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A REVIEW PAPER ON CORONA PRODUCT OF GRAPHS

GAJENDRA PRATAP SINGH, APARAJITA BORAH and SANGRAM RAY

School of Computational and Integrative Sciences
Jawaharlal Nehru University
New Delhi 110067, India
E-mail: gajendra@mail.jnu.ac.in

Department of Mathematics
National Institute of Technology Sikkim
Sikkim-737139, India
E-mail: aparajita.juri@gmail.com

Department of Computer Science and Engineering
National Institute of Technology Sikkim
Sikkim-737139, India
E-mail: sray.cse@nitsikkim.ac.in

Abstract

The paper basically contains a brief elucidation about the notion of Corona Product of Graphs, inter alia, its shrouded potentialities in unriddling the puzzles whether scientific or mathematical. It is by and large a review paper on recent findings and works on corona product of graphs. As the very concept is yet to gain ground within the scientific community, it is our attempt to take up a concise study and unfold the nuances of corona product of graphs. Further, we will also pore out how the very notion of Corona Product of graph enjoys leverage over other concurrent methodologies. In one word, the paper will consolidate the idea of Corona Product of graphs in augmenting its future applications. Towards the end of this paper we also highlighted some open problems to proceed the research in this field concerned.

1. Introduction

The concept of Corona Product is a recent inclusion to mathematical vocabulary. It is still predominantly in its evolution stage and therefore demands further refinements. Albeit many aspects of the concept are not

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completely unwrapped till date, nevertheless, its potentiality in disentangling the most critical puzzles in future cannot be ignored altogether. Not only in Mathematics, it can be applied in myriad fields of human activity such as biology, social science, chemistry and even astronomy. In recent times, the researchers are more fascinated towards the graph operations such as its product. Among many existent graph operations researchers are more inclined towards the corona product because of its complex but unique structure since the corona product of one graph on itself generates replicas of the same graph forging a large structure akin to a giant star.

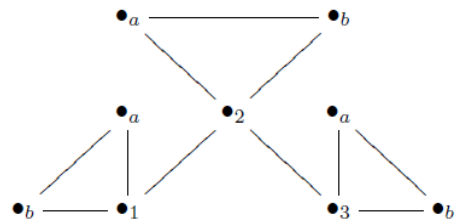
Therefore, it is intriguing but inevitable to study its structural properties and applications. In layman's terminology, the Corona Product can be branded as the amalgamation of two or more similar or disparate structures irrespective of number and degree of graphs to generate a gigantic network of nodes.

It is worthwhile to note that there are umpteen numbers of methodologies already available for creating a larger network. The enigma, however, may stem regarding the utmost emphasis that has been put on this novice concept. One can easily create a large network from a given set of nodes and links by simply augmenting one after another. However, such a methodology consumes a huge amount of time and incurs a bucket of costs. Hence, the aforementioned concept is thought to be lucrative in that aspect.

In current times the researchers are more fascinated towards the graph operations such as product. Among many existent graph operations researchers are showing more interest in corona product because of its structure. Since corona product one graph to itself creates replicas of the graph which generates a star like structure. Therefore it is interesting to study the structural properties and application of this type of graph.

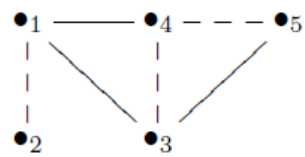
The formal mathematical definition of Corona product of two graphs is as follows Corona product of two graphs, say G_1 and G_2, was introduced by Frucht and Harary in [5] is a graph constructed by taking n instances of G_2 and each such G_2 gets connected to each node of G_1, where n is the number of nodes of G_1. In this paper we generalize this definition of corona product to get some graphs.

For example, let $G_1 = \bullet_1 \to \bullet_2 \to \bullet_3$ and $G_2 = \bullet_a \to \bullet_b$. Then $G_1 \circ G_2$ is given by



2. Definition and Preliminaries

Here Signed graphs are graphs $G = (V, E)$ with a sign function $\sigma : E \to \{+1, -1\}$. The function σ is called a signature for the corresponding signed graph. The sign $\sigma(e)$ of an edge $e \in E$ is interpreted as a characteristic of the edge e, for example, it provides a positive (if $\sigma(e) = +1$) or negative (if $\sigma(e) = -1$) relationship between the adjacent vertices which was introduced in social psychology. For example Σ_1:



Signed graph mostly depicts the behavioural property social-network. We can easily visualize social relationship among friends and enemies through signed graph [3].

