzy set. We introduce intuitionistic fuzzy translation of intuitionistic fuzzy subfield and some properties are investigated.

2. Preliminaries

Definition 2.1. Let X be a non empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.

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Definition 2.2. Let $(F, +, \cdot)$ be a field. A fuzzy subset A of F is said to be a fuzzy subfield (FSF) of F if the following conditions are satisfied:

(i) $A(x - y) \geq \min\{A(x), A(y)\}$, for all x and y in F,

(ii) $A(xy^{-1}) \geq \min\{A(x), A(y)\}$, for all x and y in $F - \{0\}$.

Definition 2.3. An intuitionistic fuzzy subset (IFS) A of a set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\upsilon_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$.

Definition 2.4. Let A and B be any two intuitionistic fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\upsilon_A(x), \upsilon_B(x)\}\rangle / x \in X\}$.

(ii) $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\upsilon_A(x), \upsilon_B(x)\}\rangle / x \in X\}$.

Definition 2.5. Let $(F, +, \cdot)$ be a field. An intuitionistic fuzzy subset A of F is said to be an intuitionistic fuzzy subfield (IFSF) of F if the following conditions are satisfied:

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in F,

(ii) $\mu_A(-x) \geq \mu_A(x)$, for all x in F,

(iii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in F,

(iv) $\mu\mu_A(x^{-1}) \geq \mu_A(x)$, for all x in $F - \{0\}$,

(v) $\upsilon_A(x + y) \leq \max\{\upsilon_A(x), \upsilon_A(y)\}$, for all x and y in F,

(vi) $\upsilon_A(-x) \leq \upsilon_A(x)$, for all x in F,

(vii) $\upsilon_A(xy) \leq \max\{\upsilon_A(x), \upsilon_A(y)\}$, for all x and y in F,

(viii) $\upsilon_A(x^{-1}) \leq \upsilon_A(x)$, for all x in $F - \{0\}$.

Definition 2.6. Let A be an intuitionistic fuzzy subset of X and α and β in

$[0, 1 - \sup\{\mu_A(x) + \upsilon_A(x) : x \in X, 0 < \mu_A(x) + \upsilon_A(x) < 1\}]$. Then $T = T^A_{(\alpha, \beta)}$ is called an intuitionistic fuzzy translation of A if $\mu_T(x) = \mu^A_\alpha(x) = \mu_A(x) + \alpha$, $\upsilon_T(x) = \upsilon^A_\beta(x) = \upsilon_A(x) + \beta, \alpha + \beta \leq 1 - \sup\{\mu_A(x) + \upsilon_A(x) : x \in X, 0 < \mu_A(x) < 1\}$, for all x in X.

Example 2.7. Consider the set $X = \{0, 1, 2, 3, 4\}$. Let $A = \{A = (0, 0.5, 0.1), (1, 0.4, 0.3), (2, 0.6, 0.05), (3, 0.45, 0.2), (4, 0.2, 0.5)\}$ be an intuitionistic fuzzy subset of X and $\alpha = 0.25, \beta = 0.05$. The intuitionistic fuzzy translation of A is $T = T^A(0.25, 0.05) = \{(0, 0.75, 0.15), (1, 0.65, 0.35), (2, 0.85, 0.1), (3, 0.7, 0.25), (4, 0.45, 0.55)\}$.

3. Properties

Theorem 3.1. *If M and N are two intuitionistic fuzzy translations of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$ then their intersection $M \cap N$ is an intuitionistic fuzzy translation of A.*

Proof. Let x and y belong to F. Let $M = T^A_{(\alpha, \beta)} = \{\langle x, \mu_A(x)^+\alpha, \upsilon_A(x)^+\beta\rangle / x \in F\}$ and $N = T^A_{(\gamma, \delta)} = \{\langle x, \mu_A(x)^+\gamma, \upsilon_A(x)^+\delta\rangle / x \in F\}$ be two intuitionistic fuzzy translations of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$.

Let $C = M \cap N$ and $C = \{\langle x, \mu_C(x)^+, \upsilon_C(x)^+\rangle / x \in F\}$, where $\mu_C(x) = \min\{\mu_A(x)^+\alpha, \mu_A(x)^+\gamma\}$ and $\upsilon_C(x) = \max\{\upsilon_A(x)^+\beta, \upsilon_A(x)^+\delta\}$.

