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BINGHAM PLASTIC FLUID FLOW ANALYSIS BETWEEN TWO MOVING INFINITE PARALLEL PLATES

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Abstract

In this paper, the velocity and Temperature effects are analysed between two parallel plates in the presence of Bingham plastic fluid. The upper and lower parallel Plates are thought to be travelling in the same direction and at the same speed. The incompressible two-dimensional flow between two parallel plates is examined using Bingham plastic fluid. The temperature and velocity profile are studied by solving the simultaneous system of nonlinear equation. In three cases, the impact of the temperature profile is explored. The equations formed by motion, continuity and energy equation are solved using conventional analytical techniques and then fourth order Runge-Kutta method was used to solve the problem numerically. The result for velocity and temperature is obtained and present graphically with respect to various physical parameter. The goal of this research is to look into the influence of pressure and temperature on various fluid parameters. Finally, a comparison with a number of previously published results was reviewed.

1. Introduction

The fluid flow problem between two parallel plates has got enough application in the field of science and technology. Hydrodynamic lubrication, petroleum industry, biomechanics are some of the application areas. Many researchers have discussed about this flow with different geometry with different flow parameters. Bingham plastic fluid can be used to replicate the rheological behaviour of a variety of lubricants used for different purposes.

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Lampaert, Stefan GE et al. [1]. A thin layer lubrication for a Bingham plastic fluid was discussed. It has been explained that the model does not require any additional approximations in deriving the Bingham plastic fluid model does not require any numerical regularization because it is based on the generic Reynolds equation. Shirkhani, M. R., et al. [2] studied the incompressible Newtonian fluid between two parallel plates numerically and analytically. Author has used collocation method and perturbation method also finally used RK method to solve the problem and presented the velocity field.

Derakhshan, R., et al. [3] studied between two plates that are parallel, heat and mass transmission occur using nano fluid. AGM, and RK methods are compared with various parameter discuss the effect of thermophoretic parameter towards Brownian motion.

Salehi, Sajad, et al., [4] discussed Magneto hydrodynamic squeezing Nano fluid flow between two infinity parallel plates: flow and heat transfer. The non-linear equations were solved using Akbari-Ganji's method, and the results were compared using a numerical method, and the velocity profile was investigated.

Miranda, Luiz Paulo Borges et al. [5], the chosen geometry is that of a planar channel with a sudden expansion and contraction. Heat transfer happens exclusively in the cavity generated between the expansion and contraction planes because the channel walls are kept insulated.

Sobamowo, M. G et al., [6]. The regular perturbation approach is used to investigate the squeezing flow of Cu-water and Cu-kerosene under the influence of a magnetic field with a pressure gradient. The analytical solutions obtained are utilized to explore the squeezing phenomena of the nanofluid when the plates move apart and together. Abbas, Zaheer, HasnainJafar and Muhammad AsifJafar [7], the flow of a non-Newtonian(Casson) fluid is studied between parallel discs moving in various directions in planar motion. The fluid motion is caused by the top disc radially stretching or shrinking at a rate a and the bottom spinning disc at an angular velocity Ω, with plate spacing h.

Berabou, Welid, et al. [8], examined the laminar forced convection heat transfer for a simplified Casson fluid flow across a horizontal circular pipe and between two parallel plates kept at the same temperature. Many researchers have done many investigations with different fluid flow with various parameter between two parallel plates and in this work, we have considered between two parallel plates, the Bingham plastic fluid flow, where both plates are considered to be in the same direction with same velocity. The velocity and temperature profile has been studied discussed in graphs and table. The findings are also compared to the literature, which shows that they are in good accord.

2. Governing Equations

In Cartesian coordinates, this topic examines a two-dimensional incompressible non-Newtonian fluid flowing between two infinite length rectangular parallel plates separated by h. Figure 1, shows the geometry of the problem under consideration. It is also considered that both the plates are moving in same direction and with same velocities. Upper plate is at $y = h/2$ and lower plate is positioned at $y = -h/2$. $Y = 0$ is the midpoint of both the plates. The film thickness is h which separates the both plates.

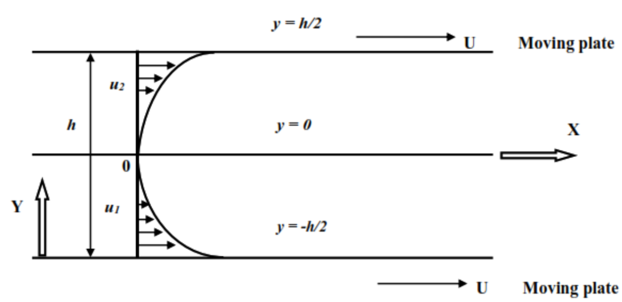

Figure 1. Geometry of the problem.

