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# A CLOSED LOOP SUPPLY CHAIN INVENTORY MODEL WITH DISTRIBUTION-FREE APPROACH IN ENVIRONMENTAL INVESTIGATION

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## Abstract

This paper presents a centralized manufacturing stock version for limitless making planes horizon of multi echelon closed loop supply chain, consisting a supplier, manufacturer, remanufacturer, retailer and collector. The retailer's demand is satisfied by new and remanufactured goods obtained from the manufacturer respectively. The proposed model considers the retailer's demand and returns to the remanufacturer as random. The manufacturer produces the product in a limited quantity, and they supply it in multiple batches to the retailer as an alternative. In this model we also examine environmental impacts in terms of exploration, carbon emissions, and energy usage, as well as annual disposals. The model will guide relevant institutions industry planning CLSC (closed loop supply chain) inventory management system

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with environment considerations. A computer code using the software Matlab is developed to derive the optimal solution and numerical example is presented to illustrate the procedures of algorithm.

## I. Introduction

A supply chain is a network of business players and suppliers that collaborate to deliver goods, services, and information to consumers in a more effective and efficient manner Akdogan and Coskun [1]. The company could achieve what we call a "optimal supply chain system" with good supply chain management planning and design.

Supply chain management is divided into two categories: open-loop supply chain and closed-loop supply chain. There are no product returns from the consumer to the initial producer in an open-loop supply chain. A closed loop supply chain, on the other hand, is distinguished by the presence of product returns that are managed by the manufacturer or supplier by performing the recovery process on the returned product prior to resale. The salvage process can be divided into repair, remanufacturing, cannibalization and recycling.

Remanufacturing establishes the inspiration for CLSC. According to Guide and Van Wassenhove [9], CLSC is described as "the design, control, and operation of a device to maximize cost advent over the complete lifestyles cycle of a product with dynamic recuperation of cost from different sorts and volumes of returns overtime".

The main motivation of this research is set through the gaps in the current literature in the field of multi-tier CLSC, particularly the growing industry and government concerns about the recycling of various products such as batteries, photocopiers, televisions and others at the end of their useful life. The topic of remanufacturing has only recently become important in developing countries such as India.

The shortening of the product life cycle due to changing customer preferences, technological advances, product exchange promotional programs, etc., has created tremendous opportunities for reuse, remanufacturing, cannibalization, and the restoration of returned products.

Not only remanufacturing help address environmental concerns, it also lowers the cost of the end product. A remanufactured or refurbished component is invisible to customers, it shows them the same value as the new component. This helps manufacturers capture more market share through price competition.

The first to work on CLSC, where he developed a deterministic model to find the EOQ for a repairable inventory system considering one procurement batch and at least one repairable batch in each production cycle assuming instant recovery. Subsequently, attempts have been made to develop variants of EOQ/EPQ models considering product returns under different assumptions such as complete backorder Konstantaras et al. [11], partial backorder Hasanov et al. [8], lost sales Jaber et al. [9], also developed different product quality of recovered and new products. Switching cost Saadany et al. [3], variable return rate of used product Saadany et al. [4], effect of learning in manufacturing and remanufacturing processes Tsai [19], different batch sizes for remanufacturing Schulz and Voigt [18]. A growing literature on modeling of production-inventory problems with product return justifies the importance of this area. Mawandiya [17] developed considering finite production and remanufacturing rates with full backordering at the manufacturer and remanufacturer, which pose significant challenge to synchronize the ordering of the retailer with the production cycle of the manufacturer and remanufacturing cycle of the remanufacturer. Jauhari et al. [10] developed a closed-loop supply chain inventory model considering limited number of remanufacturing generation and environmental investigation.

Zanoni et al. [20] studied a vendor-buyer system with remanufacturing and consignment stock policy. Hariga et al. [7] also addressed the consignment stock strategy for a single vendor-buyer system and developed a model for minimizing the total cost to determine the production cycle length, number and the sequence of manufacturing and remanufacturing batches. Mawandiya et al. ([15], [16]) extended the work of Lee [12] and developed production inventory models in a two-echelon CLSC by incorporating remanufacturer for satisfying customer demand by remanufactured product in addition to the new manufactured product. Maiti and Giri [14] proposed a two-echelon CLSC model considering two ways collection of used product through the retailer such as 1. A fraction of the new product sold through the forward supply chain and 2. The exchange of used product with new one under exchange offers.

In this study, we develop a multi-echelon closed-loop supply chain inventory model consisting of a supplier, a manufacturer, and a retailer and a collector of used goods from the market. We assume that defined including the number of renewable generations and inventory costs and carbon emission costs generated from routine manufacturing process, remanufacturing process and transportation activities.

