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MINIMAL AND MAXIMAL OPEN SETS IN GITS

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Abstract

In this article, minimal and maximal μ_I -open sets in generalized intuitionistic topological spaces were introduced. Also, some of their basic properties are investigated. In addition, the new structure of minimal and maximal μ_I -open sets were classified by type-I, type-II and type-III. Regarding these sets, some minimal and maximal μ_I -continuous functions in generalized intuitionistic topological spaces were introduced and studied in detail.

1. Introduction

The specialization of an intuitionistic fuzzy set was given by Coker [6]. After that time, intuitionistic topological spaces were introduced [14]. A. Csaszar [4] introduced many closed sets in generalized topological spaces based on their basics. We have introduced a new type of topology called as generalized intuitionistic topological spaces with the help of intuitionistic closed sets. And then we have investigated and studied some μ_I -maps in GITS. In 2001, minimal and maximal open sets were introduced and some applications of these sets were studied by Nakaoka and Oda [8]. Thereafter, minimal μ-open sets in generalized topological spaces were introduced [5].

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The subject like minimal and maximal continuity, minimal and maximal irresolute etc [1] were investigated on basic topological spaces. The aim of this paper is to introduce minimal and maximal μ_I-open sets in GITS. Also minimal and maximal μ_I-continuous functions were introduced and studied in detail. In addition, we give some examples and counterexamples for support this work.

2. Preliminaries

Definition 2.1. A μ_I-topology on T_x is a family of intuitionistic subsets of T_x satisfying the following axioms:

1. $\Phi_\sim \in \mu_I$

2. Union of elements of μ_I belongs to μ_I.

For a GITS (T_x, μ_I), the mates of μ_I are called μ_I-open sets(briefly μ_I-opss) and the complement of μ_I-opss are known as μ_I-closed sets(briefly μ_I-cdss).

Note: $I_{\mu_I}(\Phi_\sim) = \Phi_\sim$, $I_{\mu_I}(T_{x\sim}) \neq T_{x\sim}$, $C_{\mu_I}(\Phi_\sim) \neq \Phi_\sim$, $C_{\mu_I}(T_{x\sim}) = T_{x\sim}$.

Results.

1. Arbitrary intersection of μ_I-cdss are μ_I-cd.

2. Union of two μ_I-cdss is not necessarily μ_I-cd.

Properties. Let W_K and K_0 a ICS (T_x). Then,

1. If $W_K \subseteq B \Rightarrow C_{\mu_I}(W_K) \subseteq C_{\mu_I}(K_0)$.

2. $C_{\mu_I}(C_{\mu_I}(W_K)) = C_{\mu_I}(W_K)$.

3. $C_{\mu_I}(W_K \cap K_0) \subseteq C_{\mu_I}(W_K) \cap C_{\mu_I}(K_0)$.

4. $C_{\mu_I}(W_K) \cup C_{\mu_I}(K_0) = C_{\mu_I}(W_K \cup K_0)$.

5. $T_x - C_{\mu_I}(W_K) = I_{\mu_I}(T_x - W_K)$.

6. $T_x - I_{\mu_I}(W_K) = C_{\mu_I}(T_x - W_K)$.

7. If $W_K \subseteq K_0 \Rightarrow I_{\mu_I}(W_K) \subseteq I_{\mu_I}(K_0)$.

8. $I_{\mu_I}(I_{\mu_I}(W_K)) = I_{\mu_I}(W_K)$.

9. $I_{\mu_I}(W_K \cup K_0) = I_{\mu_I}(W_K) \cup I_{\mu_I}(K_0)$.

10. $I_{\mu_I}(W_K \cap K_0) \subseteq I_{\mu_I}(W_K) \cap I_{\mu_I}(K_0)$.

Definition 2.2. Let (T_x, μ_I) be a GITS. Then the ICS $W_K(T_x)$ is said to be a

1. pre μ_I-cds(briefly $\mathbb{P}\mu_I$-cds) if $C_{\mu_I}(I_{\mu_I}(W_K)) \subseteq W_K$.

2. semi μ_I-cds(briefly $\mathbb{S}\mu_I$-cds) if $I_{\mu_I}(C_{\mu_I}(W_K)) \subseteq W_K$.

3. $\alpha\mu_I$-cds if $C_{\mu_I}(I_{\mu_I}(C_{\mu_I}(W_K))) \subseteq W_K$.

