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SOLVING FUZZY TRANSPORTATION PROBLEM BY GRAPHICAL TECHNIQUE OF LINEAR PROGRAMMING APPROACH

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Abstract

Transportation model is a distinctive type of LPP in which the objective is to minimize the cost of allocating an item for consumption from a number of sources to specific destinations. Here, the transportation model is considered with fully fuzzy form. Transportation problem in which the sources and destinations are represented as heptagonal FN and nonagonal FN. Based on ranking function, the sources and destinations are converted to the crisp form. The problem is then transformed into linear programming approach to get an optimal solution using graphical method. The benefit of LPP for the decision-maker is it's easy to explain and can be implemented in real life transportation. The overall concept is illustrated with the help of a solved numerical.

Abbreviation: Linear Programming (LP), Fuzzy Linear Programming (FLP), Fuzzy Number (FN), Triangular Fuzzy Number (TFN), Heptagonal Fuzzy Number (HFN), Nonagonal Fuzzy Number (NFN).

1. Introduction

Transportation is one of the issues that the organisations encounter. It originally referred to the difficulty of transporting/shipping items from the factory to the customer at the lowest cost considering supply and demand restrictions in mind. This is a type of LP technique built specifically for systems with linear objective and constraint functions.

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Deshmukh et al. [2] compared a new two ranking technique for normal HFN to solve FLPP with hexagonal FN. The fuzzy multiple objective programming model is treated by Hussein and Mitlif [4] using two ranking functions. Following that, two types of membership functions are used, namely, ordinary membership function and weighted membership function to solve fuzzy multiple objective problem using TFN. Pathade et al. [5] unravelled a new concept of BCM for deciphering mixed constraints fuzzy balanced and transportation problem using trapezoidal and trivial FNs.

Rajarajeswari and Menaka [6] suggested a new ranking technique to discover an optimum result for FTP using octagonal FTP with BCM method to find the best minimum value of optimal solution. In addition, three strategies were compared: robust ranking technique, centroid ranking technique, and suggested ranking methodology. Shanmuga [9] developed a modest technique, to solve FFLP problem with no need to convert it into classical FFLP problem. Sami Kadhemkareem Althabhawi [10] compared various ranking methods to find minimum total transportation cost.

In this paper, a LP model is derived from transportation problem and it is then solved by graphical method, and compared with the existing method (Column Minima Method).

2. Preliminaries

2.1 FN: [3]

2.2 TFN: [1]

2.3 HFN: [8]

A FN $\widetilde{A}_H$ of R is said to be a heptagonal FN denoted by $\widetilde{A}_H = (g_1, g_2, g_3, g_4, g_5, g_6, g_7)$ if its membership function $\mu_{\widetilde{A}_H} R \to [0, 1]$ is defined

$$\mu_{\widetilde{A}_H}(x) = \begin{cases} 0, & \text{for } x \le g_1 \\ \frac{1}{2}\left(\frac{x - g_1}{g_2 - g_1}\right), & \text{for } g_1 \le x \le g_2 \\ \frac{1}{2}, & \text{for } g_2 \le x \le g_3 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - g_3}{g_4 - g_3}\right), & \text{for } g_3 \le x \le g_4 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{g_5 - x}{g_5 - g_4}\right), & \text{for } g_4 \le x \le g_5 \\ \frac{1}{2}, & \text{for } g_5 \le x \le g_6 \\ \frac{1}{2}\left(\frac{g_7 - x}{g_7 - g_6}\right), & \text{for } g_6 \le x \le g_7 \\ 0, & \text{for } x \ge g_7 \end{cases}$$

where $0 < k < 1$.