Case (i). $\alpha \leq \gamma$ and $\beta \leq \delta$.

Now,

$$\mu_C(x - y) = \min\{\mu_M(x - y), \mu_M(x - y)\} = \min\{\mu_A(x - y) + \alpha, \mu_A(x - y)\}$$

$$= \mu_A(x - y) + \alpha$$

$$= \mu_M(x - y), \text{ for all } x \text{ and } y \text{ in } F.$$

And

$\mu_C(xy^{-1}) = \min\{\mu_M(xy^{-1}), \mu_M(xy^{-1})\} = \min\{\mu_A(xy^{-1}) + \alpha, \mu_A(xy^{-1}) + \gamma\}$

$= \mu_A(xy^{-1}) + \alpha$

$= \mu_M(xy^{-1})$, for all x and $y \neq 0$ in F.

Now,

$\upsilon_C(x - y) = \max\{\upsilon_M(x - y), \upsilon_N(x - y)\} = \max\{\upsilon_A(x - y) + \beta, \upsilon_A(x - y) + \delta\}$

$= \upsilon_A(x - y) + \delta$

$= \upsilon_N(x - y)$, for x and y in F.

And

$\upsilon_C(xy^{-1}) = \max\{\upsilon_M(xy^{-1}), \upsilon_M(xy^{-1})\} = \max\{\upsilon_A(xy^{-1}) + \beta, \upsilon_A(xy^{-1}) + \delta\}$

$= \upsilon_A(xy^{-1}) + \delta$

$= \upsilon_N(xy^{-1})$, for all x and $y \neq 0$ in F.

Therefore $C = T^A_{(\alpha, \delta)} = \{\langle x, \mu_A(x)^+\alpha, \upsilon_A(x)^+\delta\rangle / x \in F\}$ is an intuitionistic fuzzy translation of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$.

Case (ii). $\alpha \geq \gamma$ and $\beta \geq \delta$.

Now,

$\mu_C(x - y) = \min\{\mu_M(x - y), \mu_N(x - y)\} = \min\{\mu_A(x - y) + \alpha, \mu_A(x - y) + \gamma\}$

$= \mu_A(x - y) + \gamma$

$= \mu_N(x - y)$, for all x and y in F.

And

$\upsilon_C(xy^{-1}) = \min\{\mu_M(xy^{-1}), \mu_N(xy^{-1})\} = \min\{\mu_A(xy^{-1}) + \alpha, \mu_A(xy^{-1}) + \gamma\}$

$= \upsilon_A(xy^{-1}) + \gamma$

$= \mu_N(xy^{-1})$, for all x and $y \neq 0$ in F.

Now,

$$v_C(x-y) = \max\{v_M(x-y), v_N(x-y)\} = \max\{v_A(x-y)+\beta, v_A(x-y)+\delta\}$$

$$= v_A(x-y)+\beta$$

$$= v_M(x-y), \text{ for } x \text{ and } y \text{ in } F.$$

And

$$v_C(xy^{-1}) = \max\{v_M(xy^{-1}), \mu_M(xy^{-1})\} = \max\{v_A(xy^{-1})+\beta, v_A(xy^{-1})+\delta\}$$

$$= v_A(xy^{-1})+\beta$$

$$= v_M(xy^{-1}), \text{ for all } x \text{ and } y \neq 0 \text{ in } F.$$

Therefore $C = T^A_{(\gamma, \beta)} = \{\langle x, \mu_A(x)^+\gamma, v_A(x)^+\beta\rangle / x \in F\}$ is an intuitionistic fuzzy translation of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$.

Case (iii). $\alpha \geq \gamma$ and $\beta \geq \delta$.

Clearly $C = T^A_{(\alpha, \beta)} = \{\langle x, \mu_A(x)^+\alpha, v_A(x)^+\beta\rangle / x \in F\}$ is an intuitionistic fuzzy translation of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$.

Case (iv). $\alpha \geq \gamma$ and $\beta \geq \delta$.

