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PICTURE FUZZY LATTICES

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Abstract

Picture fuzzy set is the extension of fuzzy set and intuitionistic fuzzy set. In this paper, we introduce the concept of picture fuzzy lattices with respect to picture fuzzy partial order relation. Ideals of picture fuzzy lattices are also defined.

1. Introduction

Fuzzy set theory was familiarized to generalize the classical notion of set theory [8]. Since then it was developed in variety of theoretical and application oriented fields of Mathematics. Subsequently, the classical areas such as Group theory, Graph theory, Topology, Rings, Matrix theory, etc. are fuzzified. Many algebraists bared their interest in fuzzifying the formal algebraic structures. The development of various fuzzy algebraic structures has been protracted by many authors. Azriel Rosenfeld [5] proposed the fundamental theory of fuzzy groups. N. Ajmal and K. V. Thomas [1] proved the structural theorems for fuzzy lattices. U. M. Swamy [6] introduced fuzzy ideals on lattices. The theory of fuzzy lattice ordered ideals was studied in [2].

The concept of intuitionistic fuzzy sets and intuitionistic fuzzy relations are introduced to extend the notion of fuzzy sets. Then the idea of picture fuzzy sets was proposed to generalize the theory of fuzzy sets. Picture fuzzy relations and Picture fuzzy soft sets were discussed in [3].

The distinctive types of relations like equivalence relations and partially ordered relations have significant applications in Mathematics. The notion of fuzzy partial order relation was applied to lattice by Inheung Chon [4] to

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construct fuzzy lattice. B. K. Tripathy [7] introduced intuitionistic fuzzy lattices and intuitionistic Boolean algebras.

In this paper, we introduce picture fuzzy partial order relation to build the structure of picture fuzzy lattices.

In section 2, we report preliminary definitions and results on fuzzy lattices. In section 3, we define picture fuzzy partial order relation and picture fuzzy lattice. Ideals of picture fuzzy lattice are also defined.

2. Preliminaries

Definition 2.1 [8]**.** A fuzzy set of a non-empty set X is a function $f : X \to [0, 1]$.

Definition 2.2 [6]**.** A intuitionistic fuzzy set A on universe X is an object of the form $A = \{(x, \mu_A(x), \gamma_A(x)/x \in X)\}$ where $\mu_A(x) \in [0, 1]$ is called the degree of membership of x in A and $\gamma_A(x) \in [0, 1]$ is called the degree of non-membership of x in A and $\mu_A(x) + \gamma_A(x) \leq 1$ for $x \in X$.

Definition 2.3 [3]**.** A picture fuzzy set A on universe is an object in the form of $A = \{(x, \mu_A(x), \eta_A(x), \gamma_A(x)/x \in X)\}$ where $\mu_A(x) \in [0, 1]$ is called the degree of positive membership of x in A, $\eta_A(x) \in [0, 1]$ is called the degree of neutral membership of x in A and $\gamma_A(x) \in [0, 1]$ is called the degree of negative membership of x in A. Also $\mu_A(x) + \gamma_A(x) \leq 1$ for $x \in X$.

Definition 2.4 [4]**.** Let X be a set. A function $A : X \times X \to [0, 1]$ is called a fuzzy relation in X. The fuzzy relation A in X is reflexive if $A(x, x) = 1$ for all x in X. A is transitive if $A(x, z) \geq \sup\min(A(x, y), A(y, z))$ and A is anti-symmetric if $A(x, y) > 0$ implies $x = y$. A fuzzy relation is fuzzy partial order relation if A is reflexive, anti-symmetric and transitive. Any set X with fuzzy partial order relation is said to be a fuzzy partially ordered set (poset).

Definition 2.5 [4]**.** Let (X, A) be a fuzzy poset and $B \subseteq X$. An element u in X is said to be an upper bound for a subset B if $A(b, u) > 0$ for all b in B. An upper bound u_0 for B is the least upper bound of B if $A(u_0, u) > 0$ for every upper bound u for B. An element v in X is said to be a lower bound for a subset B if $A(v_0, v) > 0$ for all b in. A lower bound v_0 for B is the greatest lower bound of the set B if $A(v_0, v) > 0$ for every lower bound v for B.

Definition 2.6 [3]**.** Let X, Y, Z be ordinary nonempty sets. A picture fuzzy relation is a picture fuzzy set of $X \times Y$ i.e., R given by $R = \{(x, \mu_R(x), \eta_R(x), \gamma_R(x))/x \in X, y \in Y\}$ where $\mu_R : X \times Y \to [0, 1]$, satisfy the condition $\eta_R : X \times Y \to [0, 1], \gamma_R : X \times Y \to [0, 1]$ for every (x, y) in $X \times Y$.