2.4 NFN: [7]

A nonagonal FN $\widetilde{A}_N$ represented as, $\widetilde{A}_H = (g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9)$ and membership function is defined as

$$\mu_{\widetilde{A}_N}(x) = \begin{cases} 0, & \text{for } x \le g_1 \\ \frac{1}{4}\left(\frac{x - g_1}{g_2 - g_1}\right), & \text{for } g_1 \le x \le g_2 \\ \frac{1}{4} + \frac{1}{4}\left(\frac{x - g_2}{g_3 - g_2}\right), & \text{for } g_2 \le x \le g_3 \\ \frac{1}{2} + \frac{1}{4}\left(\frac{x - g_3}{g_4 - g_3}\right), & \text{for } g_3 \le x \le g_4 \\ \frac{3}{4} + \frac{1}{4}\left(\frac{x - g_4}{g_5 - g_4}\right), & \text{for } g_4 \le x \le g_5 \\ 1 - \frac{1}{4}\left(\frac{x - g_5}{g_6 - g_5}\right), & \text{for } g_5 \le x \le g_6 \\ \frac{3}{4} - \frac{1}{4}\left(\frac{x - g_6}{g_7 - g_6}\right), & \text{for } g_6 \le x \le g_7 \\ \frac{1}{2} - \frac{1}{4}\left(\frac{x - g_7}{g_8 - g_7}\right), & \text{for } g_7 \le x \le g_8 \\ \frac{1}{4}\left(\frac{g_9 - x}{g_9 - g_8}\right), & \text{for } g_8 \le x \le g_9 \\ 0 & \text{for } x \ge g_9 \end{cases}$$

2.5 Ranking Function:

1. Heptagonal FN: [8]

If $(g_1, g_2, g_3, g_4, g_5, g_6, g_7)$ be a heptagonal FN, then the ranking function $R(\widetilde{A}_H)$ is defined as:

$$R(\widetilde{A}_H) = \int_0^1 0.5(a_{g\alpha}^L, a_{g\alpha}^U)d\alpha$$

$$\int_0^1 0.5(g_2 - g_1)\alpha + g_1, g_4 - (g_4 - g_3)\alpha, (g_6 - g_5)\alpha + g_5, g_7 - (g_7 - g_5)\alpha d\alpha;$$

$\forall \alpha \in 0, 1 \ldots$ (2.5.1)

2. Nonagonal FN: [7]

If $(g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9)$ be a nonagonal FN, then the ranking function $R(\widetilde{A}_N)$ is defined as:

$$R(\widetilde{A}_N) = \int_0^1 0.5(a_{g\alpha}^L, a_{g\alpha}^U)d\alpha$$

Where $(a_{g\alpha}^L, a_{g\alpha}^U)$ is the α-level cut of FN $\widetilde{A}_N$

$$= \int_0^1 0.5(g_2 - g_1)\alpha + g_1, g_4 - (g_4 - g_3)\alpha, (g_6 - g_5)\alpha + g_5, g_7$$

$$- (g_7 - g_6)\alpha, -(g_7 - g_6)\alpha + g_6, g_9 - (g_9 - g_8)\alpha d\alpha;$$

$\forall \alpha \in 0, 1 \ldots$ (2.5.2)

2.6 Arithmetic Operations on FN

1. Heptagonal FN: [8]

Let $\widetilde{A}_H = (u_1, u_2, u_3, u_4, u_5, u_6, u_7)$ and $\widetilde{B}_H = (q_1, q_2, q_3, q_4, q_5, q_6, q_7)$ be two heptagonal FN then addition and subtraction can be performed as

Addition:

$$\widetilde{A}_H + \widetilde{B}_H = (u_1 + q_1, u_2 + q_2, u_3 + q_3, u_4 + q_4, u_5 + q_5, u_6 + q_6, u_7 + q_7)$$

Subtraction:

$$\widetilde{A}_H - \widetilde{B}_H = (u_1 - q_7, u_2 - q_6, u_3 - q_5, u_4 - q_4, u_5 - q_3, u_6 - q_2, u_7 - q_1)$$

2. Nonagonal FN: [7]

Let $\widetilde{A}_H = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9)$ and $\widetilde{B}_H = (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9)$ be two nonagonal FN then addition and subtraction can be performed as

Addition:

$$\widetilde{A}_H + \widetilde{B}_H =$$

$$(u_1 + q_1, u_2 + q_2, u_3 + q_3, u_4 + q_4, u_5 + q_5, u_6 + q_6, u_7 + q_7, u_8 + q_8, u_9 + q_9)$$

Subtraction:

$$\widetilde{A}_H - \widetilde{B}_H =$$

$$(u_1 - q_9, u_2 - q_8, u_3 - q_7, u_4 - q_6, u_5 - q_5, u_6 - q_4, u_7 - q_3, u_8 - q_2, u_9 - q_1)$$