4. $\beta\mu_I$-cds if $I_{\mu_I}(C_{\mu_I}(I_{\mu_I}(W_K))) \subseteq W_K$.

Definition 2.3. Let (T_x, μ_I) be a GITS. Then the IOS $L_K(T_x)$ is said to be a

1. pre μ_I-ops(briefly $\mathbb{P}\mu_I$-ops) if $L_K \subseteq I_{\mu_I}(C_{\mu_I}(L_K))$

2. semi μ_I-ops(briefly $\mathbb{S}\mu_I$-ops) if $L_K \subseteq C_{\mu_I}(I_{\mu_I}(L_K))$

3. $\alpha\mu_I$-ops if $L_K \subseteq I_{\mu_I}(C_{\mu_I}(I_{\mu_I}(L_K)))$

4. $\beta\mu_I$-ops if $L_K \subseteq C_{\mu_I}(I_{\mu_I}(C_{\mu_I}(L_K)))$

Results.

1. Every μ_I-cds(resp. μ_I-ops) is a $\mathbb{P}\mu_I$-cds(resp. $\mathbb{P}\mu_I$-ops).

2. Every μ_I-cds(resp. μ_I-ops) is a $\mathbb{S}\mu_I$-cds(resp. $\mathbb{S}\mu_I$-ops).

3. Every μ_I-cds(resp. μ_I-ops) is a $\alpha\mu_I$-cds(resp. $\alpha\mu_I$-ops).

4. Every μ_I-cds(resp. μ_I-ops) is a $\beta\mu_I$-cds(resp. $\beta\mu_I$-ops).

5. Every $\mathbb{P}\mu_I$-cds(resp. $\mathbb{P}\mu_I$-ops) is a $\beta\mu_I$-cds(resp. $\beta\mu_I$-ops).

6. Every $\alpha\mu_I$-cds(resp. $\alpha\mu_I$-open) is a $\mathbb{S}\mu_I$-cds(resp. $\mathbb{S}\mu_I$-ops).

Definition 2.4. A mapping $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ is said to be μ_I-continuous (briefly μ_I-cts) if the inverse image of μ_I-opss in (L_x, σ_I) is μ_I-op in (T_x, μ_I).

Definition 2.5. A mapping $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ is said to be semi (resp. pre μ_I, $\alpha\mu_I$ and $\beta\mu_I$) μ_I-cts (shortly $\mathbb{P}\mu_I$-cts, $\mathbb{S}\mu_I$-cts) if the inverse image of μ_I-opss in (L_x, σ_I) is semi (resp. pre μ_I, $\alpha\mu_I$ and $\beta\mu_I$) μ_I-op in (T_x, μ_I).

Results.

1. Every μ_I-cts is P(resp. S, α and β) μ_I-cts.

2. Every $\mathbb{P}\mu_I$-cts is $\beta\mu_I$-cts.

3. Every $\alpha\mu_I$-cts is $\mathbb{S}\mu_I$-cts.

Definition 2.6 [8]**.** Let T_x be a topological space.

1. A proper non-void ops U_K of T_x is said to be a minimal ops if any ops which contained in U_K is Φ or U_K.

2. A proper non-empty cds U_{K_o} of T_x is said to be a minimal cds if any cds which is contained in U_{K_o} is Φ or U_{K_o}.

Definition 2.7 [7]**.** Let (T_x, τ_I) be an ITS.

1. A proper IOS V_{K_o} in T_x is said to be a minimal IOS if any IOS which is contained in V_{K_o} is $\Phi_\sim$ or V_K.

2. A proper IOS V_{K_o} in T_x is said to be a maximal IOS if any IOS which is contains V_{K_o} is $T_{x\sim}$ or V_{K_o}.

3. Minimal and Maximal μ_I-open Sets

Definition 3.1. Let T_x be a μ_I-ts.

1. A proper non-void μ_I-ops J of (T_x, μ_I) is said to be a minimal μ_I-ops (briefly mn-μ_I-ops) if any μ_I-ops that is contained in J is $\Phi_\sim$ or J.

2. A proper non-void μ_I-cds $K(\neq M^c_{\mu_I})$ of (T_x, μ_I) is said to be a minimal μ_I-cds(briefly mn-μ_I-cds) if any μ_I-cds that is contained in K is $M^c_{\mu_I}$ or K.

