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SOLVING FUZZY ASSIGNMENT PROBLEM USING MINIMUM ALGORITHM

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Abstract

Assignment Problem is a specific case of transportation problem. In assignment problem, we allocate the different resources to the same number of activities on one to one basis to minimize the total cost and it is used to solve real valued problems. Suppose the cost values of the Assignment Problem are fuzzy numbers then we transform the fuzzy numbers to crisp values and then only solve the assignment problem easily. In this content we consider the Fuzzy Assignment Problem with triangular fuzzy numbers. First the triangular fuzzy numbers are changed into crisp values using Robust ranking method [9]. Then using our new Minimum Algorithm the optimum schedule of the Fuzzy Assignment Problem is obtained. This process is explained with a numerical example.

1. Introduction

A specific case of transportation problem with equal number of resources and activities is known as Assignment Problem. We have to find the optimal solution to assign the various resources to the same number of activities at a least cost. First the term Assignment problem was introduced in Vo Taw and Orden (1952). All the algorithms which are used to find the optimal solution

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of Transportation problem are suitable to the assignment problem. But Kuhn [1] introduced a specific algorithm to find the optimal solution for assignment problem which is degenerate in nature named as Hungarian Algorithm. In real world situations, the boundary of Assignment Problem are not a fixed real numbers because the task by various persons or machines take the different time and cost due to different reasons.

Lotfi A. Zadeh [2] introduced the fuzzy set theory in 1965. Chen [3] suggested a fuzzy assignment model that did not consider the differences of characters. Using Graph theory Wang [4] solved the same model. Dubois and Fortemphs [5] developed a algorithm with the combination of fuzzy theory and multiple criteria decision making. In 2004 C. J. Lin and W. P. Wen [6] introduced a labelling method for solving the fuzzy assignment problem.

Feng and Yang [7] investigated a two objective cardinality assignment problem. Liu and Gao [8] proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi job assignment problem. Nagarajan and Solairaju [9] solved the fuzzy assignment problem by converting the fuzzy numbers into crisp numbers using Robust ranking techniques. K. Kalaiarasi, S. Sindhu and M. Arunadevi [10] proposed a method to solve the triangular fuzzy assignment problem using robust ranking method and Hungarian one algorithm. In this paper the fuzzy assignment problem has been converted into crisp assignment problem using Robust Ranking Method and then we can find the optimal solution using New Minimum.

2. Preliminaries

Definition 2.1 (Fuzzy Set)**.** Let A be the subset of X where, X is the universe of objects then the characteristic function μ_A of a crisp set A defined by $\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$

The *fuzzy set* $\widetilde{A}$ is the function $\mu_{\widetilde{A}}$ that carries X into $[0, 1]$ i.e. $\mu_{\widetilde{A}} : X \to [0, 1]$, $\mu_{\widetilde{A}}$ is called the *membership function* which is a value on the unit interval that measures the degree to each $x \in \widetilde{A}$.

Definition 2.2. A Fuzzy set defined on universal of real numbers R is said to be a *fuzzy number* has the following characteristics:

(i) $\widetilde{A}$ is convex i.e., $\mu_{\widetilde{A}}(\lambda a + (1 - \lambda)b) \geq$ minimum $(\mu_{\widetilde{A}}(a), \mu_{\widetilde{A}}(b)) \forall a$, $b \in R$ and $0 \leq \lambda \leq 1$.

(ii) $\widetilde{A}$ is normal, (i.e.) the membership function of $\widetilde{A}$ has at least one element $x \in X$ whose

Membership value is unity.

(iii) $\mu_{\widetilde{A}}$ is continuous except at a finite number of points in its domain.

Definition 2.3. A fuzzy number $\widetilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & x = b \\ \dfrac{(x-c)}{(b-c)} & b \leq x \leq c \end{cases}$$

where $a, b, c \in R$.

Definition 2.4 (λ-cut of a Triangular fuzzy number) [11]**.** The λ-cut of a fuzzy number $A(x)$ is defined as $A(\lambda) = \{x : \mu(x) \geq \lambda, 0 \leq \lambda \leq 1\}$.

Definition 2.5 (Addition of two fuzzy numbers)**.**

If $\widetilde{A} = (a_1, b_1, c_1)\widetilde{B} = (a_2, b_2, c_2)$ then $\widetilde{A} + \widetilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$.

3. Robust's Ranking Technique [10]

Robust's ranking technique which satisfies linearity, and additives properties and provides results which are consistent with human intuition. Give a convex fuzzy number $\widetilde{A}$, the Robust's Ranking Index is defined by $R(\widetilde{A}) = \int_0^1 0.5(a_\lambda^L a_\lambda^U) d\lambda$ where $(a_\lambda^L a_\lambda^U)$ is the λ-level cut of the fuzzy number. In this paper we use this method for ranking the objective values. The

Robust's ranking index $R(\widetilde{A})$ gives the representative value of the fuzzy number $\widetilde{A}$. It satisfies the linearity and additive property.

Defuzzification [11]**.** Defuzzification of the fuzzy set is the process of converting the fuzzy set with precise quantity instead of fuzzy quantity. Here Robust Ranking method is used to defuzzify the triangular fuzzy numbers because of its clarity and accuracy.

