Advances and Applications in Mathematical Sciences
Volume 22, Issue 2, December 2022, Pages 537-555

SOLVING INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM USING NEW GM RANKING METHOD

NALLA VEERRAJU[1,*] and V. LAKSHMI PRASANNAM[3,4]

[1]Research Scholar
Department of Mathematics
Krishna University
Machilipatnam, A. P., India

[*]Department of Mathematics
S. R. K. R. Engineering College
Bhimavaram, A. P., India

[3]Research Supervisor
Krishna University
Machilipatnam, A. P., India

[4]Department of Mathematics
P. B. Siddhartah College of Arts and Sciences
Vijayawada, A. P., India
Email: drvlp@rediffmail.com

Abstract

The decision making concerned with the data of linear programming problem (LPP) of real world, sometimes, involve uncertainty (vagueness or impreciseness). The fuzzy set theory is observed to tackle well with uncertainty efficiently making use of its membership function. However, there have been phenomena in many problems, LPPs in particular, where, along with degree of acceptance, non-acceptance of the data for parameters is to be understood, considered and utilised in decision making. The intuitionistic fuzzy sets (IFS) are noticed to be significant in addressing logically this type of situations of real time.

2020 Mathematics Subject Classification: 90C05, 03E72, 90C70, 03F55.
Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy number, Ranking and Intuitionistic fuzzy linear programming problem.
[*]Corresponding author; Email: veerrajunalla@gmail.com
Received October 31, 2021; Accepted December 4, 2021

In this paper, a new method is proposed to solve intuitionistic fuzzy linear programming problem (IFLPP). Since ranking method plays a pivotal role in providing the possible best values for IFLPP, a new ranking method, based on support and resultant membership function of an intuitionistic fuzzy number (IFN), is also proposed for ordering IFNs. To assert the authenticity of the proposed methods, the methods are studied with numerical examples available in the domain.

I. Introduction

The data related to most of the decision-making problems often include vagueness or impreciseness. The fuzzy sets (FS), since their introduction by Zadeh [27], have been applied to represent vagueness or impreciseness in a logical and meaningful way with the help of its membership function. This representation has been examined to be effective in solving decision making problems. However, in some decision-making problems, data, related to parameters, has to be defined and formulated taking both membership and non-membership values into consideration. It has been observed that this kind of problems have been addressed and handled well by the IFS. The IFS, introduced by Atanassov [2] as a generalisation of FS, got due significance and developed by many others in the course of time. In IFS, along with the degree of membership indicating belongingness of an element to a set, the degree of non-membership indicating non belongingness of an element to a set, is denoted. In IFS, the membership degree and the non-membership degree are not complemented to each other and they are related such that sum of its two degrees must be less than or equal to one.

Ranking (or ordering) of fuzzy numbers is known to be a difficult task as those are not linearly ordered (i.e., those could not be mapped onto real line). However, it is most inevitable and essential in decision making. This ranking is important and significant in IFNs also, as they have to be ordered in real-time decision-making problems. A number of ranking techniques have been forwarded from all over the world for ranking of IFNs based on different characteristics since its inception.

Viewing and understanding each IFN as a union of ordinary fuzzy number through a statistical view point, Mitchell [14] brought forward a method for ordering IFNs. By generalising characteristic values of membership function and non-membership function as fuzzy quantities, Nehi

[19] introduced a method for ranking IFNs. Having based on weighted expected values, Ye [26] introduced expected value method to rank trapezoidal intuitionistic fuzzy numbers (TrIFNs). An ordering method, founded on centroid concept that uses geometric centre, was added to the domain by Arun praksh et al. [1] to rank both TrIFNs and triangular intuitionistic fuzzy numbers (TIFNs). A total ordering method, using the idea of selected lower and upper dense sequence on the basis of decomposition theorem for the class of IFNs, was given by Nayagam et al. [11]. Making use of axiomatic approach which is supported by a set of eight different scores, a linear (total) ranking procedure is forwarded by Nayagam et al. [12] for ranking the class of TrIFNs. Having rooted on the concepts of eight different score functions which explores IFNs in different ways, a total ordering method on the class of TrIFNs was put forward by Nayagam et al. [13]. Bharati [3] brought to the fore a method which ranks given TIFNs depending on the distance for each IFN from fuzzy origin. Chutia and Saikia [5] forwarded a method supported by the concepts of values and ambiguity at different levels for ranking of TrIFNs. Having relied on area lying on left side and alpha and beta cuts of an IFN, a method is proposed by Darehmiraki [7] to rank IFNs. Suresh Mohan et al. [15] introduced a method for ranking IFNs basing on means of magnitude. Chutia [6] developed a ranking method depending on index of optimism in which value and multiple of ambiguity inclusion function concepts are utilised.

In optimisation, LPP is most important and most sought-after tool. Most of the times, the LPPs have to use the data which is inherently filled with vagueness or impreciseness. As IFS are noticed to have been addressing vagueness and impreciseness in a meaningful and logical way, hence naturally, “Intuitionistic Fuzzy Linear Programming (IFLP)” has been gaining the attention and importance from researchers.

