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A BINARY-TYPE ESTIMATOR FOR ESTIMATING POPULATION MEAN IN SIMPLE RANDOM SAMPLING

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Abstract

This paper suggests binary-type estimator for estimating finite population mean under simple random sampling using information on single auxiliary variable. The proposed estimator includes number of estimators as its member which have been listed. Mathematical expressions for the bias and the mean square error of proposed estimator have been derived. To show the applicability and efficiency of the proposed estimator, an empirical study is carried out by using real life population data population data sets and found that the proposed estimator performed better than other estimators including linear regression estimator.

1. Introduction

Cochran [4] introduced the use of auxiliary variable at the estimation stage and proposed the ratio estimator for the population mean. The ratio estimator provides better estimate of population parameter when the line of regression of variable of interest Y on auxiliary variable X is linear and passes through origin but in practical situation it is not real so. Sometimes the line of regression of Y on X may be linear but may not pass through origin, in such situation, the difference and the regression estimator were introduced. The fact that ratio and product estimators have superiority over sample mean estimator, when the correlation between study variable and auxiliary variable in the population is either positively or negatively high, led

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the statisticians to focus their attention on the modification of such conventional estimators so that the modified estimators can work efficiently even if the correlation is low. Such modified estimators are generally developed either using one or more unknown constants or introducing a convex linear combination of sample and population means of auxiliary characteristic with unknown weights. In both the cases, optimum choices of unknown parameters are made by minimizing the mean square error of modified estimators so that they become superior to the conventional one. In the sequence of suggesting modification over classical estimators Walsh [23] , Ray et al. [11], Srivenkataramana and Tracy [17], Vos [22] and Srivenkataramana [18] considered the use of weighted mean of $\bar{X}$ and $\bar{x}$ in place of $\bar{x}$ in classical ratio and product estimators. Isaki [7], Shah and Gupta [12], Singh and Singh [15], Singh and Audu [14], Subzar et al. [19], Audu et al., [2], Singh et al., [13] and Audu and Singh [3] did some other remarkable works in this direction. Singh and Shukla [16] suggested a factor-type estimator which includes sample mean, ratio, product, and Srivenkataramana [18] estimator as particular cases. Audu and Adewara [1] studied the factor-type estimator under two phase sampling scheme. Yadav and Zaman [24] improved the efficiency of ratio-type estimators by using some conventional and non-conventional parameters of auxiliary variable. Uraiwan and Nuanpan [21] modified two families of ratio-type estimators by adjusting the Khoshnevisan, et al. [9] and Kumar [10] estimators then they suggested a combined family of ratio-type estimator by taking linear combination of the two modified families of ratio-type estimators. Motivated by the work done by Singh and Shukla [16] and Uraiwan and Nuanpan [21], in the present paper a binary-type estimator of population mean of study characteristic has been suggested which generates the number of ratio-type estimators and thus, enable to make a unified study of several estimators.

2. Material and Methods

Consider a finite population $U = (U_1, U_2, \ldots, U_N)$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i and x_i denote the values of characteristic under study and auxiliary characteristic respectively for the i^{th} unit

$(i = 1, 2, \ldots, N)$ in the population. Let y_i and x_i denote the value of characteristic under study and auxiliary variable which are included in the sample at i^{th} draw $(i = 1, 2, \ldots, n)$. Further let $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of characteristic under study.

$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ The population mean of auxiliary variable. $S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2$; The population mean square of characteristic under study. $S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2$; The population mean square of auxiliary variable. $C_Y = \frac{S_Y}{\overline{Y}}$; The coefficient of variation of characteristic under study. $C_X = \frac{S_X}{\overline{X}}$; The coefficient of variation of auxiliary variable. $\rho = \frac{S_{YX}}{S_Y S_X}$; The correlation coefficient between the auxiliary variable and characteristic under study.

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$; The sample mean of study variable. $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$; The sample mean of auxiliary variable.