3. Problem Formulation

Consider fuzzy TP which includes m sources and n destination in where $\widetilde{p}_l$, $\widetilde{q}_m$ and $\widetilde{d}_{lm}$ are heptagonal FN or nonagonal FN. The problem is to find the optimum values of y_{lm} such that

$$MinZ = \sum_{l=1}^{r} \sum_{m=1}^{r} \widetilde{d}_{lm} y_{lm}$$

Subject to constraint

$$\sum_{m=1}^{s} y_{lm} \le \widetilde{p}_l, \; l = 1, 2, 3, \ldots, r$$

$$\sum_{m=1}^{r} y_{lm} \leq \tilde{q}_m, \; m = 1, 2, 3, \ldots, s \tag{3.1}$$

Where $y_{lm} \geq 0$, $l = 1, 2, 3, \ldots, s.$ and $m = 1, 2, 3, \ldots, s.$

A TP with one or more fuzzy parameters is defined as a fuzzy transportation problem as on definition 2.5 problem (3.1) can be converted into its corresponding crisp as

$$\text{Minimize} Z = R \sum_{l=1}^{r} \sum_{m=1}^{r} \tilde{d}_{lm} y_{lm}$$

Subject to constraints

$$\sum_{m=1}^{r} y_{lm} \leq R(\tilde{p}_l), \; l = 1, 2, 3, \ldots, r$$

$$\sum_{m=1}^{r} y_{lm} \leq R(\tilde{q}_m), \; m = 1, 2, 3, \ldots, s \tag{3.2}$$

Where $y_{lm} \geq 0$, $l = 1, 2, 3, \ldots, r$ and $m = 1, 2, 3, \ldots, s.$

The model is transformed into a LP model.

The LP version for problem (3.2) is as follows:

$$\text{Maximize or minimize } Z = \sum_{m=1}^{r} d_m y_m$$

$$\sum_{m=1}^{s} p_{lm} y_m (\leq, =, \geq) q_l, \; l = 1, 2, 3, \ldots, r,$$

and

$$y_m \geq 0, \; m = 1, 2, 3, \ldots s. \tag{3.3}$$

Here, variables $y_m \geq 0, (m = 1, 2, 3, \ldots s)$ are known as decision variables and d_m, d_{lm} and $q_l (l = 1, 2, 3, \ldots, r); (m = 1, 2, 3, \ldots s)$ are constants.

4. Solution Technique

This section establishes a solution technique process for resolving the TP in the following stages:

Step 1. Construct fuzzy transportation problem (FTP).

Step 2. Transform the model (3.1) into the corresponding crisp transportation model (3.2) based on the ranking function as in definition (2.5).

Step 3. Apply linear programming approach for the model (3.2).

Step 4. Use graphical method to elucidate linear programming model (3.3).

5. Numerical Examples

1. Heptagonal FN:

Model 1: Consider the following TP with heptagonal FN:

Distribution Centre Plants	D_1	D_2	D_3
P_1	(2,3,4,5,6,7,8)	(3,4,5,6,7,8,9)	(11,12,13,14, 15,16,17)
P_2	(1,2,3,4,5,6,7)	(9,10,11,12,13, 14,15)	(6,7,8,9,10, 11,12)

and the fuzzy availability of the supply are (6,7,8,9,10,11,12), (9,10,11,12,13,14,15) and the fuzzy availability of the demand are (7,8,9,10,11,12,13), (5,6,7,8,9,10,11), (3,4,5,6,7,8,9) respectively.

Solution:

Create the fuzzy transportation table for the given problem

Table 1.

Distribution Centre Plants	D_1	D_2	D_3	Supply
P_1	(2,3,4,5, 6,7,8)	(3,4,5,6, 7,8,9)	(11,12,13, 14,15,16,17)	(6,7,8,9,10, 11,12)
P_2	(1,2,3,4, 5,6,7)	(9,10,11,12, 13,14,15)	(6,7,8,9,10,1 1,12)	(9,10,11,12, 13,14,15)
Demand	(7,8,9,10, 11,12,13)	(5,6,7,8,9, 10,11)	(3,4,5,6,7, 8,9)	

Using ranking function (2.5.1)

Table 2.