Dubey and Mehra [8] forwarded a method making use of the concept of value and ambiguity in order to solve IFLPPs with data consisting of TIFNs. Nagoorgani and Ponnalagu [16] came up with a method to deal with IFLPPs in which, scoring function is used to rank TIFNs and accuracy function is defined to defuzzify TIFNs. Parvathi and Malathi [20] forwarded a ranking method to rank symmetric TrIFNs and the same is used to solve IFLPP with the help of intuitionistic fuzzy simplex method. Nagoorgani and Ponnalagu

[17] put forward a score function for ranking TIFNs and an algorithm to deal with IFLPP using single step method. Having based on possibility, credibility and necessity measures, Chakraborty et al. [4] introduced an intuitionistic fuzzy chance constraints model method and applied their method to solve IFLPP. The method, introduced by Suresh et al. [25], ranked TIFNs by means of magnitude and this method is used to solve IFLPP by simplex method. A method, established on matrix analysis, was utilised by Nagoorgani et al. [18] to solve IFLPP. By modifying Suresh et al. [25] method, Sidhu and Kumar [22] introduced a method and applied it to solve IFLPP. Sidhu and Kumar [23] observed that the method given by Parvathi and Malathi [20] fails if any one of the coefficients in objective function or constraints is negative real number and if variables and right-hand side vectors are represented by non-symmetric TrIFNs. Hence Sidhu and Kumar [23] modified the method given by Parvathi and Malathi [20] and forwarded modified method as Meher method to solve IFLPP which overcame the deficiencies of Parvathi and Malathi [20]. To solve IFLPP, a method called separation and addition method, was introduced by Jayalakshmi et al. [9]. To solve IFLPP, Kabiraj et al. [10] introduced a method based on Zimmermann [28] ranking technique. Boris et al. [21] analysed and modified Singh and Yadav [24] method and introduced a method to solve IFLPP.

It is very obvious that there have been a good number of ranking methods available in the literature. Owing to the different typical features of IFNs like the uncertainty in nature, the complexity in visualisation, understanding and interpretation and the difficulty in ranking, all the existing methods may not deal with all types of problems concerned with ranking of IFNs in all contexts. Along with these points, most of the existing methods have some drawbacks such as dealing only with a particular type of IFNs, lengthy computations, complex calculations and inconsistency i.e., sometimes being not able to rank. These contexts surely motivate and provide an ample opportunity to venture for proposing a new ranking method which may play its own significant role. The proposed ranking method has its roots from the concepts-support and resultant membership of given IFN which are used to find crisp value associated with that IFN through geometric mean.

It is worthy and logical to note that the efficiency of the IFLPP methods depends on the preciseness of the ranking methods (or ranking functions). As long as a ranking method has a drawback, surely it will influence the outcome of that particular ranking method and hence the solution of IFLPP may not be optimal. The better the ranking method is the better the result from IFLPP. Some of the existing methods produce fuzzy optimal solution while other give crisp optimal solution. As for as decision making is concerned, in order to obtain better alternatives, solution of the crisp form would be of a great advantage for decision makers. Consequently, having kept these core ideas in mind, a method to solve IFLPP which produces crisp solution, is presented here basing on the proposed ranking method.

The rest of the paper is organised as following: Section II is devoted to mentioning necessary preliminaries. Section III is dedicated for proposed ranking method and its discussion. Section IV is assigned for proposed method to solve IFLPP. Finally, Section V is kept for conclusions.

II. Preliminaries

Definition 2.1 ([27])**.** A fuzzy set $\widetilde{A}$, X being a universe of discourse and $\mu_{\widetilde{A}}(x)$ being a membership function, is defined by $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)), x \in X\}$ and $\mu_{\widetilde{A}}(x) : X \to [0, 1]$.

Definition 2.2 ([23])**.** An intuitionistic fuzzy set $\widetilde{A}^I$ in X is defined as a set of the form $\widetilde{A}^I = \{(x, \mu_{\widetilde{A}^I}(x), \vartheta_{\widetilde{A}^I}(x)) : x \in X\}$ where the function $\mu_{\widetilde{A}^I}(x) : X \to [0, 1]$ and $\vartheta_{\widetilde{A}^I}(x) : X \to [0, 1]$ define the degree of membership and non-membership of the element $x \in X$ respectively and for every $x \in X$ in $\widetilde{A}^I$, $0 \le \mu_{\widetilde{A}^I}(x) + \vartheta_{\widetilde{A}^I}(x) \le 1$ holds.

Definition 2.3 ([23])**.** An intuitionistic fuzzy set $\widetilde{A}^I = \{(x, \mu_{\widetilde{A}^I}(x), \vartheta_{\widetilde{A}^I}(x)) : x \in X\}$ is said to be an intutitionistic fuzzy normal if there exists two points $x_0, x_1 \in X$ such that $\mu_{\widetilde{A}^I}(x_0) = 1$, $\vartheta_{\widetilde{A}^I}(x_1) = 1$.

Definition 2.4 ([23])**.** An intuitionistic fuzzy set $\widetilde{A}^I$ is said to be an intuitionistic fuzzy number if it is

(a) Intutitionistic fuzzy normal

(b) Convex for the membership function $\mu_{\tilde{A}^I}(x)$ i.e. $\mu_{\tilde{A}^I}(x)(\lambda x_1 + (1-\lambda) x_2) \geq \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in X, \lambda \in [0, 1]$.

(c) Concave for the non-membership function $v_{\tilde{A}^I}(x)$ i.e. $\vartheta_{\tilde{A}^I}(x)(\lambda x_1 + (1-\lambda) x_2) \leq \max(\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in X, \lambda \in [0, 1]$.