The governing equation for the problem considered as

$$\frac{\partial u}{dx} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$\frac{dp}{dx} + \frac{d\tau}{dx}, \text{ where } \tau = \tau_0 + \mu \frac{du}{dy}; \tag{2}$$

The boundaries are as follows:

$$\left.\begin{array}{l} u_1 = U \text{ at } y = -h/2 \\ u_2 = U \text{ at } y = -h/2 \\ \dfrac{du}{dy} = 0 \text{ at } y = 0 \end{array}\right\} \tag{3}$$

The profile of the velocity for each region is presented below.

$$\left.\begin{array}{l} \dfrac{du_1}{dy} \leq 0, \text{ for } -\dfrac{h}{2} \leq y \leq 0; \\ \dfrac{du_2}{dy} \geq 0, \text{ for } 0 \leq y \leq \dfrac{h}{2}; \end{array}\right\} \tag{4}$$

Use of the above boundary conditions equation (3) in equation(2), it gives the velocity distribution of fluid as

$$\therefore u = U + \left(\frac{-1}{\mu}\frac{dp}{dx}\right)\left(\frac{h^2}{8}\ \frac{y^2}{2}\right), \text{ for } -\frac{1}{2} \leq \frac{y}{h} \leq \frac{1}{2}; \tag{5}$$

Assuming that there are no pressure gradients $p'(x) = 0$, equation (5) becomes

$$u = U, \text{ for } -\frac{h}{2} \leq y \leq \frac{h}{2};$$

The velocity of the fluid for both lower and upper regions are

$$u = U + \left(\frac{-1}{\mu}\frac{dp}{dx}\right)\left(\frac{h^2}{8} - \frac{y^2}{2}\right), \text{ for } -\frac{h}{2} \leq y \leq \frac{h}{2}; \tag{6}$$

Total Discharge of fluid per unit width of plates:

$$Q = \int_{-h/2}^{h/2} u dy.$$

$$Q = Uh + \left(\frac{-1}{\mu}\frac{dp}{dx}\right)\left(\frac{h^3}{12}\right).$$

Average Velocity of fluid:

$$u_a = U + \left(\frac{-1}{\mu}\frac{dp}{dx}\right)\left(\frac{h^2}{12}\right).$$

Energy equation:

$$k\left(\frac{d^2T}{dy^2}\right)\tau\left(\frac{du}{dy}\right) = 0. \tag{7}$$

Simplifying the above equation, then we will get

$$\therefore\ T = -\frac{16\mu A^2U^2}{3kh^4}y^4 + \frac{4\tau_0 AU}{3kh^2}y^3 + cy + d,\ \text{for } -\frac{h}{2} \le y \le \frac{h}{2}; \tag{8}$$

Plates are kept at same temperature:

Boundary conditions are $T_1 = T_L$ at $y = -h/2$ and $T_2 = T_L$ at $y = h/2$.

Also using $T_1 = T_L$ at $y = 0$ as the interface temperature condition and at $y = 0$, the interface temperature gradient $\frac{dT_1}{dy} = \frac{dT_2}{dy}$.

Using boundary conditions to solve the above equation (8), we arrive at

$$T = T_L + \frac{\tau_0 AUy}{3y}\left(4\frac{y^2}{h^2} - 1\right) - \frac{\mu A^2U^2}{3k}\left(16\frac{y^4}{h^4} - 1\right),\ \text{for } -\frac{h}{2} \le y \le \frac{h}{2};$$

Let T_m denote the channel's temperature in the midpoint, that is to say, $T_1 = T_2 = T_m$ when $y = 0$.

$$T_m - T_L = \frac{\mu A^2U^2}{3k}.$$

Plates are kept at different Temperature:

Boundary conditions are $T_1 = T_L$ at $y = -\frac{h}{2}$ and $T_2 = T_U$ at $y = \frac{h}{2}$.