Distribution Centre Plants	D_1	D_2	D_3	Supply
P_1	10.25	12.25	28.25	18.25
P_2	8.25	24.25	18.25	24.25
Demand	20.25	16.25	12.25	

The problem is converted into a linear programming problem (LPP).

Formulation of LPP:

Key decision is to find the quantities that are transferred from each plant to each distribution centre. Let x_1, x_2 be the quantity of transported from plant I to distribution centre no. I and II correspondingly. The transportation table that resulted is presented below.

Table 3.

Distribution Centre Plants	D_1	D_2	D_3	Supply
P_1	x_1	x_2	$18.25 - x_1 - x_2$	18.25
P_2	$20.25 - x_1$	$16.25 - x_2$	$24.25 - (20.25 - x_1)$ $(16.25 - x_2)$	24.25
Demand	20.25	16.25	12.25	

Minimize $z = 10.25x_1 + 12.25x_2 + 28.25(18.25 - x_1 - x_2) + 8.25(20.25 - x_1)$ $+ 24.25(16.25 - x_2) + 18.25(24.25 - (20.25 - x_1) - (16.25 - x_2)$

Constraints are $18.25 - x_1 - x_2 \geq 0$, $20.25 - x_1 \geq 0$

$16.25 - x_2 \geq 0$, $24.25 - (20.25 - x_1) - (16.25 - x_2) \leq 0$

Minimize $z = 853.12 - 8x_1 - 22x_2$

Subject to constraints: $x_1 + x_2 \leq 18.25$, $x_1 \leq 20.25$

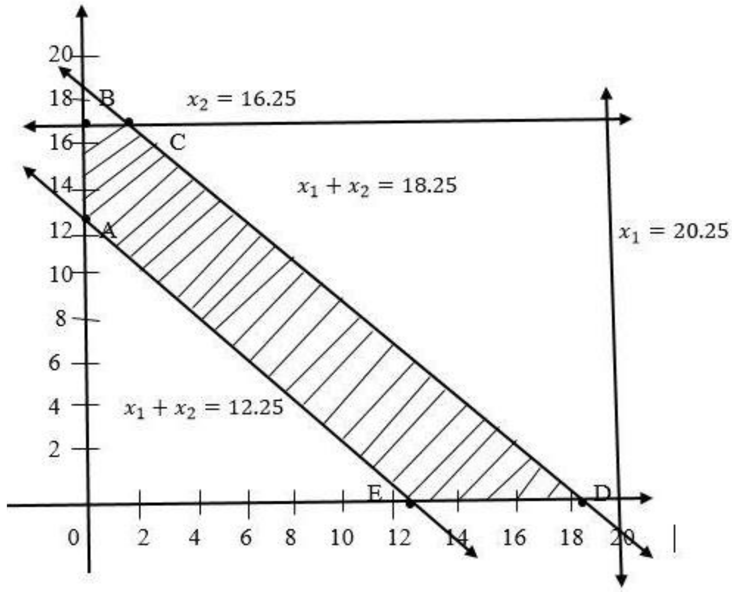
$x_2 \leq 16.25$, $x_1 + x_2 \geq 12.25$

$x_1, x_2 \geq 0$

Using graphical method

The first constraint $x_1, x_2 \leq 18.25$ can be shown by plotting the straight line $\frac{x_1}{18.25} + \frac{x_2}{18.25} = 1$.

This cuts a x_1−intercept and x_2−intercept of 18.25 each the area below this line symbolizes the feasible region. Similarly, the another constraints are depicted by plotting the straight lines corresponding to the equations $x_1 = 20.25$, $x_2 = 16.25$, $x_1 + x_2 = 12.25$. Here, the area below the first two lines and beyond the third line represents the feasible are a with regard to these three limitations.



Graph-1

As a result, feasible area for given model is as illustrated in graph 1. Co-ordinates of the extreme points are: A (0, 12.25), B (0, 16.25), C (2, 16.25), D (18.25, 0) and E (12.25, 0). The extreme points Z-values are as follows:

Table 4.