Definition 2.5 ([1]). A generalised triangular intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 (\leq, \geq) a_2 \leq a_3 \leq b_3$ is denoted as

$$\tilde{A}^I = ((a_1, a_2, a_3; w), (b_1, b_2, b_3; v))$$

Where the membership is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} w \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ w \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x < a_3 \\ 0, & \text{otherwise} \end{cases}$$

non-membership is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b_2 - x + v(x - b_1)}{b_2 - b_1}, & b_1 < x < b_2 \\ \dfrac{x - b_2 + v(b_3 - x)}{b_3 - b_2}, & b_2 < x < b_3 \\ 1, & \text{otherwise} \end{cases}$$

and w is the maximum membership value and v is the minimum non-membership value

Such that $\mu_{\tilde{A}^I}(x) \leq w$ and $\vartheta_{\tilde{A}^I}(x) \geq v$ for all x, $0 \leq w \leq 1, 0 \leq v \leq 1$ and $0 \leq w + v \leq 1$.

If $w = 1$ and $v = 0$ in the definition of generalised triangular intuitionistic fuzzy number, then it is called triangular intuitionistic fuzzy number (TIFN) and it is denoted as

$$\tilde{A}^I = ((a_1, a_2, a_3), (b_1, b_2, b_3))$$

where the membership is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x < a_3 \\ 0, & \text{otherwise} \end{cases}$$

and non-membership is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b_2 - x}{b_2 - b_1}, & b_1 < x < b_2 \\ \dfrac{x - b_2}{b_3 - b_2}, & b_2 < x < b_3 \\ 1, & \text{otherwise.} \end{cases}$$

Definition 2.6 ([1]). A generalised trapezoidal intuitionistic fuzzy number with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ is denoted as $\tilde{A}^I = ((a_1, a_2, a_3, a_4; w), (b_1, b_2, b_3, b_4; v))$.

Where the membership is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} w\dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ w, & a_2 < x < a_3 \\ w\dfrac{a_4 - x}{a_4 - a_3}, & a_3 < x < a_4 \\ 0, & \text{otherwise} \end{cases}$$

non-membership is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b_2 - x + v(x - b_1)}{b_2 - b_1}, & b_1 < x < b_2 \\ v, & b_2 < x < b_3 \\ \dfrac{x - b_3 + v(b_4 - x)}{b_4 - b_3}, & b_3 < x < b_4 \\ 1, & \text{otherwise} \end{cases}$$

and w is the maximum membership value and v is the minimum non-membership value. Such that $\mu_{\tilde{A}^I}(x) \leq w$ and $\vartheta_{\tilde{A}^I}(x) \geq v$ for all x $0 \leq w \leq 1$, $0 \leq v \leq 1$ and $0 \leq w + v \leq 1$.

In the above definition, if we let $b_2 = b_3$ (and hence $a_2 = a_3$), then generalised trapezoidal intuitionistic fuzzy number becomes generalised triangular intuitionistic fuzzy number.

If $w = 1$ and $v = 1$ in the definition of generalised trapezoidal intuitionistic fuzzy number, then it is called trapezoidal intuitionistic fuzzy number (TrIFN) and it is denoted as $\tilde{A}^I = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4))$. Where the membership is given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ 1, & a_2 < x < a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 < x < a_4 \\ 0, & \text{otherwise} \end{cases}$$

and non-membership is given by

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b_2 - x}{b_2 - b_1}, & b_1 < x < b_2 \\ v, & b_2 < x < b_3 \\ \dfrac{x - b_3}{b_4 - b_3}, & b_3 < x < b_4 \\ 1, & \text{otherwise} \end{cases}$$

III. Proposed Ranking Method

Fuzzy set theory, whose membership value ranges from 0 to 1, is a generalization of crisp set theory whose membership value is '1'. Intuitionistic fuzzy set theory is known to be the generalization of fuzzy set theory. Further, in intuitionistic fuzzy set theory, along with membership function, non-membership function is also defined. In many decision-making problems in fuzzy environment, the crisp value associated with fuzzy number is to be identified either by defuzzification or ranking method. Thus, it is also essential to find crisp value associated with an IFN to rank them properly. In intuitionistic fuzzy set theory, the non-membership function is involved to describe the degree of non-belongingness along with membership function that describes the degree of belongingness. Hence resultant membership degree is to be identified for the belongingness of an element to the set.

Definition 3.1. Resultant membership function. Let $\mu_{\widetilde{A}^I}(x)$ and $\vartheta_{\widetilde{A}^I}(x)$ be membership and non-membership functions of an intuitionistic fuzzy number $\widetilde{A}^I$, then the resultant membership function $R_{\widetilde{A}^I}(x)$ is defined to be

$$R_{\widetilde{A}^I}(x) = \mu_{\widetilde{A}^I}(x) - \vartheta_{\widetilde{A}^I}(x)$$

Based on the Resultant membership function, the support of an intuitionistic fuzzy number can be defined as following.

Definition 3.2. Support of a intuitionistic fuzzy number. The support of an intuitionistic fuzzy number $\widetilde{A}^I$ is $S(\widetilde{A}^I) = \{x : R_{\widetilde{A}^I}(x) \geq 0\}$.

