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PROPERTIES OF INTUITIONISTIC *L* FUZZY SUB ℓ-GROUPS

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Abstract

In this paper we introduce the properties of intuitionistic L fuzzy sub ℓ- groups.

1. Introduction

L. A. Zadeh introduced the notion of a fuzzy subset A of a set X as a function from X into $I = [0, 1]$. Rosenfeld applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy subsemigroupoids respectively. J. A. Goguen replaced the valuations set [0, 1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. In fact it seems

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in order to obtain a complete analogy of crisp mathematics in terms of fuzzy mathematics, it is necessary to replace the valuation set by a system having more rich algebraic structure. These concepts ℓ-groups play a major role in mathematics and fuzzy mathematics. G. S. V Satya Saibaba introduced the concept of L-fuzzy ℓ-group and L-fuzzy ℓ-ideal of ℓ-group. K. T. Atanassov introduced the concept of intuitionistic fuzzy set, which deals with the degree of membership and non membership of each element in the fuzzy set. Li Xiaoping introduced the concept intuitionistic fuzzy group and its homomorphic image. Senthil kumar and Muthuraj introduced the properties of L-fuzzysub ℓ-group. In this article, we initiate the study of Intuitionistic L-fuzzy sub ℓ-groups and its properties

2. Preliminaries

In this section we site the fundamental definitions that will be used in sequel. Throughout this section we mean $(G, *)$ is a group, e is the identity of G and xy as $x * y$.

Definition 2.1. An L-fuzzy subset A of G is called an L-fuzzy subgroup (LFS) of G if for every $x, y \in G$

(i) $A(xy) \geq A(x) \wedge A(y)$

(ii) $A(x^{-1}) = A(x)$.

Definition 2.2. An L-fuzzy subset A of G is said to be an L-fuzzy sub ℓ-group ($LFS\ \ell\ G$) of G if for any $x, y \in G$

(iii) $A(xy) \geq A(x) \wedge A(y)$,

(iv) $A(x^{-1}) = A(x)$,

(v) $A(x \vee y) \geq A(x) \wedge A(y)$

(vi) $A(x \wedge y) \geq A(x) \wedge A(y)$.

Definition 2.3. A set A of G is said to be an intuitionistic fuzzy set (IFS) of G if for any $x, y \in G$, the triad formed as $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in G\}$, where the functions $\mu_A : G \to [0, 1]$ denote the degree of membership and

$\gamma_A : G \to [0, 1]$ denote the degree of non membership of each element $x \in G$ to the set A.

Definition 4.2. An L-fuzzy subset A of G is said to be an intuitionistic L-fuzzy sub ℓ-group ($ILFS\ell G$) of G if for any $x, y \in G$

(i) $\mu_A(xy^{-1}) \geq \mu_A(xy), \gamma_A(xy^{-1}) \leq \gamma_A(xy),$

(ii) $\mu_A(x \vee y) \geq \mu_A(x) \wedge \mu_A(y), \gamma_A(x \wedge y) \leq \gamma_A(x) \vee \gamma_A(y),$

(iii) $\mu_A(x \wedge y) \geq \mu_A(x) \wedge \mu_A(y), \gamma_A(x \vee y) \leq \gamma_A(x) \vee \gamma_A(y) \ \forall x, y \in G$

3. Properties of Intuitionistic *L*-Fuzzy Sub ℓ-Group

Theorem 3.1. *let A be an intuitionistic L-fuzzy sub ℓ-group of an sub ℓ-group G then*

(i) $\mu_A(x) \leq \mu_A(e), \gamma_A(x) \geq \gamma_A(e) \ \forall x, y \in G$

(ii) *The subset* $H = \{x \in G / \mu_A(x) = \mu_A(e), \gamma_A(x) = \gamma_A(e), \forall x, e \in G$ *is a sub ℓ-group*

Proof.

(i) Let $x \in G$

$$\mu_A(x) = \mu_A(x) \wedge \mu_A(x)$$

$$= \mu_A(x) \wedge \mu_A(x^{-1})$$

$$\leq \mu_A(xx^{-1})$$

$$= \mu_A(e)$$

$$\mu_A(x) \leq \mu_A(e)$$

$$\gamma_A(x) = \gamma_A(x) \vee \gamma_A(x)$$

$$\geq \gamma_A(x) \vee \gamma_A(x^{-1})$$

$$= \gamma_A(xx^{-1})$$

$\gamma_A(x) \geq \gamma_A(e)$ for all $x, e \in G$.

(ii) Let $H = \{x \in G / \mu_A(x) = \mu_A(e), \gamma_A(x) = \gamma_A(e), \forall x, e \in G$ is a sub ℓ-group. Clearly H is nonempty.

Then $\mu_A(x) = \mu_A(y) = \mu_A(e), \gamma_A(x) = \gamma_A(y) = \gamma_A(e)$

$$\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$$

$$\geq \mu_A(x) \wedge \mu_A(y)$$

$$= \mu_A(e) \wedge \mu_A(e)$$

$$= \mu_A(e)$$

$$\therefore \mu_A(xy^{-1}) \geq \mu_A(e).$$

By part (i) $\therefore \mu_A(xy^{-1}) \leq \mu_A(e)$.

Hence $\mu_A(xy^{-1}) = \mu_A(e)$

$$\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y^{-1})$$

$$= \gamma_A(x) \vee \gamma_A(y)$$

$$= \gamma_A(e) \vee \gamma_A(e)$$

$$= \gamma_A(e)$$

i.e. $\gamma_A(xy^{-1}) \leq \gamma_A(e)$

by part (i) $\gamma_A(xy^{-1}) \geq \gamma_A(e)$

$$\gamma_A(xy^{-1}) = \gamma_A(e).$$

Hence $\mu_A(xy^{-1}) = \mu_A(e), \gamma_A(xy^{-1}) = \gamma_A(e)$, then $xy^{-1} \in H$.