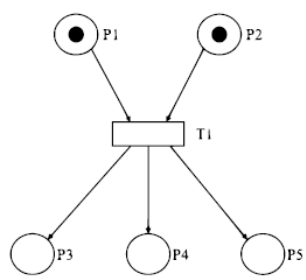


Figure 1. Petri net.

Petri Nets are exclusive directed structure of connectivity between two solely different types of nodes. It is basically a collection of directed arcs joining nodes called places and transitions. The places may or may not hold tokens. [17] In graph theory, Petri nets a directed bipartite graph where places and transitions are the partition sets of nodes. Here figure 1 is simple petri net containing its all components.

3. Review

Frucht and Harary [5] in 1970 introduced the binary product of two graphs named Corona. Here they defined the corona product and how to create a new graph as a product of two ordinary graphs. The main and strong property which is proved through this paper is that the group of newly contrived graph is isomorphic to the wreath product of the groups of the two graphs.

Many researchers have been working on various properties of corona product. Extensively the properties of spectrum and laplacian spectrum of corona product have been studied. Out of all these works, most significant one is in [1].

S. Barik, et al. in [1] raised a result on adjacency spectrum of corona product of two graphs using the adjacency spectrum of two graphs with a special case of one of the graph being regular. Also they showed results for laplacian spectrum of the corona product in terms of laplacian spectrum of the two graphs along with two applications to create infinitely many pairs of non-isomorphic graphs with same spectrum and laplacian spectrum. In addition they provided a result with a property R (reciprocal) and SR (strongly reciprocal). Also one result on algebraic connectivity and characteristic set of corona product is being shown here. After works of Barik, et.al many researchers have been trying to find more results on spectrum.

In [6] the author defined a new class of graphs called Neighbourhood Corona and he studied the spectrum of this new graph. This work is completely inspired by Bariks work where the author uses the same techniques and methods to find the spectrum of newly defined corona. We can visualize this is an extended work of [1].

Similarly in [8] the authors again defined a new operation called Edge Corona and also find the spectrum of this special kind of graphs using the same technique. In [12] the authors developed a new and explicit method to find the spectrum of corona product of two graphs by introducing a term called coronal. With this approach we can deftly find the spectrum of the product graph explicitly in terms of spectrum of two individual graphs and the Coronal. Also they computed coronals of some specific graphs such as Regular, complete bipartite, path graphs etc. Comparably in [4] the author introduced a new invariant M-coronal to obtain the spectrum of corona and edge-corona commingling the results in [1], [8] and [12]. Motivated by Corona and Edge Corona, in [10] the authors defined subdivision-vertex and subdivision-edge neighborhood coronae by using subdivision graph. Here they are also finding the spectra basically for both adjacency and laplacian by using M-Coronal defined in [4]. The notable application of all these special type corona (Neighbourhood Corona, Edge Corona, subdivision-vertex and subdivision-edge neighbourhood coronae) to produce infinitely many pairs of cospectral graphs.

In [2] authors defined initially two special types of matrices namely super corona matrix and super neighbourhood corona matrix and describe all the eigenvalues and corresponding eigen vectors. Further, they defined some new corona graphs such as subdivision double corona, Q-graph double corona, R-graph double corona, total double corona, subdivision double neighbourhood corona, Q-graph double neighbourhood corona, R-graph double neighbourhood corona and total double neighbourhood corona. They provided a technique by which we can easily obtain the nonzero laplacian eigenvalues of the corona graph from the spectrum of super corona matrix. Also they showed a result to compute the adjacency and the signless laplacian spectrum for the double corona graphs from super corona and super neighbourhood corona matrix.

In [16] authors extended a result on Behzad and Vizing conjecture on total coloring. They found a tight bound for total chromatic number of corona product of two graphs G and H with H being a cycle, complete and a bipartite graph.

The recent work on Corona Product is in [18] where the authors have collectively studied the algebraic structure of join and corona product of

graphs. The main motive of this paper is to obtain explicit formulae of degree sequence of both the graph operations join as well as corona product of graphs by using mathematical induction. They found compact formulae for degree sequence of the resulting graph in terms of vertices, vertex degrees and their frequencies which is convenient for further research.

4. Discussion

Corona product is of immense importance in Data Analytics where huge chunks of data need to be analyzed in a handful of time in order to arrive at a certain decision. In biotechnology, it can be used in DNA Sampling and forensic study [11]. In chemistry [9], it can be applied to understand the structures of chemical compounds. In social science, it can be helpful in understanding the behavioral pattern of a group of people or a community. In astronomy, it can be applied to understand the creation of the universe and anticipate the future changes, if any. Further, the corona product can be utilized in enhancing the recent emergence of science such as Internet of Things, Virtual Reality and Augmented Reality etc. Hence, Corona Product is of Prime Importance in our study.