Proposition 3.1. *If* $\widetilde{A}^I = ((a_1, a_2, a_3, a_4; w), (b_1, b_2, b_3, b_4; v))$ *be a TrIFN then the support of* $\widetilde{A}^I$ *is* $S(\widetilde{A}^I) = [MP_1, MP_2]$ *where*

$$MP_1 = \frac{wa_1(b_2 - b_1) + (a_2 - a_1)(b_2 - vb_1)}{w(b_2 - b_2) + (1 - v)(a_2 - a_1)} \text{ and}$$

$$MP_2 = \frac{wa_4(b_4 - b_3) + (a_4 - a_3)(b_3 - vb_4)}{w(b_4 - b_3) + (1 - v)(a_4 - a_3)}.$$

Proposition 3.2. *If* $\widetilde{A}^I = ((a_1, a_2, a_3; w), (b_1, b_2, b_3; v))$ *be a TIFN, then the support of* $\widetilde{A}^I$ is $S(\widetilde{A}^I) = [MP_1, MP_2]$ *where*

$$MP_1 = \frac{a_1 w(b_2 - b_1) + (a_2 - a_1)(b_2 - vb_1)}{w(b_2 - b_2) + (1 - v)(a_2 - a_1)} \text{ and}$$

$$MP_2 = \frac{wa_3(b_3 - b_2) + (a_3 - a_2)(b_2 - vb_3)}{w(b_3 - b_2) + (1 - v)(a_3 - a_2)}.$$

Geometric Mean-Ranking (GM-R) Method:

If $\widetilde{A}^I$ be an intuitionstic fuzzy number with $\mu_{\widetilde{A}^I}(x)$ and $\vartheta_{\widetilde{A}^I}(x)$ be membership and non-membership functions respectively and $S(\widetilde{A}^I)$ be the support of $\widetilde{A}^I$. Then the crisp value associated with $\widetilde{A}^I$, denoted as $G_{\widetilde{A}^I}$, is defined to be

$$G_{\widetilde{A}^I} = Exp\left[\frac{\int_{S(\widetilde{A}^I)} R_{\widetilde{A}^I}(x).\ln x\, dx}{\int_{S(\widetilde{A}^I)} R_{\widetilde{A}^I}(x)\, dx}\right] \tag{1}$$

Where $R_{\widetilde{A}^I}(x)$ is resultant membership function of $\widetilde{A}^I$.

Ranking procedure:

Let $\widetilde{A}^I$ and $\widetilde{B}^I$ be two IFNs.

Step 1. Calculate the Supports $S(\widetilde{A}^I)$ and $S(\widetilde{B}^I)$ of given $\widetilde{A}^I$ and $\widetilde{B}^I$ respectively using. Proposition 3.1 or Proposition 3.2 according to the given type of IFNs.

Step 2. Calculate crisp values of $\widetilde{A}^I$ and $\widetilde{B}^I$ using the formula (1) and denote them as $G_{\widetilde{A}^I}$ and $G_{\widetilde{B}^I}$ respectively.

Step 3. Identify the raking order using the following cases

(i) If $G_{\widetilde{A}^I} < G_{\widetilde{B}^I}$ then $\widetilde{A}^I \prec \widetilde{B}^I$.

(ii) If $G_{\widetilde{A}^I} > G_{\widetilde{B}^I}$ then $\widetilde{A}^I \succ \widetilde{B}^I$.

(iii) If $G_{\widetilde{A}^I} = G_{\widetilde{B}^I}$ then $\widetilde{A}^I \sim \widetilde{B}^I$.

Example 3.1. Consider two TrIFNs $\widetilde{A}^I = ((0.4, 0.5, 0.7, 0.9; 1); (0.3, 0.4, 0.8, 0.9; 0))$ and $\widetilde{B}^I = ((0.4, 0.5, 0.7, 0.8; 1); (0.2, 0.3, 0.7, 0.85; 0))$ which were studied by Nayagam et al. [11]

Using the proposed GM-R method, the following are obtained.

Step 1. The supports of $\widetilde{A}^I$ and $\widetilde{B}^I$ are calculated using proposition 3.1 and they are $S(\widetilde{A}^I) = [0.4, 0.8333]$ and $S(\widetilde{B}^I) = [0.35, 0.76]$.

Step 2. The crisp values associated with $\widetilde{A}^I$ and $\widetilde{B}^I$ are obtained by the formula (1) and they are respectively $G_{\widetilde{A}^I} = 0.6034$ and $G_{\widetilde{B}^I} = 0.5248$.

Step 3. It is clear that $G_{\widetilde{B}^I} < G_{\widetilde{A}^I}$ follows that $\widetilde{B}^I \prec \widetilde{A}^I$.

Nayagam et al. [11] used upper and lower dense sequence and obtained the result as $\widetilde{B}^I \prec \widetilde{A}^I$ which coincides with the result obtained by the proposed GM-R method.

Example 3.2. Consider the following sets of IFNs which were treated by Chutia [5]

Set 1. $\widetilde{P}^I = ((0.2, 0.3, 0.5, 0.6; 1); (0.1, 0.2, 0.6, 0.7; 0))$

$\widetilde{Q}^I = ((0.1, 0.3, 0.3, 0.5; 1); (0.0, 0.3, 0.3, 0.6; 0))$.

Set 2. $\widetilde{P}^I = ((0.2, 0.3, 0.5, 0.6; 1); (0.1, 0.2, 0.6, 0.7; 0))$

$\widetilde{R}^I = ((0.2, 0.4, 0.4, 0.6; 1); (0.1, 0.4, 0.4, 0.7; 0))$.

Set 3. $\widetilde{P}^I = ((0.2, 0.3, 0.5, 0.6; 1); (0.1, 0.2, 0.6, 0.7; 0))$

$\widetilde{S}^I = ((0.3, 0.5, 0.5, 0.7; 1); (0.2, 0.5, 0.5, 0.8; 0))$.