Remark 3.2. Union of two mn-μ_I-opss need not be mn-μ_I-op. It will be explained in the forthcoming example. Also, since intersection of two μ_I-opss need not be μ_I-op, intersection of two mn-μ_I-opss need not be mn-μ_I-op.

Example 3.3. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{c_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle\}$. Here, $\langle T_x, \Phi, \{c_{k_0}\}\rangle$ and $\langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle$ are the mn-μ_I-opss. Take $J = \langle T_x, \Phi, \{c_{k_0}\}\rangle$ and $H = \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle \Rightarrow J \cup H\langle T_x, \{b_{k_0}\}, \Phi\rangle$ is not mn-μ_I-ops.

Theorem 3.4.

1. *If J is a mn-μ_I-ops $\Leftrightarrow$ J^c is a mx-μ_I-cds.*

2. *If J is a mx-μ_I-ops $\Leftrightarrow$ J^c is a mn-μ_I-cds.*

Proof (1)**.** Let J be a mn-μ_I-ops. Then any μ_I-ops which is contained in J is $\Phi_\sim$ (or) J. Applying compliments we have any μ_I-cds which contains $J^c = K$ is $T_{x\sim}$ (or) K. Hence K is a mx-μ_I-cds.

Proof (2)**.** The proof follows from proof (1).

Remark 3.5. Let J be a mn-μ_I-ops and W be a μ_I-ops. Then both the conditions $J \cap W = \Phi_\sim$ and $J \subset W$ fails.

Example 3.6. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}, d_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{b_{k_0}\}\rangle,$

$\langle T_x, \{a_{k_0}, c_{k_0}\}, \Phi\rangle, \langle T_x, \Phi, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{c_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle,$ $\langle T_x, \Phi, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \Phi\rangle\}$. Let $J = \langle T_x, \Phi, \{b_{k_0}\}\rangle$ and $W = \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle \Rightarrow J$ is the mn-μ_I-ops $J \cap W = \langle T_x, \Phi, \{b_{k_0}, d_{k_0}\}\rangle$. Here $J \cap W \neq \Phi_\sim$ and $J \not\subset W$.

Definition 3.7. 1. Let J be a mn-μ_I-ops of Type-I then $J \cap W = \Phi_\sim$ (or) $J \subset W$ for any $W \in \mu_I$.

2. Let J be a mn-μ_I-ops of Type-II then $(J \cap W)_T = \Phi_\sim$ (or) $J \subset W$ for any $W \in \mu_I$.

3. Let J be a mn-μ_I-ops of Type-III then $(J \cap W)_F = \Phi_\sim$ (or) $J \subset W$ for any $W \in \mu_I$.

Theorem 3.8. *Let J be a mn-μ_I-ops of Type-I then J is a mn-μ_I-ops.*

Example 3.9. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{b_{k_0}\}\rangle,$ $\langle T_x, \{b_{k_0}\}, \{a_{k_0}, c_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle, \langle T_x, \{c_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}, c_{k_0}\}, \Phi\rangle\}$. Let $J = \langle T_x, \Phi, \{b_{k_0}\}\rangle$ and $W = \langle T_x, \{b_{k_0}\}, \{a_{k_0}, c_{k_0}\}\rangle \Rightarrow J \cap W = \Phi_\sim$ and other μ_I-opss are the supersets of J. Hence J is the mn-μ_I-ops of Type-I.

Example 3.10. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{a_{k_0}\}\rangle,$ $\langle T_x, \{c_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \Phi, \Phi\rangle, \langle T_x, \Phi, \{a_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \Phi\rangle$. Let $J = \langle T_x, \Phi, \{a_{k_0}\}\rangle$ and $W = \langle T_x, \Phi, \{b_{k_0}\}\rangle \Rightarrow J \cap W = \langle T_x, \Phi, \{a_{k_0}, b_{k_0}\}\rangle$ and other μ_I-opss are the supersets of J. Hence J is the mn-μ_I-ops of Type-II.