2.1 Related Existing Estimators

It is well known that sample mean estimator is denoted by $\bar{y}$ is an unbiased estimator of population mean and its variance is given by;

$$Var(\bar{y}) = \overline{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)C_Y^2 \tag{1}$$

The ratio estimator of population mean $\overline{Y}$ is defined as;

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}} \cdot \overline{X} \text{ Where } \overline{X} \text{ is known and } \bar{x} \neq 0 \tag{2}$$

The Bias and MSE of $\bar{y}_r$ is given as;

$$Bias(\bar{y}_r) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)(C_X^2 - \rho C_X C_Y) \tag{3}$$

$$MSE(\bar{y}_r) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)(C_Y^2 + C_X^2 - 2\rho C_X C_Y) \tag{4}$$

Product estimator is analogous to the ratio estimator and its theory and treatment are similar to ratio estimator. The product estimator for estimating the population mean is defined as,

$$\bar{y}_p = \frac{\bar{y} \cdot \bar{x}}{\bar{X}}, \text{ Where } \bar{X} \text{ is known and } \bar{X} \neq 0 \tag{5}$$

The Bias and MSE of $\bar{y}_p$ is given as;

$$Bias(\bar{y}_p) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\rho C_X C_Y \tag{6}$$

$$MSE(\bar{y}_p) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)(C_Y^2 + C_X^2 + 2\rho C_X C_Y) \tag{7}$$

The classical regression estimator is given as

$$\bar{y}_l = \bar{y} + \hat{\beta}(\bar{X} - \bar{x}) \tag{8}$$

Where $\hat{\beta} = \frac{S_{yx}}{S_x^2}$ is the sample regression coefficient $S_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ is the sample variance $S_{yx} = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})$ is the sample covariance between x and y.

The bias and mean square error of linear regression estimator up to the first order of approximation is given by:

$$Bias(\bar{y}_l) = -\left(\frac{1}{n} - \frac{1}{N}\right)\hat{\beta}\left(\frac{\mu_{12}}{\mu_{11}} - \frac{\mu_{03}}{\mu_{02}}\right) \tag{9}$$

$$MSE(\bar{y}_l) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2 C_Y^2(1 - \rho^2) \tag{10}$$

Where $\mu_{rs} = \frac{1}{N} \sum_{i=1}^{N} (Y_i - \bar{Y})^r (X_i - \bar{X})^s$

2.2 Adapted Estimator:

2.2.1 Singh and Shukla [16] Estimator

Singh and Shukla [16] suggested a factor-type estimator for population mean as

$$\bar{y}_{FT} = \frac{(A + C)\bar{X} + fB\bar{x}}{(A + fB)\bar{X} + C\bar{x}} \tag{11}$$

$$Bias(\bar{y}_{FT}) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right) K\{\theta_2 C_X^2 + \rho C_Y C_X\} \tag{12}$$

$$MSE(\bar{y}_{FT}) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)(C_Y^2 + K^2 C_X^2 + 2K\rho C_Y C_X) \tag{13}$$

Where $A = (d-1)(d-2)$, $B = (d-1)(d-4)$, $C = (d-2)(d-3)(d-4)$

$\theta_1 = \dfrac{fB}{A + fB + C}$ $\theta_2 = \dfrac{C}{A + fB + C}$, $K = \theta_1 - \theta_2$ and $d > 0$

In their work, it was observed that factor-type estimator $\bar{y}_{FT}$ was more efficient than classical ratio estimator $\bar{y}_r$ if $K > \dfrac{2\rho C_Y}{C_X} - 1$

Particular cases of proposed by Singh and Shukla [16] estimator

When $d = 1$ proposed estimator reduces to classical ratio estimator and given as

$$(\bar{Y}_{FT})_{d=1} = \frac{\bar{y}}{\bar{x}}\bar{X} = \bar{y}_r$$

When $d = 2$ proposed estimator reduces to classical product estimator and given as

$$(\bar{Y}_{FT})_{d=2} = \frac{\bar{y}\cdot\bar{x}}{\bar{X}} = \bar{y}_p$$

When $d = 3$ proposed estimator reduces to Srivenkataramana [18] estimator and given as

$$(\bar{Y}_{FT})_{d=3} = \bar{y}\,\frac{N\bar{X} - n\bar{x}}{(N - n)\bar{X}} = \bar{y}_s$$

When $d = 4$, proposed estimator reduces to sample mean estimator and given as

$$(\overline{Y}_{FT})_{d=4} = \overline{y}.$$

2.2.2 Uraiwan and Nuanpan [21] Estimator:

Uraiwan and Nuanpan [21] modified two families of ratio-type estimators t_R and t_{Reg} by adjusting the Khoshnevisan et al. [9] and Kumar [10] estimators then Uraiwan and Nuanpan [21] suggested a combined family of ratio-type estimator by taking linear combination of the two modified families of ratio-type estimators.

Now the modified estimators t_R and t_{Reg} are given as.

$$t_R = \overline{y}\left(\frac{a\overline{X} + c}{a\overline{x} + c}\right) \text{ and } t_{\text{Reg}} = \{\overline{y} + b(\overline{X} - \overline{x})\}\left(\frac{d\overline{X} + h}{d\overline{x} + h}\right)$$

Where c and h are either real numbers or functions of known parameters of an auxiliary variable.