Using the above boundary conditions in equation (8), we get

$$T = T_L + \left(\frac{T_U - T_L}{2}\right) + \left(\frac{T_U - T_L}{h}\right)y + \frac{\tau_0 AUy}{3k}\left(4\frac{y^2}{h^2} - 1\right)$$
$$-\frac{\mu A^2U^2}{3k}\left(16\frac{y^4}{h^2} - 1\right),\ \text{for } -\frac{h}{2} \le y \le \frac{h}{2};$$

One plate is kept at adiabatic and other at constant Temperature:

Boundary conditions are $\frac{dT_1}{dy} = 0$ at $y = -\frac{h}{2}$ and $T_2 = T_U$ at $y = \frac{h}{2}$.

The outcome of using the boundary conditions in equation (8) is

$$T = T_U + \frac{\tau_0 AUh}{3k}\left(4\frac{y^3}{h^3} - 3\frac{y}{h} + 1\right) - \frac{\mu A^2 U^2}{3k}\left(16\frac{y^4}{h^4} + 8\frac{y}{h} - 5\right), \quad \text{for}$$

$-\frac{h}{2} \le y \le \frac{h}{2}$;

3. Result and discussion

The numerical values used for computing the solution for the given system of equation are $\tau_0 = 0.1$, $y/h = 1$, $\mu = 0.001$, $T_L = 0.4$, $A = -2, -1, 0, 1, 2$, $P = 0, 2, 4, 6, 8$ where $P = \frac{\tau_0 Uh}{3k(T_U - T_L)}$ and $A = \left(\frac{-h^2}{8\mu U}\right)\frac{dp}{dx}$.

Figure 2, represents the velocity profile which shows that for different values of A, with respect to $\bar{y}$. The velocity of fluid $\bar{u}$ decreases when $\bar{y}$ increases in the lower region, whereas it increases in the upper region. The velocity distribution at different values of A is as shown in below Table 1. All the Figures represents the temperature profile of the fluid except Figure 1 and 2. Figure 3, it is considered when both the plates are of same temperature with different values of A. In the lower region, the temperature of the fluid $\bar{T}$ rises with $\bar{y}$, while in the upper zone, it lowers with $\bar{y}$. Figure 4, shows the varying temperature with respect to A where both the plates are with different temperature. When $A = -1$ as $\bar{y}$ increases, the temperature of fluid $\bar{T}$ drops in the lower part of the body and increases in the upper part of the body. This behaviour is true for all N. The only difference among them is their magnitudes. Matches well with the results of that of Brandi. A. C and Danane, Fetta, et al. [10, 11]. The same trend has been observed for the temperature at $A = 0$ and $A = 1$ shown in Figure 5 and 6. For the both lower and upper regions, the temperature of fluid $\bar{T}$ decreasing throughout with $\bar{y}$ increasing. Similar profile for temperature has been presented by Hassan, A. R and Mollah, Md. Tusher. [9, 12] for Couette flows as shown in the below Figure7. In this case, it is adiabatic in the lower plate, while the temperature of the upper plate is kept constant.

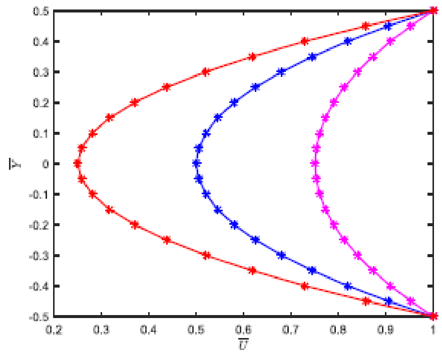

Figure 2. Profile of velocity.

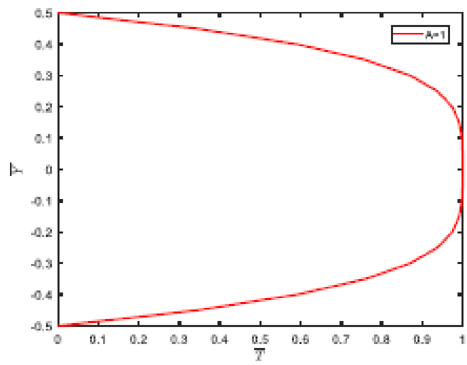

Figure 3. Temperature profile with same temperature at $A = 1$.

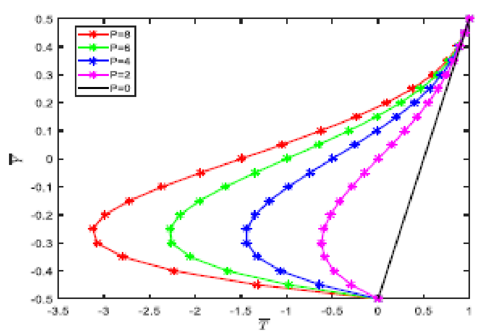

Figure 4. Temperature profile with different temperature at $A = -1$.