Example 3.11. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}, d_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \{a_{k_0}, d_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}, d_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, b_{k_0}, d_{k_0}\}, \Phi\rangle\}$. Let $J = \langle T_x, \{b_{k_0}, d_{k_0}\}, \Phi\rangle$ and $W = \langle T_x, \{a_{k_0}, d_{k_0}\}, \Phi\rangle \Rightarrow J \cap W = \langle T_x, \{d_{k_0}\}, \Phi\rangle$ and other μ_I-ops$_s$ are the supersets of J. Hence J is the mn-μ_I-ops of Type-III.

Lemma 3.12. *Let J and H be two mn-μ_I-ops of Type-I then $J \cap H = \Phi_\sim$ (or) $J = H$.*

Proof. Given that J and H are the two mn-μ_I-opss of Type-I. To prove, $J \cap H = \Phi_\sim$. Suppose $J \cap H \neq \Phi_\sim$. Now to prove, $J = H$. Since J and H are the two mn-μ_I-opss of Type-I, $J \subset H$ and $H \subset J$ by definition 3.7(1). Hence $J = H$.

Definition 3.13. Let T_x be a μ_I-ts, $x \in T_x$ and let $N_o \in IS(T_x)$. Then n_x is known as μ_I-neighbourhood(briefly μ_I-nhd) of x_I, if there exists a μ_I-ops J in T_x s.t. $x_I \in J \subset n_x$. The notation of all μ_I-nhd's is $N_{\mu_I}(x_I)$.

Example 3.14. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \Phi, \{b_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}, c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle$ $\langle T_x, \{b_{k_0}\}, \Phi\rangle\}$. Let $x_I = \langle T_x, \{b_{k_0}\}, \{a_{k_0}, c_{k_0}\}\rangle$ be an IP of $T_x = N_{\mu_I}(x_I) = \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}, c_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, b_{k_0}, c_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{b_{k_0}, c_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}, c_{k_0}\}\rangle\}$.

Results. Let T_x be a μ_I-ts and let $x \in T_x$.

1. If $N_0 \in N_{\mu_I}(x_I) \Rightarrow x_I \in N_0$.

2. If $N_0 \in N_{\mu_I}(x_I)$ and $N_0 \subset \mathbb{L} \Rightarrow \mathbb{L} \in N_{\mu_I}(x_I)$.

3. If $N_0 \in N_{\mu_I}(x_I)$ then there exists $\mathbb{L} \in N_{\mu_I}(x_I)$ s.t $N_0 \in N_{\mu_I}(y_I)\ \forall\ y_I \in \mathbb{L}$.

Remark 3.15. Intersection of two μ_I-nhd's of x_I need not be a μ_I-nhd of x_I.

Example 3.16. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle\}$. Let $x_I = \langle T_x, \{a_{k_0}\}, \{b_{k_0}, c_{k_0}\}\rangle$ be an IP of $T_x \Rightarrow N_{\mu_I}(x_I) = \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, b_{k_0}, c_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \{c_{k_0}\}\rangle,$

$\langle T_x, \{a_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}\}, \{c_{k_0}\}\rangle\}$. Let $N_0 = \langle T_x, \{a_{k_0}, c_{k_0}\}$, and let $\mathbb{L} = \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle \Rightarrow N_0 \cap \mathbb{L} = \langle T_x, \{a_{k_0}\}, \{b_{k_0}, c_{k_0}\}\rangle \notin N_{\mu_I}(x_I)$.

Proposition 3.17. *Let J be a mn-μ_I-ops of Type-I. If $x_I \in J$ then $J \cap W = \Phi_{\sim}$ (or) $J \subset W$ for any μ_I-open neighbourhood (shortly μ_I-op nhd) W of x_I.*

Proof. Obvious.

Proposition 3.18. *Let J be a mn-μ_I-ops of Type-I. Then $J = \bigcap\{W/W$ is a μ_I-op nhd of $x_I\}$ for any x_I of J.*

Proof. By proposition 3.17 and J is a μ_I-op nhd of x_I, $J = \bigcap\{W/W$ is a μ_I-op nhd of $x_I\}$.