Definition 2.7 [3]**.** Let R be the picture fuzzy relation on $X \times Y$. Then R^{-1} between Y and X is defined by $\mu_{R^{-1}}(y, x) = \mu_R(x, y)$, $\mu_{R^{-1}}(y, x) = \eta_R(x, y)$, $\gamma_{R^{-1}}(y, x) = \gamma_R(x, y)$ for (x, y) in $X \times Y$.

Definition 2.8 [3]**.** Let R and P be two picture fuzzy relations between X and Y. Then for every (x, y) in $X \times Y$,

(i) $R \vee P = \{((x, y), \mu_R(x, y) \times \mu_P(x, y), \eta_R(x, y) \times \eta_P(x, y), \psi_R(x, y) \times \gamma_P(x, y)/x \in X, y \in Y\}$.

(ii) $R \vee P = \{((x, y), \mu_R(x, y) \times \mu_P(x, y), \eta_R(x, y) \times \eta_P(x, y), \gamma_R(x, y) \times \gamma_P(x, y)/x \in X, y \in Y\}$.

Definition 2.9 [4]**.** Let (X, A) be a fuzzy poset, (X, A) is a fuzzy lattice iff $x \vee y$ and $x \wedge y$ exist for all $x, y \in X$.

3. Picture Fuzzy Lattices

Definition 3.1. A picture fuzzy relation R defined on $X \times X$ by $R = \{(x, \mu_R, \eta_R(x), \gamma_R(x))/x \in X)\}$ is

(i) Reflexive if for every x in $X, \mu_R(x, x) = 1, \eta_R(x, x) \in (0, 1), \gamma_R(x, x) = 0$

(ii) Anti-symmetric if for every $(x, y) \in X \times X, \mu_R(x, y) > 0$ and $\mu_R(y, x) > 0$ (or) $\eta_R(x, y) = \eta_R(y, x)$ (or) $\gamma_R(x, y) < 1$ and $\gamma_R(x, y) < 1$ implies $x = y$.

(iii) Transitive if

$$\mu_R(x, z) \geq \max[\min\{\mu_R(x, y), \mu_R(y, z)\}]$$

$$\mu_R(x, z) \geq \max[\min\{\eta_R(x, y), \eta_R(y, z)\}]$$

$$\eta_R(x, z) \geq \max[\min\{\eta_R(x, y), \eta_R(y, z)\}].$$

Definition 3.2. A picture fuzzy relation R on X is said to be picture fuzzy partial order relation if R is reflexive, anti-symmetric and transitive. Let X be a set with picture fuzzy partial order relation R defined on X. Then (X, R) is called a picture fuzzy partial order set.

Definition 3.3. A non-empty set X on which a picture fuzzy partial order relation R is defined is said to be a picture fuzzy lattice if for any two element x, y in X, the least upper bound and greatest lower bound exists in X.

Definition 3.4. Let P be the picture fuzzy lattice with picture fuzzy partial order relation R. Then $I \subseteq P$ is an ideal if for $p \in P$ and $a \in I$, $R(p, a) > 0$ implies $p \in I$. That is, $\mu_R(p, a) > 0$, $\eta_R(p, a) \in (0, 1)$, $\gamma_R(p, a) < 1$ implies $p \in I$.

Definition 3.5. Let P be the picture fuzzy lattice and I, J be the ideals of P. Then $I \cup J = \{x, \max(\mu_I(x), \mu_J(x)), \min(\eta_I(x), \eta_J(x)), \min(\gamma_I(x), \gamma_J(x)$ for $x \in P)\}$ $I \cup J = \{x, \min(\mu_I(x), \mu_J(x)), \min(\eta_I(x), \eta_J(x)), \max(\gamma_I(x), \gamma_J(x)$ for $x \in P)\}$.

By the above definitions, we can prove the following propositions:

Proposition 3.6. *Intersection of two ideals of picture fuzzy lattice is also an ideal.*

Proposition 3.7. *If I, J are two fuzzy ideals of a picture fuzzy lattice P with picture fuzzy partial order relation R, then*

(i) $(R_I \vee R_J)^{-1} = R_I^{-1} \vee R_J^{-1}$

(ii) $(R_I \vee R_J)^{-1} = R_I^{-1} \wedge R_J^{-1}$.

Proposition 3.8. *If I, J are two fuzzy ideals of a picture fuzzy lattice P*

with picture fuzzy partial order relation R, then

(i) $\overline{(R_I R_J)} = \overline{R_I R_J}$

(ii) $\overline{(R_I R_J)} = \overline{R_I R_J}$.

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