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FURTHER RESULTS ON 2-ODD LABELING OF GRAPHS

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Abstract

Graph labeling is one of the most interesting concepts in graph theory which has a numerous applications in different fields. The notion of labeling the nodes or lines or both subject to some conditions in graphs has been proposed with various types of labeling being applied to different graphs by the research community, one among them being a 2-odd labeling. A 2-odd labeling assigns distinct integers to the nodes of G in such a manner that the positive difference of adjacent nodes is either 2 or an odd integer, $2k \pm 1$, $k \in N$ and so G is a 2-odd graph iff it permits a 2-odd labeling. The motivation behind the development of this article is to apply the concept of 2-odd labeling to diverse types of graphs, namely, diamond graph, double fan graph, double alternate triangular snake graph, antiprism graph, and king graph.

1. Introduction

A graph labeling is actually a function that assigns integers to the lines or nodes, or both of G under some conditions. One can understand the importance of graph labeling because it has numerous applications in many areas like circuit design, radar, communication network addressing, etc. For more details, see [2, 3, 6, 12]. The graphs we are using for the present study are simple, finite, undirected and connected. Let Z be the set of integers and Da subset of $+ve$ integers, respectively. The integer distance graph is defined as a graph $G(Z, D)$ with node set Z and any 2 nodes s and t are adjacent if and only if $|s - t| \in D$ [4].

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A 2-odd labeling of G was introduced in [8] as an $1-1$ function $h : V(G) \to Z$ so that the absolute difference between any adjacent nodes, x_1 *and* x_2, i.e. $|h(x_1) - h(x_2)|$ is either $2n \pm 1; n \in N$ or exactly 2. Thus G is a 2-odd iff there is a 2-odd labeling of G. Note that, in this labeling, $h(V(G))$ are unique, but line labels are not unique. This labeling is derived for some graphs like wheel, butterfly, fan, helm, path, cycle graphs. For more information on this labeling, one can see [1, 8, 9]. One can also relate the concept of 2-odd labeling with prime distance labeling (PDL) which is actually a stronger version of 2-odd labeling. For further study on PDL, one can see [8, 9]. Observe that all the prime distance graphs are clearly 2-odd graphs but the converse need not be true [8]. First we recollect a few relevant definitions from the literature. By WOLG, DG, DFG, DATS, ATS, KG we mean without loss of generality, diamond graph, double fan graph, double alternate triangular snake, alternate triangular snake, king graph, respectively.

Definition 1 [5, 7, 11]**.** Let $L_n \cong P_n \times P_2, n \geq 2,$ with $V(L_n) = \{v_i, u_i : i = 1, 2, \ldots, n\}$. If we add lines $u_i v_{i+1}, i = 1, 2, \ldots, n-1,$ to L_n and delete u, which are incident to both lines $u_n v_{n-1}$ and $u_i v_n$, by deleting the node we get a $T(L_n)$ $ADG\, Br_{n,}$ $n \geq 3,$ is formed by connecting a node t to all nodes $v_i, i = 1, 2, \ldots, n,$ of $T(L_n)$. The node set of Br_n, is $V(Br_n) = \{t\} \bigcup \{u_i : i = 1, 2, \ldots, n-1\} \bigcup \{v_i : i = 1, 2, \ldots, n\}$.

Definition 2 [5]**.** A DFG $F_{2,n}$ is $\overline{K_2} + P_n$, where $\overline{K_2}$ denotes 2 nodes without edge and P_n is a path.

Definition 3 [10]**.** A DATS, $DA(T_n)$ consists of 2 ATSs having a common path. i.e., $DA(T_n)$ is constructed from the path $u_i : 1 \leq i \leq n$ by connecting u_i and u_{i+1} to the two new nodes v_i and w_i.

Definition 4 [13]**.** The antiprism graph $A_n, n \geq 3$ contains an outer and inner cycles C_n, while the 2 cycles joined by lines $v_i u_i$ and $v_i u_{1+i0(\bmod n)}$ for $i = 1, 2, 3, \ldots, n.$

Definition 5 [13]**.** The KG, $K_{m,n} \forall\ m, n \geq 2$ has mn nodes where every node denotes a square in the $m \times n$ chessboard, and every line refers to a logical move by a king.