The bias and MSE of t_R and t_{Reg} up to first order of approximation are given as:

$$Bias(t_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\overline{Y}(w_1^2 C_x^2 - w_1 \rho C_X C_Y) \tag{14}$$

$$MSE(t_R) = \overline{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)(C_Y^2 + w_1^2 C_X^2 + w_1 2\rho C_Y C_X) \tag{15}$$

$$Bias(t_{\text{Reg}}) = \overline{Y}\left(\frac{1}{n} - \frac{1}{N}\right) w_2\{(w_2 + bk)C_X^2 - \rho C_Y C_X\} \tag{16}$$

$$MSE(t_{\text{Reg}}) = \overline{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\{C_Y^2 + (w_2 + bk)^2 C_X^2 - 2(w_2 + bk)\,\rho C_Y C_X\} \tag{17}$$

Where $w_1 = \dfrac{a\overline{X}}{a\overline{X} + b}$, $w_2 = \dfrac{d\overline{X}}{d\overline{X} + h}$ and $k = \dfrac{\overline{X}}{\overline{Y}}$

Uraiwan and Nuanpan [21] combined the two estimator and proposed a combined family of ratio-type estimator as:

$t_{RC} = \alpha t_R + (1 - \alpha) t_{\text{Reg}}$ where α is a constant.

$$t_{RC} = \alpha \bar{y} \left(\frac{a\bar{X} + c}{a\bar{x} + c} \right) + (1 - \alpha) \{ \bar{y} + b(\bar{X} - \bar{x}) \} \left(\frac{d\bar{X} + h}{d\bar{x} + h} \right) \tag{18}$$

The bias and MSE of t_{RC} up to first order of approximation are given as:

$$Bias(t_{RC}) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)[\{\alpha w_1^2 + (1 - \alpha) w_2^2 + (1 - \alpha) bkw_2\} C_X^2$$

$$- \{\alpha w_1 + (1 - \alpha) w_2\} \rho C_Y C_X] \tag{19}$$

$$MSE(t_{RC}) = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N}\right) \begin{bmatrix} C_Y^2 + \{\alpha w_1 + (1 - \alpha) w_2 + (1 - \alpha) bk\}^2 C_X^2 \\ - 2\{\alpha w_1 + (1 - \alpha) w_2 + (1 - \alpha) bk\} \rho C_Y C_X \end{bmatrix} \tag{20}$$

The optimum value of α is obtained and given as

$$\alpha_{opt} = \frac{\rho C_Y - (w_2 + bk) C_X}{(w_1 - w_2 - bk) C_X} \text{ and } MSE(t_{RC})_{opt} = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N}\right) C_Y^2 (1 - \rho^2) \tag{21}$$

That is a linear regression estimator.

2.3 Proposed Estimator:

By utilizing the concept of Singh and Shukla [16] and Uraiwan and Nuanpan [21] a binary type estimator of finite population mean has been suggested which is as follows.

$$t_{BT} = \left[p_1 \bar{y}_\beta + p_2 \bar{y} \left\{ \frac{(\alpha + \beta)\bar{X} + g\delta\bar{x}}{(\alpha + g\delta)\bar{X} + \beta\bar{x}} \right\} \right] \tag{22}$$

where p_1 and p_2 are constants to be determined. α, β and δ take values 0 and 1. $g = \frac{n}{N - n}$.

By substituting the values of α, β and δ members of the proposed estimator are given as:

Table 1. Members of proposed estimator t_{BT}.

Estimators	p_1	p_2	α	β	δ
$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$	0	1	0	1	0
$\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$	0	1	0	0	1
$\bar{y}_d = [\bar{y} + \beta(\bar{X} - \bar{x})]$	1	0	-	-	-
$\bar{y}_{BH} = W[\bar{y} + \beta(\bar{X} - \bar{x})]$	w	0	-	-	-
$t_{BT1} = \left[p_1\bar{y}_\beta + p_2\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	p_1	p_2	0	1	0
$t_{BT2} = \left[p_1\bar{y}_\beta + p_2\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)\right]$	p_1	p_2	0	0	1
$t_{BT3} = \left[p_1\bar{y}_\beta + p_2\bar{y}\left(\frac{\bar{X} + g\bar{x}}{g\bar{X} + \bar{x}}\right)\right]$	p_1	p_2	0	1	1
$t_{BT4} = \left[p_1\bar{y}_\beta + p_2\bar{y}\left(\frac{2\bar{X}}{X + \bar{x}}\right)\right]$	p_1	p_2	1	1	0
$t_{BT5} = [p_1\bar{y}_\beta + p_2\bar{y}]$	p_1	p_2	1	0	0
$t_{BT6} = \left[p_1\bar{y}_\beta + p_2\bar{y}\left(\frac{\bar{X} + g\bar{x}}{(1+g)\bar{X}}\right)\right]$	p_1	p_2	1	0	1