Clearly $C = T^A_{(\gamma, \delta)} = \{\langle x, \mu_A(x)^+\gamma, v_A(x)^+\delta\rangle / x \in F\}$ is an intuitionistic fuzzy translation of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$.

Hence all cases, intersection of any two intuitionistic fuzzy translations of intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$ is an intuitionistic fuzzy translation of A.

Theorem 3.2. *If T is an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$, then*

(i) $\mu_T(x-y) = v_T(0)$ implies $\mu_T(x) = v_T(y)$, for all x and y in F and $\mu_T(xy^{-1}) = \mu_T(1)$ implies $\mu_T(x) = \mu_T(y)$, for all x and $y \neq 0$ in F,

(ii) $v_T(x-y) = v_T(0)$ implies $v_T(x) = v_T(y)$, for all x and y in F and

$\upsilon_T(xy^{-1}) = \upsilon_T(1)$ implies $\upsilon_T(x) = \upsilon_T(y)$, for all x and $y \neq 0$ in F, where 0 and 1 are identity elements of F.

Proof. Let x and y in F.

Now,

$$\begin{aligned}
\mu_T(x) = \mu_A(x) + \alpha = \mu_A(x - y + y) + \alpha &\geq \min\{\mu_A(x - y), \mu_A(y)\} + \alpha \\
&\geq \min\{(\mu_A(x - y) + \alpha), (\mu_A(y) + \alpha)\} \\
&= \min\{\mu_T(x - y), \mu_T(y)\} \\
&= \min\{\mu_T(0), \mu_T(y)\} \\
&= \mu_T(y) = \mu_A(y) + \alpha \\
&= \mu_A(y - x + x) + \alpha \\
&\geq \min\{\mu_A(y - x), (\mu_A(x)\} + \alpha \\
&= \min\{(\mu_A(y - x) + \alpha), (\mu_A(x) + \alpha)\} \\
&= \min\{\mu_T(x - y), \mu_T(x)\} \\
&= \min\{\mu_T(0), \mu_T(x)\} \\
&= \mu_T(x).
\end{aligned}$$

Therefore, $\mu_T(x) = \mu_T(y)$, for all x and y in F.

Now,

$$\begin{aligned}
\mu_T(x) = \mu_A(x) + \alpha = \mu_A(xy^{-1}) + \alpha &\geq \min\{\mu_A(xy^{-1}), \mu_A(y)\} + \alpha \\
&\geq \min\{(\mu_A(xy^{-1}) + \alpha), (\mu_A(y) + \alpha)\} \\
&= \min\{\mu_T(xy^{-1}), \mu_T(y)\} \\
&= \min\{\mu_T(1), \mu_T(y)\} \\
&= \mu_T(y) = \mu_A(y) + \alpha \\
&= \mu_A(yx^{-1}x) + \alpha
\end{aligned}$$

$$\geq \min\{\mu_A(yx^{-1}), (\mu_A(x)\} + \alpha$$

$$= \min\{(\mu_A(yx^{-1}) + \alpha), (\mu_A(x) + \alpha)\}$$

$$= \min\{\mu_T(yx^{-1}), \mu_T(x)\}$$

$$= \min\{\mu_T(1), \mu_T(x)\}$$

$$= \mu_T(x).$$

Therefore, $\mu_T(x) = \mu_T(y)$, for all x and $y \neq 0$ in F.

And

$$\mu v_T(x) = v_A(x) + \beta = v_A(x - y + y) + \beta \leq \max\{v_A(x - y), v_A(y)\} + \beta$$

$$= \max\{(v_A(x - y) + \beta), (v_A(y) + \beta)\}$$

$$= \max\{v_T(x - y), v_T(y)\}$$

$$= \max\{v_T(0), v_T(y)\}$$

$$= v_T(y) = v_A(y) + \beta$$

$$= v_A(y - x + x) + \beta$$

$$\leq \max\{v_A(y - x), (v_A(x)\} + \beta$$

$$= \max\{(v_A(y - x) + \beta), (v_A(x) + \beta)\}$$

$$= \max\{v_T(y - x), v_T(x)\}$$

$$= \max\{v_T(0), v_T(x)\}$$

$$= v_T(x).$$

Therefore, $v_T(x) = v_T(y)$, for all x and y in F.