Extreme Points	(x_1, x_2)	$Z853.12 - 8x_1 - 22x_2$
A	(0, 12.25)	583.62
B	(0, 16.25)	495.62
C	(2, 16.25)	479.62
D	(18.25, 0)	707.12
E	(12.25, 0)	755.12

Hence the optimum solution is $x_1 = 2, x_2 = 16.25$ and minimize $Z = 479.62$.

2. Nonagonal FN:

Model 1: Consider the following FTP with nonagonal FN:

Destinations Sources	D_1	D_2	D_3
S_1	(0,1,2,3,4,5, 6,7,8)	(2,3,4,5,6,7, 8,9,10)	(-6,1,2,3,4,5,6,7,8)
S_2	(-1,0,1,2,3, 4,5,6,7)	(3,4,5,6,7,8, 9,10,11)	(1,2,3,4,5,6,7,8,9

and the fuzzy availability of the supply are (-6,1,2,3,4,5,6,7,8), (1,2,3,4,5,6,7,8,9) and the fuzzy availability of the demand are (-2,-1,0,1,2,3,4,5,6), (-1,0,1,2,3,4,5,6,7), (0,1,2,3,4,5,6,7,8) respectively.

Solution:

Create the fuzzy transportation table for the given problem

Table 1.

Destinations Sources	D_1	D_2	D_3	Supply
S_1	(0,1,2,3,4, 5,6,7,8)	(2,3,4,5,6, 7,8,9,10)	(-1,0,1,2, 3,4,5,6,7)	(-6,1,2,3, 4,5,6,7,8)
S_2	(-0,1,2,3, 4,5,6,7)	(3,4,5,6,7, 8,9,10,11)	(1,2,3,4,5, 6,7,8,9)	(1,2,3,4,5, 6,7,8,9
Demand	(-2,-1,0,1, 2,3,4,5,6)	(-1,0,1,2,3, 4,5,6,7)	(0,1,2,3,4, 5,6,7,8)	

Using ranking function (2.5.2)

Table 2.

Destinations Sources	D_1	D_2	D_3	Supply
S_1	13	19	10	14
S_2	10	22	16	16
Demand	7	10	13	

The problem is converted into a linear programming problem (LPP).

Formulation of LP model:

Key decision is to find the quantities that are transferred from each source to each location. Let x_1, x_2 be the quantity of transported from source S_1 to destination no. D_1 and D_2 correspondingly. The transportation table that resulted is presented below.

Table 3.

Destinations Sources	D_1	D_2	D_3	Supply
S_1	x_1	x_2	$14 - x_1 - x_2$	14
S_2	$7 - x_1$	$10 - x_2$	$16 - (7 - x_1)$ $-(10 - x_2)$	16
Demand	7	10	13	

Minimize $z = 13x_1 + 19x_2 + 10(14 - x_1 - x_2) + 10(7 - x_1) + 22 - (10 - x_2)$
$+16(16 - (7 - x_1) - (10 - x_2)$

Constraints are $14 - x_1 - x_2 \geq 0,\ 7 - x_1 \geq 0,$

$10 - x_2 \geq 0,\ 16 - (7 - x_1) - (10 - x_2) \leq 0,$

Minimize $z = 9x_1 + 3x_2 + 414$

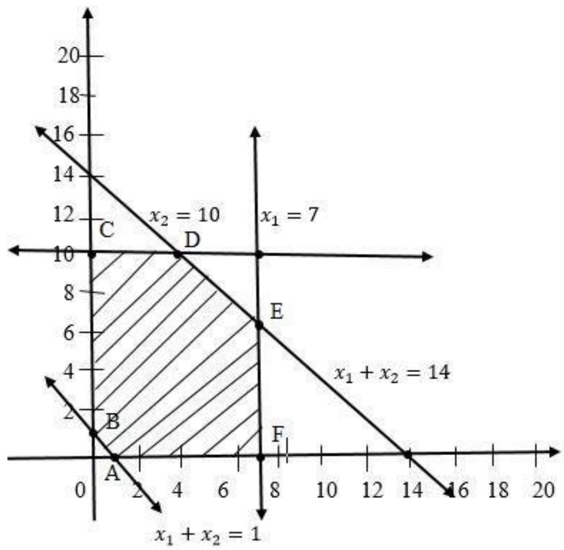
Subject to constraints: $x_1 + x_2 \leq 14,\ x_1 + 0x_2 \leq 7,$

$x_1 + x_2 \geq 1,\ 0x_1 + x_2 \leq 10,$

$x_1, x_2 \geq 0$

Using graphical method.