Set 4. $\widetilde{L}^I = ((0.0, 0.4, 0.7, 0.8; 1); (0.0, 0.3, 0.8, 0.9; 0))$

$\widetilde{M}^I = ((0.2, 0.5, 0.5, 0.9; 1); (0.2, 0.5, 0.5, 1; 0))$

$\widetilde{N}^I = ((0.0, 0.6, 0.6, 0.8; 1); (0.0, 0.6, 0.6, 0.9; 0))$.

After applying the proposed GM-R method, the crisp values associated with the given IFNs $\widetilde{P}^I, \widetilde{Q}^I, \widetilde{R}^I, \widetilde{S}^I, \widetilde{L}^I, \widetilde{M}^I$ and $\widetilde{N}^I$ are $G_{\widetilde{P}^I} = 0.3585$, $G_{\widetilde{Q}^I} = 0.29589$, $G_{\widetilde{R}^I} = 0.39696$, $G_{\widetilde{S}^I} = 0.49758$, $G_{\widetilde{L}^I} = 0.49509$, $G_{\widetilde{M}^I} = 0.51849$ and $G_{\widetilde{N}^I} = 0.53226$ respectively.

It is observed that $G_{\widetilde{P}^I}$ is greater than $G_{\widetilde{Q}^I} \cdot G_{\widetilde{R}^I}$ is clearly larger than $G_{\widetilde{P}^I} \cdot G_{\widetilde{S}^I}$ is noticed to be higher than $G_{\widetilde{P}^I}$.

$G_{\widetilde{N}^I}$ is obtained to be greater than both $G_{\widetilde{L}^I}$, $G_{\widetilde{M}^I}$ and $G_{\widetilde{M}^I}$ is shown to be larger than $G_{\widetilde{L}^I}$.

The ranking details for the above 4 sets using existing methods Ye [26], Nayagam et al. [12], Chutia [5] and Darehmiraki [7] are furnished in the following Table 1. The ranking orders by GM-R method are also compared with the ranking orders of [5, 7, 12, 26].

Table 1. Ranking of IFNs in Example 3.2 using different methods.

Set	Ye [26]	Nayagam et al.	Chutia [5] for $\alpha = 0.1$, $\beta = 0.9$	Darehmiraki [7] for $\alpha = 0.1$, $\beta = 0.9$	Proposed method
I	$\widetilde{Q}^I \prec \widetilde{P}^I$	$\widetilde{Q}^I \prec \widetilde{P}^I$	$\widetilde{Q}^I \prec \widetilde{P}^I$	$\widetilde{Q}^I \prec \widetilde{P}^I$	$\widetilde{Q}^I \prec \widetilde{P}^I$
II	$\widetilde{P}^I \sim \widetilde{R}^I$	$\widetilde{R}^I \prec \widetilde{P}^I$	$\widetilde{P}^I \prec \widetilde{R}^I$	$\widetilde{P}^I \sim \widetilde{R}^I$	$\widetilde{P}^I \prec \widetilde{R}^I$
III	$\widetilde{P}^I \prec \widetilde{S}^I$	$\widetilde{S}^I \prec \widetilde{P}^I$	$\widetilde{P}^I \prec \widetilde{S}^I$	$\widetilde{P}^I \prec \widetilde{S}^I$	$\widetilde{P}^I \prec \widetilde{S}^I$
IV	$\widetilde{L}^I \prec \widetilde{N}^I \prec \widetilde{M}^I$	$\widetilde{N}^I \prec \widetilde{M}^I \prec \widetilde{L}^I$	$\widetilde{L}^I \prec \widetilde{N}^I \prec \widetilde{M}^I$	$\widetilde{L}^I \prec \widetilde{N}^I \prec \widetilde{M}^I$	$\widetilde{L}^I \prec \widetilde{N}^I \prec \widetilde{M}^I$

For the sets I, II, III and IV, the ranking order given by proposed GM-R method clearly coincides with the ranking order given by Chutia [5]. The ranking order obtained by GM-R method is in agreement for the set I and in disagreement for the sets II, III and IV with Nayagam et al. [12]. The ranking by GM-R method agrees with Ye [26] for the sets I and III and disagrees with IV. The proposed GM-R method ordered all the sets whereas Ye [26] could not rank set II. The proposed GM-R method ranking is in agreement with the ranking order of Darehmiraki [7] for the sets I and III and in disagreement with set IV. Darehmiraki [7] is not able to rank set II whereas proposed GM-R method ranked set II.

IV. Proposed Method for Solving Intuitionstic Fuzzy Linear Programming Problem

The mathematical formulation of the standard form of IFLPP is as follows Maximize (or) Minimize $\widetilde{Z}^I = \sum_{j=1}^{n} \widetilde{c}_j^I \otimes \widetilde{x}_j^I$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij}^{I} \otimes \tilde{x}_{j}^{I} (\leq = \geq)\, \tilde{b}_{i}^{I} \quad i = 1, 2, 3, \ldots, m$

Where $\tilde{c}_{j}^{I}$, $\tilde{a}_{ij}^{I}$ and $\tilde{b}_{i}^{I}$ are IFNs and $\tilde{x}_{j}^{I}$ are non negative IFNs.