Theorem 3.19. *Let α_0, ε_0 and η_0 be three mn-μ_I-ops of Type-I s.t $\alpha_0 \neq \varepsilon_0$. If $\eta_0 \subset \alpha_0 \cup \varepsilon_0$ then either $\eta_0 = \alpha_0$ (or) $\eta_0 = \varepsilon_0$.*

Proof. If $\eta_0 = \alpha_0$ then there is no more to prove. Suppose $\eta_0 \neq \alpha_0$ then by lemma 3.12 $\alpha_0 \cap \eta_0 = \dot{\emptyset}_{\sim}$. This implies $\eta_0 \cup \varepsilon_0 = \eta_0 \cup (\varepsilon_0 \cup \dot{\emptyset}_{\sim}) = \eta_0 \cup [\varepsilon_0 \cup (\alpha_0 \cap \eta_0)] = \eta_0 \cup [\varepsilon_0 \cup (\alpha_0 \cap \eta_0)] = (\varepsilon_0 \cup \alpha_0) \cap (\eta_0 \cup \varepsilon_0) = \varepsilon_0 \cup (\alpha_0 \cap \eta_0) = \varepsilon_0 \Rightarrow \eta_0 \subset \varepsilon_0$. Since η_0 and ε_0 are mn-μ_I-ops of Type-I, $\eta_0 = \varepsilon_0$.

Theorem 3.20. *Let α_0, ε_0 and η_0 be three mn-μ_I-ops of Type-I which are different from each other. Then $(\alpha_0 \cup \varepsilon_0) \not\subset (\alpha_0 \cap \eta_0)$.*

Proof. Suppose $(\alpha_0 \cup \varepsilon_0) \subset (\alpha_0 \cup \eta_0)$. Then $(\alpha_0 \cup \varepsilon_0) \cap (\varepsilon_0 \cup \eta_0) \subset (\alpha_0 \cup \eta_0) \cap (\varepsilon_0 \cup \eta_0) \Rightarrow \varepsilon_0 \cup (\eta_0 \cap \alpha_0) \subset \eta_0(\varepsilon_0 \cap \alpha_0)$. By lemma 3.14, we obtain $\varepsilon_0 \subset \eta_0$. Since ε_0 and η_0 are the mn-μ_I-ops of Type-I, $\varepsilon_0 = \eta_0$. This contradicts our assumption. Hence $(\alpha_0 \cup \varepsilon_0) \not\subset (\alpha_0 \cap \eta_0)$.

Theorem 3.21. *If J is a mn-μ_I-ops of Type-I then $C_{\mu_I}(J) = C_{\mu_I}(S)$ for any subset S of J.*

Proof. For any subset S of J, we have $C_{\mu_I}(S) \subset C_{\mu_I}(J)$. To prove, $C_{\mu_I}(J) \subset C_{\mu_I}(S)$. Let $x_I \in J$ then by proposition 3.17, we obtain $S = S \cap J \subset S \cap W$ for each $W \in \mathbb{N}_{\mu_I}(x_I)$. Then $S \cap W \neq \dot{\varnothing}$ and hence $x_I \in C_{\mu_I}(S)$. Hence $J \subset C_{\mu_I}(S)$ this generates $C_{\mu_I}(J) \subset C_{\mu_I}(S)$. Hence $C_{\mu_I}(J) = C_{\mu_I}(S)$.

Remark 3.22. Reversal statement of 3.21 is not necessarily true.

Example 3.23. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle\}$. Let $J = \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle$, and $S = \langle T_x, \{b_{k_0}\}, \{a_{k_0}\}\rangle \Rightarrow C_{\mu_I}(J) = \langle T_x, \{a_{k_0}, b_{k_0}, c_{k_0}\}, \Phi\rangle$ and $C_{\mu_I}(S) = \langle T_x, \{a_{k_0}, b_{k_0}, c_{k_0}\}, \Phi\rangle$ and $C_{\mu_I}(J) = C_{\mu_I}(S)$. But here J is not a mn-μ_I-ops of Type-I.

Theorem 3.24. *If $C_{\mu_I}(J) = C_{\mu_I}(S)$ for any subset S of J then J is a mn-μ_I-ops.*

Proof. Assume that J is not a mn-μ_I-ops. Then there exists a μ_I-ops W s.t $W \subset J$ and hence there exists $x_I \in J$ s.t $x_I \notin W$. Therefore $x_I \in T_x - W$ this implies $C_{\mu_I}(x_I) \subset T_x - W$. Then we obtain $C_{\mu_I}(x_I) \neq C_{\mu_I}(J)$.

Remark 3.25. Reversal statement of 3.24 is not necessarily true. It can be propounded in the below exemplum.