The first constraint $x_1, x_2 \leq 14$ can be shown by plotting the straight line $\frac{x_1}{14} + \frac{x_2}{14} = 1.$ This cuts a x_1-intercept and x_2-intercept of 6 each the area below this line symbolize the feasible region. Similarly, the another constraints are depicted by plotting the straight lines corresponding to the equations $x_1 = .7, x_2 = 10, x_1 + x_2 = 1.$ Here, the area below the first three lines and beyond the fourth line represents the feasible area with regard to these three limitations.



Graph-2

As a result, feasible area for given model is as illustrated in graph 2.Co-ordinates of the extreme points are: A (1, 0), B (0, 1), C (0, 10), D (4, 10), E (7, 6.1) and F (7,0). The extreme points Z-values are as follows.

Table 4.

Extreme Points	(x_1, x_2)	$Z = 9x_1 + 3x_2 + 414$
A	(1, 0)	423
B	(0, 1)	417
C	(0, 10)	444
D	(4, 10)	480
E	(7, 6.1)	495.3
F	(7, 0)	477

Hence the optimum solution is $x_1 = 0, x_2 = 1$ and minimize $Z = 417$.

6. Conclusion

In this paper, transportation problem having the cost, supplies and demands which are represented by HFN and NFN have been studied. A LP approach is applied to solve the same after converting it to a crisp form and the optimum solution is obtained by the given methodology. Some numericals have been solved. After comparison, it was found that, the results obtained by graphical method of LP approach are more promising than the results obtained by column minima method.

Table 5. Comparison table.

Examples	Methods	Minimize Z
1 (HFN)	Column Minima Method	558.12
	Graphical Method	479.62
1 (NFN)	Column Minima Method	216.25
	Graphical Method	209.5

References

[1] M. Deshmukh, K. Ghadle and O. Jadhav, Optimal solution of fully fuzzy LPP with symmetric HFNs, Computing in Engineering and Technology, Advances in Intelligent Systems and Computing, Springer Nature, Singapore 1025 (2020), 387-395.

[2] M. Deshmukh, K. Ghadle and O. Jadhav, An innovative approach for ranking hexagonal fuzzy numbers to solve linear programming problems, International Journal on Emerging Technologies 11(2) (2020), 385-388.

[3] K. Ghadle, M. Deshmukh and O. Jadhav, Solving equality and inequality constraints FLPP using intuitionistic fuzzy numbers with different techniques, Malaya Journal of Matematik 9(1) (2021), 385-390.

[4] I. Hussein and R. Mitlif, Ranking function to solve a fuzzy multiple objective function, Baghdad Science Journal (2020), 144-148.

[5] P. Pathade, A. Hamoud and K. Ghadle, A systematic approach for solving mixed constraints fuzzy balanced and unbalanced transportation problem, Indonesian Journal of Electrical Engineering and Computer Science 17(1) (2020), 85-90.

[6] P. Rajarajeswari and G. Menaka, Octagonal fuzzy transportation problem using different ranking method, International Journal of Trend in Scientific Research and Development 4(5) (2020), 8-13.

[7] S. Renuka and C. Jenita Nancy, Solving a transportation problem using nanogonal fuzzy number with robust ranking and Russell's method, International Journal of Engineering Technology and Computer Research 5(5) (2017), 100-104.

[8] A. Sahaya Sudha and S. Kurunambigai, Solving a transportation problem using a heptagonal fuzzy number, International Journal of Advanced Research in Science, Engineering and Technology 4(1) (2017), 3118-3125.

[9] S. Shanmuga, A novel shade to obtain an optimal solution to a fully fuzzy linear programming problem, International Journal of Scientific Research and Reviews 7(3) (2018), 2086-2102.

[10] Sami Kadhemkareem Althabhawi, Comparative study of ranking methods for fuzzy transportation, Iraqi Journal of Science 60(7) (2019), 1592-1602.