A method is proposed to solve IFLPP based on the sign present in constraints of the problem. The crisp values of all the elements in cost vector $\tilde{c}_{j}^{I}\ \forall j$ and coefficient matrix $\tilde{a}_{ij}^{I}\ \forall i, j$ are obtained using GM-R method proposed in section-III. The support of the elements $\tilde{b}_{i}^{I}\ \forall i$ are derived using either Proposition 3.1 or Proposition 3.2. Thus, using the GM-R method, the left-hand side (LHS) elements of constraints are converted into crisp values and right-hand side (RHS) values are converted into intervals. The constraints of the problem are converted to corresponding crisp constraints using the following ways.

(i) Constraint with equality sign: The value of LHS part of the constraint is considered to be lying in the support of the IFN in RHS.

(ii) Constraint with greater than sign: The value of LHS part of the constraint is considered to be greater than the infimum of Support of the IFN in RHS.

(iii) Constraint with less than sign: The value of LHS part of the constraint is considered to be less than the supremum of Support of the IFN in RHS.

Using the above steps, the IFLPP is converted into the following crisp LP problem.

Maximize (or) Minimize $Z = \sum_{j=1}^{n} c_j x_j$

Subject to

(i) Equality constraints:

$$\text{Infimum}\{S(\tilde{b}_{i}^{I})\} \leq \sum_{j=1}^{n} a_{ij} x_j \leq \text{Supremum}\{S(\tilde{b}_{i}^{I}),\ i = 1, 2, \ldots, m.$$

(ii) Constraints with greater than sign:

$$\sum_{j=1}^{n} a_{ij}x_j \geq \text{Infimum}\{S(\tilde{b}_i^I),\ i = 1, 2, \ldots, m.$$

(iii) Constraints with less than sign:

$$\sum_{j=1}^{n} a_{ij}x_j \leq \text{Supremum}\{S(\tilde{b}_i^I),\ i = 1, 2, \ldots, m.$$

Where c_j and a_{ij} are crisp values of $\tilde{c}_j^I$ and $\tilde{a}_{ij}^I$ and x_j are crisp decision variables.

Illustration of proposed method is done with the help of some examples, considered from available literature. Comparison of results is examined to showcase advantages of proposed method.

Example 4.1. Consider the following problem addressed by Boris et al. [21]

$$\text{Maximise}\left((6,\ 8,\ 10;\ 1),\ \left(\frac{11}{2},\ 8,\ \frac{21}{2};\ 0\right)\right)\tilde{x}_1^I$$

$$+\left((10,\ 12,\ 14;\ 1);\ \left(\frac{19}{2},\ 12,\ \frac{19}{2};\ 0\right)\right)\tilde{x}_2^I$$

$$+\left(\left(\frac{3}{4},\ 1,\ \frac{5}{4};\ 1\right);\ \left(\frac{1}{2},\ 1,\ \frac{3}{2};\ 0\right)\right)\tilde{x}_3^I$$

Subject to

$$\left(\left(\frac{9}{2},\ 5,\ \frac{11}{2};\ 1\right);\ \left(\frac{17}{2},\ 5,\ \frac{23}{2};\ 0\right)\right)\tilde{x}_1^I + \left(\left(\frac{9}{2},\ 5,\ \frac{11}{2};\ 1\right);\ \left(\frac{17}{4},\ 5,\ \frac{23}{4};\ 0\right)\right)\tilde{x}_2^I + \tilde{x}_3^I$$

$$= ((110,\ 155,\ 207;\ 1);\ (95,\ 155,\ 225;\ 0))$$

$$\left(\left(\frac{23}{4},\ 6,\ \frac{25}{4};\ 1\right);\ \left(\frac{11}{2},\ 6,\ \frac{13}{2};\ 0\right)\right)\tilde{x}_1^I + \left(\left(\frac{7}{4},\ 2,\ \frac{9}{4};\ 1\right);\ \left(\frac{3}{2},\ 2,\ \frac{5}{2};\ 0\right)\right)\tilde{x}_2^I$$

$$\leq ((107,\ 125,\ 147;\ 1);\ (95,\ 125,\ 160;\ 0))$$

$$\left(\left(\frac{1}{2},\ 1,\ \frac{5}{4};\ 1\right);\ \left(\frac{1}{4},\ 1,\ \frac{3}{2};\ 0\right)\right)\tilde{x}_1^I + \left(\left(\frac{15}{4},\ 4,\ \frac{9}{2};\ 1\right);\ \left(\frac{7}{2},\ 4,\ \frac{19}{4};\ 0\right)\right)\tilde{x}_2^I$$

$$\leq ((68,\ 110,\ 148;\ 1);\ (45,\ 110,\ 170;\ 0))$$

Where $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$ and $\tilde{x}_3^I$ is unrestricted

Here, the first constraint in the problem is with equality sign and the remaining two constraints are with inequality sign. These intuitionisitic fuzzy equality and inequality constraints are converted to crisp inequalities by the cases (i) and (iii) of the proposed method. Hence, the corresponding crisp LPP of the IFLPP (Example 4.1) by the proposed method is

$$\text{Maximise} 7.9871 * x_1 + 11.9914 * x_2 + 0.9977 * x_3$$

$$\text{Subject to } 129.86 \leq 4.9985 * x_1 + 4.9985 * x_2 + x_3 \leq 184.836$$

$$5.9996 * x_1 + 1.9988 * x_2 \leq 125.651$$

$$0.9505 * x_1 + 4.0433 * x_2 \leq 133.265$$

Where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

Using Mathematica 9, the solution is obtained as

$x_1 = 10.8092, x_2 = 30.4185, x_3 = -21.2402$ and optimum value is $Z_{\text{proposedmethod}} = 429.9022$.