And

$$v_T(x) = v_A(x) + \beta = v_A(xy^{-1}y) + \beta \leq \max\{v_A(xy^{-1}), v_A(y)\} + \beta$$

$$= \max\{(v_A(xy^{-1}) + \beta), (v_A(y) + \beta)\}$$

$$= \max \{\upsilon_T(xy^{-1}), \upsilon_T(y)\}$$

$$= \max \{\upsilon_T(1), \upsilon_T(y)\}$$

$$= \upsilon_T(y) = \upsilon_A(y) + \beta$$

$$= \upsilon_A(yx^{-1}x) + \beta$$

$$\leq \max \{\upsilon_A(yx^{-1}), (\upsilon_A(x)\} + \beta$$

$$= \max \{(\upsilon_A(yx^{-1}) + \beta), (\upsilon_A(x) + \beta)\}$$

$$= \max \{\upsilon_T(yx^{-1}), \upsilon_T(x)\}$$

$$= \max \{\upsilon_T(1), \upsilon_T(x)\}$$

$$= \upsilon_T(x).$$

Therefore, $\upsilon_T(x) = \upsilon_T(y)$, for all x and $y \neq 0$ in F.

Theorem 3.3. *If T is an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$ then T is an intuitionistic fuzzy subfield of F, for all x and y in F.*

Proof. Assume that T is an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of a field F. Let x and y in F and 0, 1 are identity elements of F.

We have,

$$\mu_T(x - y) = \mu_A(x - y) + \alpha \geq \min \{\mu_A(x), \mu_A(-y) + \alpha$$

$$= \min \{\mu_A(x), \mu_A(y)\} + \alpha$$

$$= \min (\{\mu_A(x) + \alpha), (\mu_A(y) + \alpha)\}$$

$$= \min \{\mu_T(x), \mu_T(y)\}.$$

Therefore, $\mu_T(x - y) \geq \min \{\mu_T(x), \mu_T(y)\}$, for all x and y in F.

We have,

$$\mu_T(xy^{-1}) = \mu_A(xy^{-1}) + \alpha \geq \min \{\mu_A(x), \mu_A(y^{-1})\} + \alpha = \min \{\mu_A(x), \mu_A(y)\} + \alpha$$

$$= \min \{(\mu_A(x) + \alpha), (\mu_A(y) + \alpha)\}$$

$$= \min \{(\mu_T(x), \mu_T(y))\}.$$

Therefore, $\mu_T(xy^{-1}) \geq \min \{\mu_T(x), \mu_T(y)\}$, for all x and $y \neq 0$ in F.

And

$$\upsilon_T(x - y) = \mu_A(x - y) + \beta \geq \max \{\upsilon_A(x), \upsilon_A(-y)\} + \beta = \max \{\upsilon_A(x), \upsilon_A(y)\} + \beta$$

$$= \max \{(\upsilon_A(x) + \beta), (\upsilon_A(y) + \beta)\}$$

$$= \max \{(\upsilon_T(x), \upsilon_T(y))\}.$$

Therefore, $\upsilon_T(x - y) \geq \max \{\upsilon_T(x), \upsilon_T(y)\}$, for all x and y in F.

And

$$\upsilon_T(xy^{-1}) = \upsilon_A(xy^{-1}) + \beta \leq \max \{\upsilon_A(x), \upsilon_A(y^{-1})\} + \beta = \max \{\upsilon_A(x), \upsilon_A(y)\} + \beta$$

$$= \max \{(\upsilon_A(x) + \beta), (\upsilon_A(y) + \beta)\}$$

$$= \max \{(\upsilon_T(x), \upsilon_T(y))\}.$$

Therefore, $\upsilon_T(xy^{-1}) \leq \max \{\upsilon_T(x), \upsilon_T(y)\}$, for all x and $y \neq 0$ in F.

Hence T is an intuitionistic fuzzy subfield of F.