2.4 Bias and Mean square of proposed estimator

To obtain the bias and MSE of the proposed of estimator t_{BT}, let us define;

$\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$

Thus, $E(e_0) = E(e_1) = 0$ and

$$E(e_0^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_Y^2,\ E(e_1^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_X^2,\ E(e_0 e_1) = \left(\frac{1}{n} - \frac{1}{N}\right) \rho C_Y C_X$$

Expressing the above estimator in terms of e's, we have

$$t_{BT} = \left[p_1 \overline{Y}(1 + e_0 - ke_1) + p_2 \overline{Y}(1 + e_0) \left\{ \frac{(\alpha + \beta)\overline{X} + g\delta(1 + e_1)\overline{X}}{(\alpha + g\delta)\overline{X} + \beta(1 + e_1)\overline{X}} \right\} \right]$$

$$t_{BT} = \left[p_1 \overline{Y}(1 + e_0 - ke_1) + p_2 \overline{Y}(1 + e_0) \left\{ (1 + \ell_1 e_1)(1 - \ell_2 e_1 + \ell_2^2 e_1^2) \right\} \right]$$

where $\ell_1 = \dfrac{g\delta}{\alpha + g\delta + \beta}$ and $\ell_2 = \dfrac{\beta}{\alpha + g\delta + \beta}$.

$$t_{BT} = \overline{Y}[p_1 + p_1 e_0 - kp_1 e_1 + p_2 - \ell_2 p_2 e_1 + \ell_2^2 p_2 e_1^2 + \ell_1 p_2 e_1 - \ell_1 \ell_2 p_2 e_1^2 + p_2 e_0$$

$$-\ell_2 p_2 e_0 e_1 + \ell_1 p_2 e_0 e_1]$$

$$t_{BT} - \overline{Y} = \overline{Y}[p_1 + p_2 - 1 + p_1 e_0 - kp_1 e_1 - \ell_2 p_2 e_1 + \ell_2^2 p_2 e_1^2 + \ell_1 p_2 e_1 - \ell_1 \ell_2 p_2 e_1^2$$

$$-\ell_2 p_2 e_0 e_1 + \ell_1 p_2 e_0 e_1] \tag{23}$$

By taking expectation both the sides, the bias of the proposed estimator is obtained as;

$$Bias(t_{BT}) = \overline{Y}\left[p_1 + p_2 - 1 + \theta p_2 \left\{ (\ell_2^2 - \ell_1 \ell_2) C_X^2 + (\ell_1 - \ell_2)\rho C_X C_Y \right\} \right],$$

where $\theta = \left(\dfrac{1}{n} - \dfrac{1}{N}\right)$ (24)

Now the MSE of the proposed estimator is obtained by squaring (23) both sides, neglecting terms of e's having power greater than two and taking expectation. Therefore, we have

$$MSE(t_{BT}) = E\left[\overline{Y} \left\{ \begin{matrix} p_1 + p_2 - 1 - kp_1 e_1 + (p_1 + p_2)e_0 + (\ell_1 - \ell_2)p_2 e_1 \\ + (\ell_1 - \ell_2)p_2 e_0 e_1 + (\ell_2^2 - \ell_1 \ell_2)p_2 e_1^2 \end{matrix} \right\} \right]^2$$

$$MSE(t_{BT}) = \overline{Y}^2 [1 - 2p_1 + p_1^2 B_1 + p_2^2 B_2 + 2p_1 p_2 B_3 - 2p_2 B_4] \tag{25}$$

where

$$B_1 = [1 + \theta\{C_Y^2 + k^2 C_X^2 - 2k\rho C_X C_Y\}]$$

$$B_2 = [1 + \theta\{C_Y^2 + (\ell_1 - \ell_2)^2 C_X^2 + 2(\ell_2^2 - \ell_1\ell_2) C_X^2 + 4(\ell_1 - \ell_2)\rho C_X C_Y\}]$$

$$B_3 = [1 + \theta\{C_Y^2 + (\ell_2^2 - \ell_1\ell_2)C_X^2 - k(\ell_1 - \ell_2)C_X^2 + 2(\ell_1 - \ell_2)\rho C_X C_Y - k\rho C_X C_Y\}]$$

$$B_4 = [1 + \theta\{(\ell_2^2 - \ell_1\ell_2)C_X^2 + (\ell_1 - \ell_2)\rho C_X C_Y\}]$$