Using the proposed method, the crisp value of the fuzzy optimum value obtained by Boris et al. [21] is $Z_{\text{Boris et al.}} = 365.664$.

Hence, the optimum value obtained by proposed method is greater than that of Boris et al. [21]. $Z_{\text{Boris et al.}} = 365.664 < Z_{\text{proposedmethod}} = 429.9022$.

As the given IFLPP is maximization problem, the proposed method performed better than the Boris et al. [21].

Example 4.2. Consider a problem discussed by Sidhu et al. [23]

$$\text{Maximise } \tilde{Z}^I = 5\tilde{x}_1^I + 4\tilde{x}_2^I$$

$$\text{Subject to } 5\tilde{x}_1^I + 4\tilde{x}_2^I \leq ((22, 23, 25, 26; 1); (20, 23, 25, 28; 0))$$

$$\tilde{x}_1^I + 2\tilde{x}_2^I \leq ((3, 5, 7, 9; 1); (1, 5, 7, 11; 0))$$

$$-\tilde{x}_1^I + \tilde{x}_2^I \leq ((-1, 3, 5, 9; 1); (-3, 3, 5, 11; 0))$$

$\tilde{x}_2^I \leq ((-1, 1, 3, 5; 1); (-3, 1, 3, 7; 0))$ and $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$

Here, all the constraints in the problem are with inequality sign. These intuitionisitic fuzzy inequality constraints are converted to crisp inequalities by the case (iii) of the proposed method. Hence, the corresponding crisp LPP of the IFLPP (Example 4.2) by the proposed method is

Maximise $5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 25.75$

$$x_1 + 2x_2 \leq 8.333$$

$$-x_1 + x_2 \leq 3.6968$$

$$x_2 \leq 4.333$$

where $x_1, x_2 \geq 0$

Using Mathematica 9, the solution is obtained as $x_1 = 2.2709$, $x_2 = 3.0312$ and optimum value is $Z_{\text{proposedmethod}} = 23.4792$.

Using the proposed method, the crisp value of the fuzzy optimum value obtained by Sidhu et al. [23] is $Z_{\text{Sidhu et al.}} = 0.4688$.

Hence, the optimum value obtained by proposed method is noticed to be greater than that of Sidhu et al. [23]. $Z_{\text{Sidhu et al.}} = 0.4688 < Z_{\text{proposedmethod}} = 23.4792$.

Example 4.3. Consider the following IFLPP problem which is studied by Nagoorgani et al. [18]

Maximise $\tilde{Z}^I = ((23, 24, 25; 1); (22, 24, 26; 0))\tilde{x}_1^I$

$$+ ((15, 16, 17; 1); 14, 16, 18; 0)\tilde{x}_2^I$$

Subject to $((48, 50, 51; 1); (47, 50, 52, 0))\tilde{x}_1^I$

$$+ ((24, 25, 6; 1); (23, 25, 28; 0))\tilde{x}_2^I(\leq (3995, 4000, 4010; 1);$$

$(3990, 4000, 4015; 0))((2, 3, 4; 1); (1, ,3 5; 0))\tilde{x}_1^I + ((8, 9, 10; 1); (7, 9, 11; 0))$

$\tilde{x}_2^I \leq ((715, 720, 723; 1); (711, 720, 725; 0))$

$((0.5, 1, 1.5; 1); (0.4, 1, 1.8; 0))\tilde{x}_1^I + ((0.9, 1, 1.1; 1); (0.8, 1, 1.2, 0))\tilde{x}_2^I \leq$

$((99, 100, 101; 1); (95, 100, 102; 0))$ where $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$

Here, all the constraints in the problem are with inequality sign. These intuitionisitic fuzzy inequality constraints are converted to crisp inequalities by the case (iii) of the proposed method. Hence, the corresponding crisp LPP of the IFLPP (Example 4.3) by the proposed method is

Maximise $23.9985 * x_1 + 15.9977 * x_2$

Subject to $49.8207 * x_1 + 25.0261 * x_2 \leq 4006$

$2.9876 * x_1 + 8.9959 * x_2 \leq 721.875$

$1.0047 * x_1 + 0.9996 * x_2 \leq 100.667$ where $x_1, x_2 \geq 0$

Using Mathematica 9, the solution is obtained as $x_1 = 60.2301$, $x_2 = 40.1699$ and optimum value is $Z_{\text{proposedmethod}} = 2088.0574$.

Using the proposed method, the crisp values of the fuzzy optimum value obtained by Nagoorgani et al. [18] is $Z_{\text{Nagoor gani et al.}} = 2080$.

Hence, the optimum value obtained by proposed method is observed to be greater than that of Nagoorgani et al. [18] $Z_{\text{Nagoor gani et al.}} = 2080$ $< Z_{\text{proposedmethod}} = 2088.0574$.

5. Conclusions

In this paper, firstly, a new ranking method, named as GM-R method, stemmed out from concepts of support and resultant membership of an IFN, is proposed to work with different types of IFNs. This method is asserted efficient with standard examples available in the existing methods. The proposed GM-R method is observed to rank properly and efficiently.

Secondly, making use of the proposed GM-R method, another new method is also proposed in this paper to solve IFLPP with both equality and inequality constraints. The proposed method has dealt with IFLPP involving

IFNs of type triangular and trapezoidal involving non-negative, negative and mixed fuzzy numbers. The proposed method is examined to exhibit better results in giving optimal solution which is a testimonial for its efficiency and authentication.