Clearly H is a sub ℓ-group

Theorem 3.2. *let A be an intuitionistic L-fuzzy sub ℓ-group of an sub ℓ-group G with identity e then*

(i) $\mu_A(xy^{-1}) = \mu_A(e) \Rightarrow \mu_A(x) = \mu_A(y)$

(ii) $\gamma_A(xy^{-1}) = \gamma_A(e) \Rightarrow \gamma_A(x) = \gamma_A(y) \quad \forall x, y \in G$.

Proof. Given A is an intuitionistic L-fuzzy sub ℓ- group of an sub ℓ- group G and

(i) $\mu_A(xy^{-1}) = \mu_A(e)$.

Then $\forall x,\ y \in G$

$$\begin{aligned} \mu_A(x) &= \mu_A(x(y^{-1}y)) \\ &= \mu_A((xy^{-1})y) \\ &\geq \mu_A(xy^{-1}) \wedge \mu_A(y) \\ &\geq \mu_A(xy^{-1}) \wedge \mu_A(y) \\ &\geq \mu_A(e) \wedge \mu_A(y) \\ &= \mu_A(y) \end{aligned}$$

i.e. $\mu_A(xy^{-1}) = \mu_A(e) \Rightarrow \mu_A(x) \geq \mu_A(y)$

i.e. $\mu_A(x) \geq \mu_A(y)$

since

$$\begin{aligned} \mu_A(yx^{-1}) &= \mu_A(yx^{-1})^{-1} \\ &= \mu_A(xy^{-1}) \\ &= \mu_A(e) \end{aligned}$$

i.e. $\mu_A(y) \geq \mu_A(x)$

$$\therefore \mu_A(x) = \mu_A(y)$$

(ii) $\gamma_A(x) = \gamma_A(x(y^{-1}y))$

$$\begin{aligned} &= \gamma_A((xy^{-1})y) \\ &\leq \gamma_A(xy^{-1}) \vee \gamma_A(y) \\ &\leq \gamma_A(e) \vee \gamma_A(y) \end{aligned}$$

i.e. $\gamma_A(x) \leq \gamma_A(y)$

since $\gamma_A(yx^{-1}) = \gamma_A(yx^{-1})^{-1}$

$= \mu_A(xy^{-1})$

$= \gamma_A(e)$

i.e. $\gamma_A(y) \geq \gamma_A(x)$

$\therefore \gamma_A(x) = \gamma_A(y)$

Theorem 3.3. *Let A be an intuitionistic L-fuzzy sub* ℓ*- group of an sub* ℓ*-group G then for* $x, y \in G$

(i) $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$

Proof. Let A be an intuitionistic L-fuzzy sub ℓ- group of an sub ℓ- group G then for $x, y \in G$

$$\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$$

$$= \mu_A(x) \wedge \mu_A(y)$$

$$\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$$

$$\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y^{-1})$$

$$= \gamma_A(x) \vee \gamma_A(x)$$

$$\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$$

Theorem 3.4. *Let A be an intuitionistic L-fuzzy subset of an sub* ℓ*-group G then for* $x, y \in G$

a. $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$

b. $\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$

then A is an intuitionistic L-fuzzy sub ℓ*-group of an sub* ℓ*-group G.*

Proof.

a. Given $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$

(i) $\mu_A(x^{-1}) = \mu_A(ex^{-1})$

$\geq \mu_A(e) \wedge \mu_A(x^{-1})$

$= \mu_A(e) \wedge \mu_A(x)$

$= \mu_A(x)$

$\therefore \mu_A(x^{-1}) \geq \mu_A(x)$

(ii) $\mu_A(xy^{-1}) \geq \mu_A(x \vee y^{-1})$

$\geq \mu_A(x) \vee \mu_A(y^{-1})$

$\geq \mu_A(x) \vee \mu_A(y)$

$\mu_A(xy^{-1}) \geq \mu_A(x) \vee \mu_A(y)$

(iii) $\mu_A(xy^{-1}) \geq \mu_A(x \wedge y^{-1})$

$\geq \mu_A(x) \wedge \mu_A(y^{-1})$

$\geq \mu_A(x) \wedge \mu_A(y)$

$\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$

b. Given $\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$

(i) $\gamma_A(x^{-1}) = \gamma_A(ex^{-1})$

$\leq \gamma_A(e) \vee \gamma_A(x^{-1})$

$\leq \gamma_A(e) \vee \gamma_A(x)$

$\leq \gamma_A(x)$

$\gamma_A(x^{-1}) \leq \gamma_A(x)$

(ii) $\gamma_A(xy^{-1}) \leq \gamma_A(x \wedge y^{-1})$

$$\leq \gamma_A(x) \vee \gamma_A(y^{-1})$$

$$\leq \gamma_A(x) \vee \gamma_A(y)$$

$$\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$$

(iii) $\gamma_A(xy^{-1}) \leq \gamma_A(x \vee y^{-1})$

$$\leq \gamma_A(x) \vee \gamma_A(y^{-1})$$

$$\leq \gamma_A(x) \vee \gamma_A(y)$$

$$\gamma_A(xy^{-1}) \leq \gamma_A(x) \vee \gamma_A(y)$$

hence A is an intuitionistic L-fuzzy sub ℓ- group of an sub ℓ- group G.

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