Example 3.26. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}, d_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{a_{k_0}\}\rangle, \langle T_x, \{d_{k_0}\}, \Phi\rangle, \langle T_x, \{d_{k_0}\}, \{a_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, d_{k_0}\}, \Phi\rangle\}$. Let $J = \langle T_x, \{a_{k_0}\}, \Phi\rangle$ and $S = \langle T_x, \{a_{k_0}\}, \{c_{k_0}, d_{k_0}\}\rangle \Rightarrow C_{\mu_I}(J) = \langle T_x, \{a_{k_0}, b_{k_0}, c_{k_0}, d_{k_0}\}, \Phi\rangle$ and $C_{\mu_I}(S) = \langle T_x, \{a_{k_0}\}, \{d_{k_0}\}\rangle$. Here J is the mn-μ_I-ops but $C_{\mu_I}(x_I) \neq C_{\mu_I}(J)$.

Theorem 3.27. *Let J be a mn-μ_I-ops of Type-I. Then any non-void subset S of J is a $\mathbb{P}\mu_I$-ops.*

Proof. By 3.21, $J \subset C_{\mu_I}(S)$. This implies $I_{\mu_I}(J) \subset I_{\mu_I}(C_{\mu_I}(S))$ for some non-empty subset S of J. Since J is a mn-μ_I-ops of Type-I, $S \subset J = I_{\mu_I}(J) \subset I_{\mu_I}(C_{\mu_I}(S))$.

4. Minimal μ_I- continuous and Maximal μ_I-continuous

Definition 4.1. Let (T_x, μ_I) and (L_x, σ_I) be the μ_I-ts. A map $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ is called,

1. minimal μ_I-continuous map(briefly mn- μ_I-cts mp) if $O_x^{-1}(J)$ is a μ_I-ops in (T_x, μ_I) for every mn-μ_I-ops J in (L_x, σ_I).

2. maximal μ_I-continuous map(briefly mx-μ_I-cts mp) if $O_x^{-1}(J)$ is a μ_I-ops in (T_x, μ_I) for every mx-μ_I-ops J in (L_x, σ_I).

Theorem 4.2. *Every μ_I-cts mp is mn-μ_I-cts.*

Proof. Let $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ be a μ_I-cts mp. Take J be a mn-μ_I-ops in (L_x, σ_I). Since every mn-μ_I-ops is a μ_I-ops, J is a μ_I-ops. Since O_x is μ_I-cts, $O_x^{-1}(J)$ is a μ_I-ops in (L_x, σ_I). Hence O_x is mn-μ_I-cts.

Remark 4.3. Reversal statement of 4.2 is not true. This condition can be explained in the below exemplum.

Example 4.4. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ and $L_x = \{u_{k_0}, v_{k_0}, w_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{a_{k_0}\}\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \{c_{k_0}\}, \Phi\rangle,$ $\langle T_x, \{c_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, c_{k_0}\}, \Phi\rangle\}$ and $\sigma_I =$ $\{\Phi_\sim, \langle L_x, \Phi, \{u_{k_0}\}\rangle, \langle L_x, \{w_{k_0}\}, \Phi\rangle, \langle L_x, \{w_{k_0}\}, \{v_{k_0}\}\rangle, \langle L_x, \{v_{k_0}, c_{k_0}\}, \Phi\rangle\}$. Define $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ by $O_x(a_{k_0}) = u_{k_0}, O_x(b_{k_0}) = v_{k_0}$, $O_x(c_{k_0}) = w_{k_0}$. Here, O_x is mn-μ_I-cts but not μ_I-cts, since $O_x^{-1}(\langle L_x, \{v_{k_0}, w_{k_0}\}, \Phi\rangle) = \langle T_x, \{b_{k_0}, c_{k_0}\}, \Phi\rangle \notin (T_x, \mu_I)$.

Theorem 4.5. *Every μ_I-cts mp is mx-μ_I-cts.*

Proof. We can prove this theorem as we have done in the theorem 4.2.

Remark 4.6. Reversal statement of 4.5 is not necessarily true. It will be rendered in the forthcoming exemplum.