In this article, 2-odd labeling of a few families of graphs like Diamond graph Br_n, Double fan graph $F_{2,n}$, Double alternate triangular snake $DA(T_n)$, King graph $K_{m,n}$, and Antiprism graph A_n.

2. Main Results

Here, we derive 2-odd labeling for some new classes of graphs. First we recall a few established results which are relevant to the current study.

Theorem 1. [1, 9] *Every bipartite graph accepts a* 2-*odd labeling*.

Proposition 1. [9] *The complete graph* K_n, $n \geq 5$ *is not* 2-*odd*.

Theorem 2. *The DG* Br_n, *admits a* 2-*odd labeling for* $n \geq 3$.

Proof. If Br_n, is the given DG, for $n \geq 3$, then clearly $|V(Br_n)| = 2n$ and $|E(Br_{n,})| = 5n - 5$. Let $V(Br_n) = \{v_o\} \cup \{v_i : 1 \leq I \leq n\} \cup \{u_j : 1 \leq I \leq n-1\}$. Define an $1-1$ function $h : V(Br_n) \to Z$ as given: WOLG, let $h(v_o) = 0$ and $h(v_1) = 1$. Then $h(v_i) = h(v_{i-1}) + 2$, for $2 \leq i \leq n-1$. Similarly, let $h(u_1) = 2$. and then $h(u_j) = h(u_{j-1}) + 2$, for $2 \leq j \leq n-1$. One can check that h induces a 2-odd labeling of Br_n, for $n \geq 3$.

Example 1. For a 2-odd labeling of Br_6, see Figure 1.

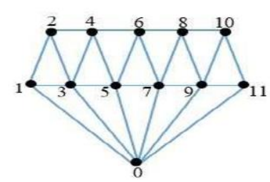

Figure 1. A 2-odd labeling of Br_6.

Theorem 3. *The DFG* $F_{2,n}$ *is a* 2-*odd graph* $\forall$ $n \geq 2$.

Proof. If $F_{2,n}$ is the given DFG, for $n \geq 2$, then clearly $|V(F_{2,n})|$

$= n+2$ and $V(F_{2,n}) = \{v_i, x, y\}$, where $\{x, y$ are the vertices above and below the vertices of $v_i\}$, for $\{1 \le i \le n\}$. Define a $1-1$ mapping $g : V(F_{2,n}) \to Z$ as follows: WOLG, let $g(x) = 2K+1, K \in N$ and $g(y) = -g(x)$. Then $g(v_i) = 2i; 1 \le i \le n$. A simple verification proves that g is a 2-odd labeling of $F_{2,n} \forall n \ge 2$.

Example 2. For a 2-odd labeling of $F_{2,8}$, see Figure 2.

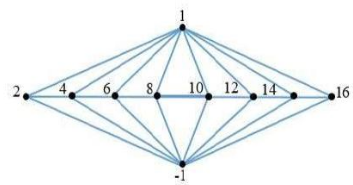

Figure 2. A 2-odd labeling of $F_{2,8}$.

Theorem 4. *The DATS,* $DA(T_n)$ *is a* 2-*odd graph* $\forall\ n \ge 2$.

Proof. If $DA(T_n)$ is the given DATS, for $n \ge 2$, then clearly, $|V(DA(T_n))| = 2n$. Let $V(DA(T_n)) = \{v_i : 1 \le i \le n\} \cup \left\{u_j : 1 \le j \le \frac{n}{2}\right\}$ $\cup \left\{w_K : 1 \le K \le \frac{n}{2}\right\}$. Define a $1-1$ mapping $g : V(F_{2,n}) \to Z$ as follows: WOLG, $g(v_i) = 2i; 1 \le i \le n$, $g(u_i) = 2i+1 : 1 \le j \le \frac{n}{2}$ and $g(w_K) = -g(u_j)$ $: 1 \le K \le \frac{n}{2}$. Clearly g is the 2-odd labeling of $DA(T_n)$, for $n \ge 2$.