5. Conclusions and Scope

The significance and importance of Corona Product rests not only on the generation of a larger network, but the speed at which such a giant network is generated. In the modern era, where data and objects are moving in such a rapid pace, Corona Product will certainly enjoys more leverage in terms of speed and cost in anchoring and augmenting the mathematical and scientific mysteries and therefore it outweighs its contemporary algorithms and methodologies on that score.

Further study has been undertaken of the following problems too.

1. Characterize the petri net ([15], [13], [14], [7]) by using graph operation such as Corona product and can we convert it to a significant reachability graph by using graph operations?

2. Our next open problem is to convert a petri net into a signed petri net by assigning signs (+ or -) to the arcs before and after firing of the token. We must need one or more conditions while assigning signs since the sign should

be consistent throughout the firing along with the pre and post conditions. This marking to the arcs will definitely add some behavioural property to the petri net models with some additional information to the data flow.

Again similarly, can we convert and characterize a signed graph into a petri net and do the comparative study on their structural properties?

3. To form n-cube: The structural properties of corona product of two graphs can be study extensively and it is so into use to create larger graph. As in higher dimensional cube, identification of square faces is difficult when the overlap among the faces is larger. By using corona product we can definitely construct some bigger structure or higher dimensional complex. So, here an open problem is that can we build such n-cube or similar structure (polyhedron) simply by using corona product without such overlapping of faces?

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References

[1] S. Barik, S. Pati and B. K. Sarma, The spectrum of corona of two graphs, SIAM Journal on Discrete Mathematics 21(1) (2007), 47-56.

[2] S. Barik and G. Sahoo, On the laplacian spectra of some variants of corona, Linear Algebra and its Applications 512(1) (2017), 32-47.

[3] Dorwin Cartwright and Frank Harary, Structural balance: a generalization of heider's theory, Psychological Review 63(5) (1956), 277.

[4] Shu-Yu Cui and Gui-Xian Tian, The spectrum and signless laplacian spectrum of coronae, Linear Algebra and Its Applications 437 (2012), 1692-1703.

[5] Roberto Frucht and Frank Harary, On the corona of two graphs, Aequationes Mathematicae 4(3) (1970), 322-325.

[6] I. Gopalapillai, The spectrum of neighbourhood corona of graphs, Kragujevac Journal of Mathematics 35(3) (2011), 493-500.

[7] G. P. Singh and S. K. Singh, Petri net recommender system for generating of perfect binary tree, International Journal of Knowledge and Systems Science 10(2) (2019), 1-12.

[8] Yaoping Hou and Wai-chee Shiu, The spectrum of the edge corona of two graphs, Electronic Journal of Linear Algebra 20 (2010), 586-594.

[9] A. Coffmann John and R. William Browne, Corona chemistry, Scientific American 212(6) (1965), 90-99.

[10] Xiaogang Liu and Pengli Liu, Spectra of subdivision-vertex and subdivision-edge neighbourhood coronae, Linear Algebra and Its Applications 438 (2013), 3547-3559.

[11] Winder Louise, Evaluation of DNA melting analysis as a tool for species identification, Methods in Ecology and Evolution 2(3) (2011), 312-320.

[12] Cam McLeman and Erin McNicholas, Spectra of coronae, Linear Algebra and Its Applications 435 (2011), 998-1007.

[13] S. Kansal, G. P. Singh and M. Acharya, On petri nets generating all the binary n-vectors, Scientific Mathematicae Japanicae 71(2) (2010), 209-216.

[14] S. Kansal G. P. Singh and M. Acharya, 1-safe petri nets generating every binary n-vector exactly once, Scientific Mathematicae Japanicae 74(1) (2011), 29-36.

[15] S. Kansal, M. Acharya and G. P. Singh, Boolen petri nets, In petri nets-Manufacturing and Computer Science, Intech Open, (2012), pages 381-406.

[16] S. Mohan, J. Geetha and K. Somasundaram, Total coloring of corona product of two graphs, Australian Journal of Combinatorics 68(1) (2017), 15-22.

[17] Murata Tadao, Petri nets Properties, analysis and applications, IEEE 77(4) (1989).

[18] V. N. Mishra, Dalan Sadik and Ismail Naci Cangul, Degree sequences of join and corona product of graphs, Electronic Journal of Mathematical Analysis and Applications 7(1) (2019), 5-30.