4. Assignment Problem

The assignment problem can be stated in the form of $n \times n$ cost matrix $[C_{ij}]$ of real numbers as given in the following table:

Jobs→ Persons↓	1	2	3	.. j..	n
1	C_{11}	C_{12}	C_{13}	.. $C_1 j$ ..	C_{1n}
2	C_{21}	C_{22}	C_{23}	.. $C_2 j$ ..	C_{2n}
-	-	-	-	-	-
i	C_{i1}	C_{i2}	C_{i3}	.. $C_i j$ ..	C_{in}
-	-	-	-	-	-
n	C_{n1}	C_{n2}	C_{n3}	.. $C_n j$ ..	C_{nn}

Mathematically assignment problem can be stated as

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$

Subject to $\sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, 3, \ldots, n$

$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, 3, \ldots, n$

Where $x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{persson assigned the } j^{th} \text{job} \\ 0 & \text{otherwise} \end{cases}$

and C_{ij} stands for the cost of assignment of person i to the job j.

When the costs $\overline{C}_{ij}$ are fuzzy numbers then the total cost becomes a fuzzy number. Then the fuzzy objective function is

$$\text{Minimize } \bar{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{C}_{ij} x_{ij}$$

where $\overline{C}_{ij} = (a, b, c)$, the triangular fuzzy numbers. It cannot be minimized directly. So first convert the fuzzy cost coefficients into crisp ones. Robust ranking method is used for defuzzification.

5. New Minimum Algorithm

Step 1. Check whether the assignment problem has equal number of rows or columns (balanced). If not add row or column with cost value 0 and make it as a balanced one.

Step 2. Select the smallest cost element from first row as well as first column and mark it.

Step 3. Similarly consider the next row as well as the next column and mark the minimum cost element continuing in this way until all the rows and columns have been checked. If there are more than one allotment in any row or column consider the least cost in that row or column.

Step 4. Check each and every row or column have the allotment.

6. Numerical Example: (Triangular Fuzzy Number)

Consider the problem of assigning four machines are available to do four various jobs. The following matrix gives the cost in rupees of producing job i on machine j and is represented by triangular fuzzy numbers.

$C_{ij} =$

	M_1	M_2	M_3	M_4
J_1	(1, 5, 9)	(3, 7, 11)	(7, 11, 15)	(2, 6, 10)
J_2	(4, 8, 12)	(1, 5, 9)	(4, 9, 13)	(2, 6, 10)

J_3	(0, 4, 8)	(3, 7, 11)	(6, 10, 14)	(3, 7, 11)
J_4	(6, 10, 14)	(0, 4, 8)	(4, 8, 12)	(–1, 3, 7)

Assign the machines to different jobs so that the total cost is minimized.

Solution. Now we convert the fuzzy cost to the crisp cost value by applying Robust ranking method [10].

The membership function of the triangular fuzzy number (1, 5, 9) is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-1)}{(5-4)} & 1 \le x \le 5 \\ 1 & x = 5 \\ \frac{(9-x)}{(5-9)} & 5 \le x \le 9 \end{cases}$$

Robust's Ranking Index is defined by $(\widetilde{A})' = \int_0^1 0.5(a_\lambda^L a_\lambda^U)d\lambda$ where is the λ-level cut of the fuzzy number.

$$(1, 5, 9) = \int_0^1 0.5((5-1)\lambda + 1, 9 - (9-5)\lambda)d\lambda = 7$$

$$(3, 7, 11) = \int_0^1 0.5((7-3)\lambda + 3, 11 - (11-7)\lambda)d\lambda = 7$$

(7,11,15)' = 11 (2,6,10)' = 6 (4,8,12)' = 8

(1,5,9)' = 5 (4,9,13)' = 8.75 (2,6,10)' = 6

(0,4,8)' = 4 (3,7,11)' = 7 (6,10,14)' = 10

(3,7,11)' = 7 (6,10,14)' =10 (0,4,8)' = 4

(4,8,12)' = 8 (-1,3,7)' = 3

The new cost table is

	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	8.75	6

J_3	4	7	10	7
J_4	10	4	8	3

Now we solve the assignment problem using our new minimum algorithm.

First consider the first row and first column. The minimum cost is 4 in the cell (1, 3). Next consider the second row and second column. The minimum cost is 4 in the cell (4, 2). Then take the third row and third column. The minimum cost is 4 but it is already allocated. Then choose the next minimum cost. Suppose we choose 7 or 8 there are allocations in that row or column with minimum cost so we consider 8.75 in the cell (2,4). Finally consider the fourth row and fourth column. The minimum element is 3. Already there is a allocation in fourth row. But comparing these two costs, 3 is minimum. So we allocate in the cell (4,4). In the similar way we reallocate the cells we get the optimal allocations are in the cells (1,3), (2,2), (3,1) and (4,4). The fuzzy optimal cost is (7,11,15) + (1,5,9) + (0,4,8) + (-1,3,9) = (7,23,39)

The optimum schedule is $J_1 \to M_3$, $J_2 \to M_2$, $J_3 \to M_1$, $J_4 \to M_4$.

The optimum assignment cost is $11 + 5 + 4 + 3 = 23$.

7. Conclusion

In this paper, the assignment cost has been considered as triangular fuzzy numbers which are more practical and general in nature. Here, the fuzzy assignment problem has been changed into crisp assignment problem using Robust ranking method [10] and the optimal solution is obtained by our new minimum algorithm. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem. Comparing the assignment cost which has been found in the above example calculated by existing method is same but our method is easy to calculate. Not only the triangular fuzzy numbers we can take any type of fuzzy numbers, first change it into a crisp one using any method and then solve it by our new minimum algorithm.

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