The optimum values of p_1 and p_2 are obtained as;

$$(p_1)_{opt} = \frac{(B_2 - B_3B_4)}{(B_1B_2 - B_3^2)} \text{ and } (p_2)_{opt} = \frac{(B_1B_4 - B_3)}{(B_1B_2 - B_3^2)}$$

Substituting the optimum values of p_1 and p_2 in (25), we get the minimum MSE of proposed estimator t_{BT} as;

$$MSE_{\min}(t_{BT}) = \bar{Y}^2\left[1 - \frac{(B_2 - 2B_3B_4 + B_1B_4^2)}{(B_1B_2 - B_3^2)}\right] \tag{26}$$

3. Numerical Illustration

In this section to show the application of proposed estimator as well as to evaluate the performance of members of proposed estimator with respect to classical estimators, five real life population data sets have been considered given in table 2.

Table 2. Discerption of population.

Population I: Source: Kadilar and Cingi [8] - Y: Apple production in 1999 and X: No. of apple trees in 1999 in Aegean region of Turkey						
N	n	$\bar{Y}$	$\bar{X}$	C_Y	C_X	ρ_{YX}
106	20	2212.59	27421.70	5.22	2.10	0.86
Population II: Source: Das, A. [6] - Y: No. of agricultural laborers for 1971 and X: No. of agricultural laborers for 1961						
278	25	39.068	25.111	1.4451	1.6198	0.7213
Population III: Source: Cochran, W.G. [5] - Y: No. of placebo children						

and X: No. of paralytic polio cases in the placebo group						
34	10	4.92	2.59	1.01232	1.07201	0.6837
Population IV: Source: Steel et al. [20] - Y: log of leaf burn in sacks and X: chlorine percentage						
30	6	0.6860	0.8077	0.7001	0.7493	-0.4996

Table 3. MSE values and PRE of members of proposed estimators and classical estimators.

Estimators		Population I	Population II	Population III	Population IV
$\bar{y}$	MSE	5411358	116.031	1.7511	0.0308
	PRE	100	100	100	100
$\bar{y}_R$	MSE	2542740	74.1901	1.1791	0.0989
	PRE	212.8156	156.3967	148.5052	31.1051
$\bar{y}_p$	MSE	10031549	449.4337	6.2503	0.0331
	PRE	53.9433	25.8171	28.0157	92.9307
$\bar{y}_l$	MSE	1409115	55.6631	0.9325	0.0231
	PRE	384.0246	208.4522	187.7743	133.2623
t_{BR1}	MSE	1341270	53.5856	0.8969	0.0224
	PRE	403.4495	216.5338	195.2428	137.1964
t_{BT2}	MSE	1383120	56.2420	0.9328	0.02228
	PRE	391.242	206.3065	187.7356	138.0088
t_{BT3}	MSE	1346514	53.5885	0.8993	0.02227
	PRE	401.8783	216.5223	194.7039	138.0457
t_{BT4}	MSE	1349085	53.8090	0.8999	0.02227
	PRE	401.1125	216.635	194.5856	138.0556
t_{BT5}	MSE	1357863	54.1927	0.9051	0.02217

	PRE	398.5193	214.1079	193.4698	138.6827
t_{BT6}	MSE	1361598	54.2853	0.9101	0.022154
	PRE	397.4264	213.743	192.4079	138.8185

The MSE and Percentage relative efficiencies (PRE) of members of proposed estimator and classical estimators have been calculated based on population data sets I-V and given in table 3. It is observed from table 3 that regression estimator and almost all members of proposed estimator always performed better than sample mean, ratio and product estimators regardless of positive or negative correlation between study variable and auxiliary variable for all population data sets considered here. Also almost all members of proposed estimator are preferable over other classical estimators due to having higher PRE.

4. Conclusion

In the present paper, a binary-type estimator of the finite population mean of study variable has been suggested which includes six estimators as its particular case. Expression for the bias and mean square error of suggested estimator have been derived and minimum mean square error has also been obtained. By numerical comparison it is found that almost all the members of the proposed estimator are more efficient than the classical estimators and Uraiwan and Nuanpan [21] estimator which is equally efficient as regression estimator.

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