Theorem 3.4. *If T is an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$, then $H = \{x \in F : \mu_T(x) = \mu_T(0) = \mu_T(1)$ and $\upsilon_T(x) = \upsilon_T(0) = \upsilon_T(1)\}$ is either empty or a subfield of F, where 0 and 1 are identity elements of F.*

Proof. If no element satisfies this condition, then H is empty.

If x and y satisfies this condition, then $\mu_T(-x) = \mu_T(x) = \mu_T(0)$, for all x in F and $\mu_T(x^{-1}) = \mu_T(x) = \mu_T(1)$, for all x in $F - \{0\}$ and $\upsilon_T(-x) = \upsilon_T(x) = \upsilon_T(0)$, for all x in F and $\upsilon_T(x^{-1}) = \upsilon_T(x) = \upsilon_T(1)$, for all x and y in $F - \{0\}$.

Therefore, $\mu_T(-x) = \mu_T(0)$, for all x in F and $\mu_T(x^{-1}) = \mu_T(1)$, for all x in

$F-\{0\}$ and $\upsilon_T(-x) = \upsilon_T(0)$, for all x in F and $\upsilon_T(x^{-1}) = \upsilon_T(1)$, for all x in $F-\{0\}$.

Hence $-x$, x^{-1} in H.

Now,

$$\begin{aligned}\mu_T(x-y) &\geq \min\{\mu_T(x), \mu_T(-y)\} \\ &\geq \min\{\mu_T(x), \mu_T(y)\} \\ &= \min\{\mu_T(0), \mu_T(0)\} \\ &= \mu_T(0).\end{aligned}$$

Therefore, $\mu_T(x-y) \geq \mu_T(0)$. (1).

And,

$$\begin{aligned}\mu_T(0) = \mu_T((x-y)-(x-y)) &\geq \min\{\mu_T(x-y), \mu_T(-(x-y))\} \\ &\geq \min\{\mu_T(x-y), \mu_T(x-y)\} \\ &= \mu_T(x-y).\end{aligned}$$

Therefore, $\mu_T(0) \geq \mu_T(x-y)$. (2).

From (1) and (2), we get $\mu_T(0) = \mu_T(x-y)$, for all x and y in F.

Now,

$$\begin{aligned}\mu_T(xy^{-1}) \geq \min\{\mu_T(x), \mu_T(y^{-1})\} &\geq \min\{\mu_T(x), \mu_T(y)\} \\ &= \min\{\mu_T(1), \mu_T(1)) \\ &= \mu_T(1).\end{aligned}$$

Therefore, $\mu_T(xy^{-1}) \geq \mu_T(1)$. (3).

And,

$$\mu_T(1) = \mu_T((xy^{-1})(xy^{-1})^{-1}) \geq \min\{\mu_T(xy^{-1}), \mu_T((xy^{-1})^{-1})\}$$

$$\geq \min \{\mu_T(xy^{-1}), \mu_T(xy^{-1})\}$$

$$= \mu_T(xy^{-1}).$$

Therefore, $\mu_T(1) \geq \mu_T(xy^{-1})$. (4).

From (3) and (4), we get $\mu_T(1) \geq \mu_T(xy^{-1})$, for all x and $y \neq 0$ in F.

Now,

$$\begin{aligned} \upsilon_T(x - y) &\geq \max \{\upsilon_T(x), \upsilon_T(-y)\} \\ &\geq \max \{\upsilon_T(x), \upsilon_T(y)\} \\ &= \max \{\upsilon_T(0), \upsilon_T(0)\} \\ &= \upsilon_T(0). \end{aligned}$$

Therefore, $\upsilon_T(x - y) \leq \upsilon_T(0)$. (5).

And,

$$\begin{aligned} \upsilon_T(0) = \upsilon_T((x - y) - (x - y)) &\leq \max \{\upsilon_T(x - y), \upsilon_T(-(x - y))\} \\ &\leq \max \{\upsilon_T(x - y), \upsilon_T(x - y)\} \\ &= \upsilon_T(x - y). \end{aligned}$$

Therefore, $\upsilon_T(0) \leq \upsilon_T(x - y)$. (6).

From (5) and (6), we get $\upsilon_T(0) = \upsilon_T(x - y)$, for x and y in F.