Example 4.7. Let $T_x = \{a_{k_0}, b_{k_0}, c_{k_0}\}$ and $L_x = \{u_{k_0}, v_{k_0}, w_{k_0}\}$ with $\mu_I = \{\Phi_\sim, \langle T_x, \Phi, \{b_{k_0}\}\rangle, \langle T_x, \{a_{k_0}\}, \Phi\rangle, \langle T_x, \{b_{k_0}\}, \Phi\rangle, \langle T_x, \{a_{k_0}\}, \{b_{k_0}\}\rangle, \langle T_x, \{b_{k_0}\}, \{c_{k_0}\}\rangle, \langle T_x, \{a_{k_0}, b_{k_0}\}, \Phi\rangle\}$ and $\sigma_I = \{\Phi_\sim, \langle L_x, \Phi, \{v_{k_0}\}\rangle, \langle L_x, \{u_{k_0}\}, \Phi\rangle, \langle L_x, \{u_{k_0}\}, \{w_{k_0}\}\rangle, \langle L_x, \{u_{k_0}, w_{k_0}\}, \Phi\rangle\}$. Define $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ by $O_x(a_{k_0}) = u_{k_0}$, $O_x(b_{k_0}) = v_{k_0}$, and $O_x(c_{k_0}) = w_{k_0}$. Here, Ô is mx-μ_I-cts but not μ_I-cts, since $O_x^{-1}(\langle L_x, \Phi, \{v_{k_0}\}\rangle) = \langle T_x, \Phi, \{a_{k_0}\}\rangle \notin (T_x, \mu_I)$.

Remark 4.8. Mn-μ_I-cts and mx-μ_I-cts mps are freer of each other.

Example 4.9. In example 4.4, f is mn-μ_I-cts but it is not mx-μ_I-cts. In example 4.7 f is mx-μ_I-cts but it is not mn-μ_I-cts.

Remark 4.10.

1. Mn-μ_I-cts and $\mathbb{P}\mu_I$-cts mps are freer of each other.

2. Mx-μ_I-cts and $\mathbb{P}\mu_I$-cts mps are freer of each other.

Theorem 4.11. *Let O_x be a mn-μ_I-cts function $\Leftrightarrow$ the inverse image of each mx-μ_I-cds in (L_x, σ_I) is a μ_I-cds in (T_x, μ_I).*

Proof. From theorem 3.4, the complement of mn-μ_I-op is mx-μ_I-cd.

Theorem 4.12. *If $O_x : (T_x, \mu_I) \to (L_x, \sigma_I)$ is μ_I-cts and $O_y : (T_x, \mu_I) \to (M_x, \rho_I)$ is mn-μ_I-cts then $O_y \circ O_x : (T_x, \mu_I) \to (M_x, \rho_I)$ is mn-μ_I-cts.*

Proof. First we take J be a mn-μ_I-ops in (M_x, ρ_I). Since $\widetilde{O}$ is mn-μ_I-cts, $O_y^{-1}(J)$ is μ_I-op in (L_x, σ_I). Also since O_x is μ_I-cts, $O_x^{-1}(O_y^{-1}(J)) = (O_y \circ O_x)^{-1}(J)$ is μ_I-op in $(T_x, \mu_I) \Rightarrow O_y \circ O_x$ is mn-μ_I-cts.

Remark 4.13. Since every μ_I-ops need not be a mn-μ_I-ops, composition of two mn-μ_I-cts maps need not be mn-μ_I-cts.

Theorem 4.14. *Let* O_x *be a mx-* μ_I *-cts function* $\Leftrightarrow$ *the inverse image of each mn-* μ_I *-cds in* (L_x, σ_I) *is a* μ_I *-cds in* (T_x, μ_I).

Proof. From theorem 3.4, the complement of mx- μ_I -op is mn- μ_I -cd.

Theorem 4.15. *If* O_x *is* μ_I *-cts and* O_y *is mx-* μ_I *-cts then* $O_y \circ O_x$ *is mx-* μ_I *-cts.*

Proof. We can prove this theorem as we have done in the theorem 4.15.

Remark 4.16. Since every μ_I -ops need not be a mx- μ_I -ops, composition of two mx- μ_I -cts maps need not be mx- μ_I -cts.

5. Conclusion

From definition 3.7, new types of minimal μ_I -open sets were introduced and gave their basic properties. In addition, two types of μ_I -continuous maps via these μ_I -open sets were defined and investigated their features. We hope that several properties of these concepts would be studied or new types of μ_I -continuities would be found.

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