The organization of this section is as follows: Section 2 deals with notations and assumptions. Section 3 describes the problem and provides the mathematical frame work of the problem. Optimal solution procedure is discussed in section 4. The model is supported by numerical results in section 5. In section 6, sensitivity analysis is conducted and section 7 concludes the work.

## II. Notations and Assumptions

### A. Notations

The proposed model is developed based on the following notations.

$H_1$ : Holding cost per unit per unit of time at manufacturer finished product inventory ($/unit/year).

$n$ : Number of shipments from manufacturer to retailer.

$d$ : Annual demand rate (units/year).

$q$ : Shipment lot size from manufacturer to retailer (unit).

$\beta$ : Nominal proportion of used items returned for remanufacturing, when the number of remanufacturing generations is unlimited $(\eta = \infty)$ with $\theta \leq \beta < 1$.

$H_4$ : Holding cost at manufacturer recoverable item inventory ($/unit/year).

$H_5$ : Holding cost at manufacturer raw material inventory ($/unit/year).

$A_1$ : Set-up cost of remanufacturing (\$/set-up).

$A_2$ : Set-up cost of manufacturing (\$/set-up).

$O_1$ : Ordering cost for manufacturer recoverable inventory (\$/order).

$K$ : Number of shipments from collector to manufacturer.

$c_{re}$ : Remanufacturing cost per unit (\$/unit).

$c_{mn}$ : Manufacturing cost per unit (\$/unit).

$F_t$ : Fixed cost per truck per trip (\$/truck).

$V$ : Annual remanufacturing rate (units/year), with $V > d$.

$\delta$ : Annual manufacturing rate (units/year), with $\delta > d$.

$t_c$ : Truck capacity (units/truck).

$c_{ghg}$ : Carbon emission cost (\$/ton CO2).

$c_T$ : Emissions function parameter for process $i$ (ton/unit), with $i = p, r$ where $i = p$ for manufacturing and $i = r$ for remanufacturing.

$a_T$ : Emissions function parameter for process $i$ (ton year2/unit3), with $i = p, r$ where $i = p$ for manufacturing activity and $i = r$ for remanufacturing activity.

$b_T$ : Emissions function parameter for process $i$ (ton year/unit2), with $i = p, r$ where $i = p$ for manufacturing activity and $i = r$ for remanufacturing activity.

$g_T$ : Number of gallons required per truck per distance travelled (gallons/truck).

$e_T$ : Amount of GHG emissions produced from one gallon of diesel-truck fuel (ton/gallon).

$l$ : Number of shipments of the remanufactured product from the remanufacturer to the retailer during each remanufacturing cycle.

$O_2$ : Ordering cost per order of the retailer for the new product.

$O_3$ : Ordering cost per order of the retailer for the remanufactured product.

$m$ : Number of shipments of the remanufactured product from the manufacturer to the retailer.

$T$ : Production (remanufacturing) cycle length of the manufacturer.

$k_m$ : Safety factor for deciding the safety stock of the remanufactured product at the retailer.

$k_n$ : Safety factor for deciding the safety stock of the manufactured product at the retailer.

$H_2$ : Inventory holding cost per unit per unit time at the retailer.

$\sigma$ : Standard deviation of demand per unit time at the retailer.

$L_m$ : Lead time of the retailer for procurement from the remanufacturer.

$L_n$ : Lead time of the retailer for procurement from the manufacturer.

$H_3$ : Holding cost per unit of time at collector inventory ($/unit/year).

$c_{ic}$ : Inspection cost ($/unit).

$c_x$ : Disposal cost per unit ($/unit).

$\rho$ : Collector defective item proportion, with $\theta \leq \rho \leq 1$.

$P_a$ : Collector used item collection cost ($/unit).

$r$ : Production rate per order.

$H_6$ : Holding cost per unit per unit time for supplier.

$P$ : Unit purchase price.

$R$ : Fixed cost to process manufacturer's order of any size.

### B. Assumption

- Reproducibility and manufacturing processes are perfect.

- The rate of reproduction is less than the rate of demand $(v < d)$.
- At the beginning of each cycle the collector will inspect all its objects.
- Equal shipment size for each party.
- Remanufactured products are of the same quality as new products (good-as-new).
- Both the retailer and the remanufacturer follow a continuous review inventory policy.
- Unnecessary demand of the retailer is completely withdrawn. Similarly, any claim by the retailer that is no satisfied by the manufacturer is fully backorder.
- Returned product identified as unrepairable is immediately rejected.

## III. Model Development

Here, we examine a CLSC system that consists of a supplier, a retailer, a manufacturer, a re-manufacturer and a used item collector. We try to get the optimal solution related to multiple inventory decisions like shipment lot size, number of shipments per year and number of remanufactures generations. The material flow between those parties is depicted in Figure 1.