Further, computing efficiency, being non-lengthy procedure and being in simpler form to apply-on are identified as better features of the proposed GM ranking method and method to solve IFLPP.

References

[1] K. Aruna Prakash, M. Suresh and S. Vengataasalam, A new approach for ranking of intuitionistic fuzzy numbers using a centroid concept, Math Sci. 10 (2016), 177-184.

[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[3] S. K. Bharati, Ranking method of intuitionistic fuzzy numbers, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 13(9) (2017), 4595-4608.

[4] D. Chakraborty, D. K. Jana and T. K. Roy, A new approach to solve intuitionistic fuzzy optimisation problem using possibility, necessity and credibility measures, International Journal of Engineering Mathematics, Article ID 593185 2014 (2014), 1-12.

[5] R. Chutia and S. Saikia, Ranking intuitionistic fuzzy numbers at levels of decision-making and its applications, Expert Systems (2018), 1-18.

[6] R. Chutia, Ordering intuitionistic fuzzy numbers by a convex combination of values and multiple of ambiguity inclusion functions with ambiguities of membership and non-membership functions, International Journal of Intelligent Systems, 18 June, (2021).

[7] M. Darehmiraki, A novel parametric ranking method for intuitionistic fuzzy numbers, Iranian Journal of Fuzzy systems 16(1) (2019), 129-143.

[8] D. Dubey and A. Mehra, Linear programming with triangular intuitionistic fuzzy number, European Society for Fuzzy Logic and Technology (2011), 563-569.

[9] M. Jayalakshmi, D. Anuradha, V. Sujata and G. Deepa, A simple mathematical approach to solve intuitionistic fuzzy linear programming problems, Recent Trends in Pure and Applied Mathematics, AIP Conf. Proc. 2177, 020025 (2019), 020025-1–020025-5.

[10] Kabiraj, P. K. Nayak and S. Raha, Solving intuitionistic fuzzy linear programming problem, International Journal of Intelligence Science 9 (2019), 44-58.

[11] V. Lakshmana Gomathi Nayagam, S. Jeevaraj and Geetha Sivaraman, Complete ranking of intuitionistic fuzzy numbers, Fuzzy Inf. Eng. 8 (2016), 237-254.

[12] V. Lakshmana Gomathi Nayagam, S. Jeevaraj and P. Dhanasekaran, A linear ordering on the class of trapezoidal intuitionistic fuzzy numbers, Expert systems with applications 60 (2016), 269-279.

[13] V. Lakshmana Gomathi Nayagam, Jeevaraj Selvaraj and Dhanasekaran Ponnialagan, A new ranking principle for ordering trapezoidal intuitionistic fuzzy numbers, Hindawi complexity, 2017 (2017), Article ID3049041, 1-25.

[14] H. B. Mitchell, Ranking-intuitionistic fuzzy numbers, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 12(3) (2004), 377-386.

[15] Suresh Mohan, Arun Prakash Kannusamy and Vengataasalam Samiappan, A new approach for ranking of intuitionistic fuzzy numbers, J. Fuzzy. Ext. Appl. 1(1) (2020), 15-26.

[16] A. Nagoorgani and K. Ponnalagu, A new approach on solving intuitionistic fuzzy linear programming problem, Applied Mathematical Sciences 6(70) (2012), 3467-3474.

[17] A. Nagoorgani and K. Ponnalagu, An approach to solve intuitionistic fuzzy linear programming problem using single step algorithm, International Journal of Pure and Applied Mathematics 86(5) (2013), 819-832.

[18] Nagoorgani, J. Kavikumar and K. Ponnalagu, The knowledge of expert opinion in intuitionistic fuzzy linear programming problem, Mathematical Problems in Engineering, Article ID 875460, 2015(2015), 1-8.

[19] H. M. Nehi, A new ranking method for intuitionistic fuzzy numbers, International Journal of Fuzzy Systems 12(1) (2010), 80-86.

[20] R. Parvathi and C. Malathi, Intuitionistic Fuzzy Simplex method, International Journal of Computer Applications (0975-888) 48(6) (2012), 39-48.

[21] Boris Perez-Canedo and E. R. Concepcion-Morales, On LR Type fully intuitionistic fuzzy linear programming with inequality constraints: Solutions with unique optimal values, Expert Systems with Applications 128 (2019), 246-255.

[22] S. K. Sidhu and A. Kumar, A note on solving intuitionistic fuzzy linear programming problems by ranking function, Journal of Intelligent and Fuzzy Systems 30(5) (2016), 2787-2790.

[23] S. K. Sidhu and A. Kumar, Mehar methods to solve intuitionistic fuzzy linear programming problems with trapezoidal intuitionistic fuzzy numbers, Performance Prediction and Analytics of Fuzzy, Reliability and Queuing Models, Asset Analytics, (2019), 265-281.

[24] V. Singh and S. P. Yadav, Development and optimisation of unrestricted LR-type intuitionistic fuzzy mathematical programming problems, Expert Systems with Applications 80 (2017), 147-161.

[25] M. Suresh, S. Vengataasalam and K. Arun Prakash, Solving intuitionistic fuzzy linear programming problems by ranking function, Journal of Intelligent and Fuzzy Systems 27 (2014), 3081-3087.

[26] J. Ye, Expected value method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems, Expert systems with applications 38 (2011), 11730-11734.

[27] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.

[28] H. J. Zimmermann, Fuzzy mathematical programming, Comput. and Ops Res. 10(4) (1983), 291-298.