Now,

$$\begin{aligned} \upsilon_T(xy^{-1}) \leq \max \{\upsilon_T(x), \upsilon_T(y^{-1})\} &\leq \max \{\upsilon_T(x), \upsilon_T(y^{-1})\} \\ &\leq \max \{\upsilon_T(x), \upsilon_T(y)\} \\ &= \max \{\upsilon_T(1), \upsilon_T(1)\} \\ &= \upsilon_T(1). \end{aligned}$$

Therefore, $\upsilon_T(xy^{-1}) \leq \upsilon_T(1)$. (7).

And,

$$\upsilon_T(1) = \upsilon_T((xy^{-1})(xy^{-1})^{-1}) \leq \max\{\upsilon_T(xy^{-1}), \upsilon_T((xy^{-1})^{-1})\}$$

$$\leq \max\{\upsilon_T(xy^{-1}), \upsilon_T(xy^{-1})\}$$

$$= \upsilon_T(xy^{-1}).$$

Therefore, $\upsilon_T(1) \leq \upsilon_T(xy^{-1})$. (8).

From (7) and (8), we get $\upsilon_T(1) \leq \upsilon_T(xy^{-1})$, for all x and $y \neq 0$ in F.

Hence $\mu_T(0) = \mu_T(x-y)$, for all x and y in F and $\mu_T(1) = \mu_T(xy^{-1})$, for all x and $y \neq 0$ in F and $\upsilon_T(0) = \upsilon_T(x-y)$, for all x and y in F and $\upsilon_T(1) = \upsilon_T(xy^{-1})$, for all x and $y \neq 0$ in F.

Therefore, $x - y$, xy^{-1} in H.

Hence H is either empty or is a subfield of F.

Theorem 3.5. *Let T be an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of a field $(F, +, \cdot)$. Then* (i) *if* $\mu_T(x-y) = 1$, *then* $\mu_T(x) = \mu_T(y)$, *for all x and y in F and if* $\mu_T(xy^{-1}) = 1$, *then* $\mu_T(x) = \mu_T(y)$, *for all x and $y \neq e$ in F,* (ii) *if* $\upsilon_T(x-y) = 0$, *then* $\upsilon_T(x) = \upsilon_T(y)$, *for all x and y in F and if* $\upsilon_T(xy^{-1}) = 0$, *then* $\upsilon_T(x) = \upsilon_T(y)$, *for all x and $y \neq e$ in F, where e and e^1 are identity elements of F.*

Proof. Let x and y in F.

(i) Now,

$$\mu_T(x) = \mu_T(x - y + y) \geq \min\{\mu_T(x-y), \mu_T(y)\}$$

$$= \min\{1, \mu_T(y)\} = \mu_T(y)$$

$$= \mu_T(-y)$$

$$= \mu_T(-x + x - y)$$

$$\geq \min \{\mu_T(-x), \mu_T(x-y)\}$$

$$= \min \{\mu_T(-x), 1\} = \mu_T(-x) = \mu_T(x).$$

Therefore, $\mu_T = \mu_T(y)$, for all x and y in F.

And,

$$\mu_T(x) = \mu_T(xy^{-1}y) \geq \min \{\mu_T(xy^{-1}y), \mu_T(y)\}$$

$$= \min \{1, \mu_T(y)\} = \mu_T(y)$$

$$= \mu_T(y^{-1})$$

$$= \mu_T(x^{-1}xy^{-1})$$

$$= \min \{\mu_T(x^{-1}), 1\} = \mu_T(x^{-1}) = \mu_T(x).$$

Therefore, $\mu_T(x) = \mu_T(y)$, for all x and $y \neq e$ in F.

(ii) Now,

$$\upsilon_T(x) = \upsilon_T(x-y+y) \leq \max \{\upsilon_T(x-y), \upsilon_T(y)\}$$

$$= \max \{1, \upsilon_T(y)\} = \upsilon_T(y)$$

$$= \upsilon_T(-y)$$

$$= \upsilon_T(-x+x-y)$$

$$\leq \max \{\upsilon_T(-x), \upsilon_T(x-y)\}$$

$$= \max \{\upsilon_T(-x), 0\} = \upsilon_T(-x) = \upsilon_T(x).$$

Therefore, $\upsilon_T(x) = \upsilon_T(y)$, for all x and y in F.