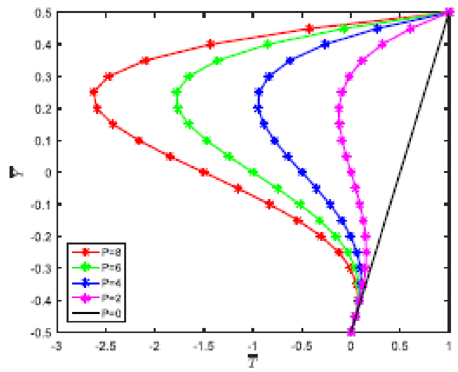

Figure 5. Temperature profile with different temperature at $A = 1$.

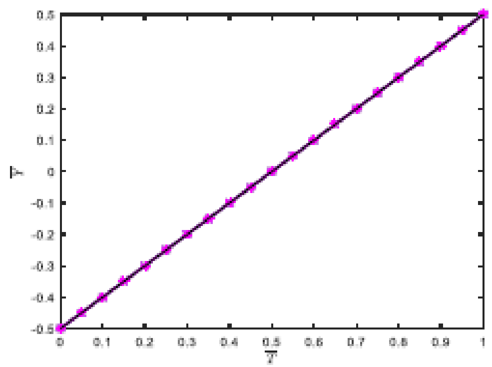

Figure 6. Temperature profile with different temperature at $A = 0$.

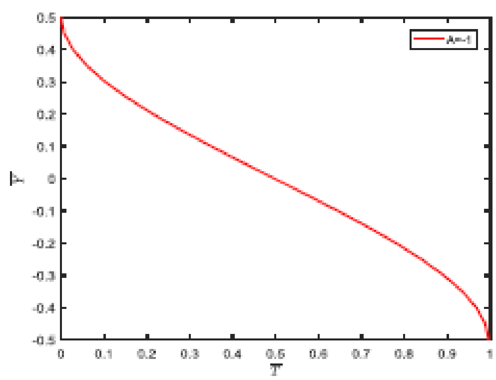

Figure 7. Temperature profile with adiabatic effect with $A = -1$.

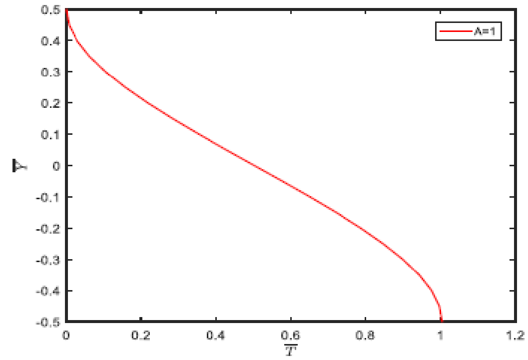

Figure 8. Temperature profile with adiabatic effect with $A = 1$.

4. Conclusion

A semi scientific methodology is made to tackle the numerical model where Bingham plastic liquid with steady state is considered. Parallel plates are taken into account throughout the problem of incompressibility, total fluid discharge, average velocity, shear stress and pressure head loss, and fluid velocity dispersion. The liquid flow between two equal rectangular parallel plates both the plates are moving with same velocity and in same direction. The velocity and temperature distribution of the liquid has been investigated by thinking about something same, different and adiabatic temperature in both the plates. The comparison reveals that the proposed solutions are highly precise and allow for the computation of flow velocities to be completed quickly. These methods are also, according to earlier papers, a powerful methodology for discovering analytical answers in scientific and engineering problems. The velocity and temperature dissemination is appeared in graphs and tables which acquired by utilizing the MATLAB programming after solving the equation of motion and energy condition with reasonable limit conditions. Also, it has been seen that every one of the outcomes are well concurrence with a portion of the experimental work [8].

The following is a synopsis of the findings:

- The profile of velocity $\overline{u}$ decreases when $\overline{y}$ increases in the lower region, whereas it increases in the upper region.

- When both plates are travelling at the same temperature, the temperature in each zone is the opposite of the velocity profile.
- When $A = -1$ as $\bar{y}$ increases, the temperature of fluid $\overline{T}$ drops in the lower part of the body and increases in the upper part of the body. This r is for different temperature.

For the both lower and upper regions, the temperature of fluid $\overline{T}$ decreasing throughout with $\bar{y}$ increasing. This is adiabatic temperature.

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