Example 3. A 2-odd labeling of $DA(T_8)$ is given in Figure 3.

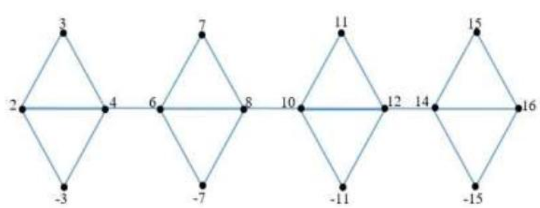

Figure 3. A 2-odd labeling of $DA(T_8)$.

Theorem 5. *The antiprism graph* A_n *is a* 2-*odd graph* $\forall\ n \ge 3$.

Proof. If A_n, $n \geq 3$ is the given antiprism graph on $2n$ nodes and $V(A_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$, then clearly $|V(A_n)| = 2n$ and $|E(A_n)| = 4n$ Define a $1-1$ function $h : V(A_n) \to Z$ as follows: WOLG, let $h(v_1) = 0, h(v_2) = 1$ and $h(v_1) = h(v_{i-1}) + 2 : 3 \leq i \leq n.$ Also, let $h(u_1) = 2$, $h(u_i) = h(u_{i-1}) + 2 : 2 \leq i \leq n-1$, then $h(u_n) = h(u_{n-1}) + 3$. Now one check that h induces a 2-odd labeling for $A_n \; \forall \; n \geq 3$.

Example 4. For a 2-odd labeling of A_8, see Figure 4.

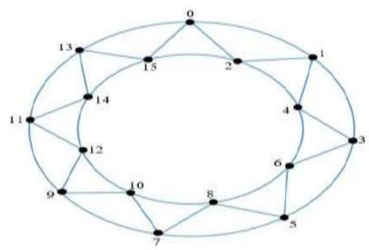

Figure 4. A 2-odd labeling of A_8.

Theorem 6. *The KG, $K_{m,n}$ allows a 2-odd labeling $\forall \; m, n \geq 2$.*

Proof. Let $K_{m,n}$ be the given KG on mn nodes with $V(K_{m,n}) = \{v_j^i : 1 \leq j \leq m, 1 \leq i \leq n\}$. Define an $1-1$ function $g : V(K_{m,n}) \to Z$ as follows: WOLG, let $g(v_i^1) = 2i : 1 \leq i \leq n$, $g(v_i^2) = g(v_i^1) + 1 : 1 \leq j \leq n$. Similarly, let $g(v_i^3) = g(v_i^2) + 1$ and $g(v_i^3) = g(v_{i-1}^3) + 2 : 2 \leq i \leq n$, $g(v_1^4) = g(v_n^3) + 1$ and $g(v_i^4) = g(v_{i-1}^4) + 2 : 2 \leq i \leq n$. By continuing thus, finally $g(v_1^m) = g(v_n^{m-1}) + 1$ and $g(v_1^m) = g(v_{i-1}^m) + 2 : 2 \leq i \leq n$. Evidently, $K_{m,n}$ admits a 2-odd labeling $\forall \; m, n \geq 2$.

Example 5. A 2-odd labeling of $K_{5,5}$ is given in the Figure 5.

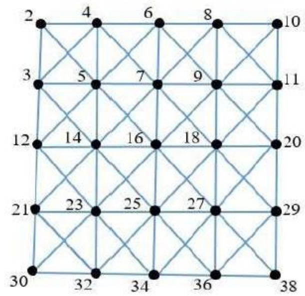

Figure 5. A 2-odd labeling of $K_{5,5}$.

Conclusion

In this present study, we have explored a few additional results on 2-odd graphs. The complete characterization of 2-odd labeling and prime distance labeling is still pending. We also believe that the concept of 2-odd labeling of graphs may be useful in graph-based cryptography.

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