And,

$$\upsilon_T(x) = \upsilon_T(xy^{-1}y) \leq \max \{\upsilon_T(xy^{-1}y), \upsilon_T(y)\}$$

$$= \max \{1, \upsilon_T(y)\} = \upsilon_T(y)$$

$$= \upsilon_T(y^{-1})$$

$$= \upsilon_T(x^{-1}xy^{-1})$$

$$\leq \max\{\upsilon_T(x^{-1}), \upsilon_T(xy^{-1})\}$$

$$= \max\{\upsilon_T(x^{-1}), 0\} = \upsilon_T(x^{-1}) = \upsilon_T(x).$$

Therefore, $\upsilon_T(x) = \upsilon_T(y)$, for all x and $y \neq e$ in F.

Theorem 3.6. *Let $(F, +, \cdot)$ be a field. If T is an intuitionistic fuzzy translation of an intuitionistic fuzzy subfield A of F, then $\mu_T(x + y) = \min\{\mu_T(x), \mu_T(y)\}$, for all x and y in F and $\mu_T(xy) = \min\{\mu_T(x), \mu_T(y)\}$, for all x and y in F and $\upsilon_T(x + y) = \max\{\upsilon_T(x), \upsilon_T(y)\}$, for all x and y in F and $\upsilon_T(xy) = \max\{\upsilon_T(x), \upsilon_T(y)\}$, for all x and y in F with $\mu_T(x) \neq \mu_T(y)$ and $\upsilon_T(x) \neq \upsilon_T(y)$ where 0 and 1 are identity elements of F.*

Proof. Let x and y belongs to F.

Assume that $\mu_T(x) > \mu_T(y)$ and $\upsilon_T(x) < \upsilon_T(y)$.

Now,

$$\mu_T(y) = \mu_T(-x + x + y) \geq \min\{\mu_T(-x), \mu_T(x + y)\}$$

$$\geq \min\{\mu_T(x), \mu_T(x + y)\} = \mu_T(x + y)$$

$$\geq \min\{\mu_T(x), \mu_T(y)\}$$

$$= \mu_T(y)$$

Therefore, $\mu_T(x + y) = \mu_T \ = \min\{\mu_T(x), \mu_T(y)\}$, for all x and y in F.

Now,

$$\mu_T(y) = \mu_T(x^{-1}xy) \geq \min\{\mu_T(x^{-1}), \mu_T(xy)\}$$

$$\geq \min\{\mu_T(x), \mu_T(xy)\} = \mu_T(xy)$$

$$\geq \min\{\mu_T(x), \mu_T(y)\} = \mu_T(y).$$

Therefore, $\mu_T(xy) = \mu_T(y) = \min\{\mu_T(x), \mu_T(y)\}$, for all x and y in F.

And,

$$\begin{aligned} \upsilon_T(y) = \upsilon_T(-x + x + y) &\le \max\{\upsilon_T(-x), \upsilon_T(x + y)\} \\ &\le \max\{\upsilon_T(y), \upsilon_T(x + y)\} = \upsilon_T(x + y) \\ &\le \max\{\upsilon_T(x), \upsilon_T(y)\} = \upsilon_T(y). \end{aligned}$$

Therefore, $\upsilon_T(x + y) = \upsilon_T(y) = \max\{\upsilon_T(x), \upsilon_T(y)\}$, for all x and y in F. And,

$$\begin{aligned} \upsilon_T(y) = \upsilon_T(x^{-1}xy) &\le \max\{\upsilon_T(x^{-1}), \upsilon_T(xy)\} \\ &\le \max\{\upsilon_T(x), \upsilon_T(xy)\} \\ &= \upsilon_T(xy) \\ &\le \max\{\upsilon_T(x), \upsilon_T(y)\} \\ &= \upsilon_T(y). \end{aligned}$$

Therefore, $\upsilon_T(xy) = \upsilon_T(y) = \max\{\upsilon_T(x), \upsilon_T(y)\}$, for all x and y in F.

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