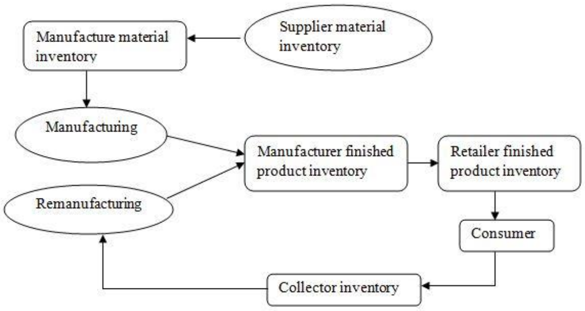

**Figure 1.** The flow diagram of product in the closed loop supply chain inventory system.

In this model, we consider the concept of reproducing generations originally proposed in Saadany et al. [5].

With this concept, items classified as recoverable will go into multiple numbers reproducing generations before they are completely disposed of as waste. This study aims to determine the shipment lot and determine the number of generations to reproduce the quantity and number of exports between the investigated parties, thereby increasing the total profit. We consider two components of cost related to environmental impacts, namely cost carbon emissions and energy use. We adopt both cost functions from Bazan et al. [2]. The objective function is to minimize the total inventory-related costs of the supply chain.

### A. Supplier inventory cost function

Supplier's average inventory levels are determined by subtracting the manufacturer's average material inventory level from the average total inventory in the supplier manufacturer inventory system and is given by,

$$TC_s = \frac{o_1 + lR}{(m+n)T} + \frac{d^2 m^2 T H_6}{2r(m+n)}\left(1 - \frac{1}{l}\right) + \frac{Pdm}{(m+n)}. \tag{1}$$

### B. Retailer inventory cost function

Retailer-related inventory costs, holding costs, order costs, back order costs and carbon emissions from transportation activities are included. The stock level of the retailer's inventory is shown in Figure 2 and is given by,

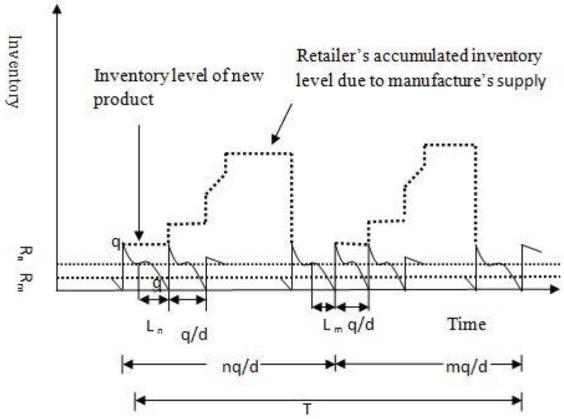

**Figure 2.** Retailer inventory.

$$TC_r = \frac{nO_2 + mO_3}{T}$$

$$+ \frac{[n(\frac{q}{2} + k_n\sigma\sqrt{L_n})\frac{q}{d} + m(\frac{q}{2} + k_m\sigma\sqrt{L_m})\frac{q}{d}]H_2}{T}$$

$$+ \frac{n\sigma\sqrt{L_n}(\sqrt{1 + k^2} - k) + m\sigma\sqrt{L_m}(\sqrt{1 + k^2} - k)\pi}{2T}$$

$$+ \frac{g_T e_T c_g hg\left(\frac{d}{t_c}\right)}{T}. \tag{2}$$

**C. Manufacture inventory cost**

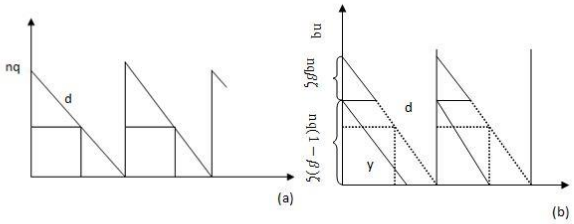

**Figure 3.** Manufacturer finished product inventory (a) and raw material recoverable item inventory (b).

Figure 3 shows the inventory levels of finished products, recoverable products, and raw materials. As mentioned earlier, the costs of the manufacturer also include the costs of storing the finished products, recoverable items and raw materials. Annual inventory cost to manufacturer consists of holding costs, ordering cost, set-up cost, manufacturing costs, transportation cost and carbon emission cost and is given by,

$$TC_m = (H_1 \frac{n}{2d} + H_4 \frac{nq}{2}\left(1 - \frac{1-\beta}{1-\beta^{(\eta+1)}}\right)^2 + H_5 \frac{nq}{2}\left(\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)^2$$

$$+ A_1 \frac{d}{nq}\left(1 - \frac{1-\beta}{1-\beta^{(\eta+1)}}\right) + A_2 \frac{d}{nq}\left(\frac{1-\beta}{1-\beta^{(\eta+1)}}\right) + O_1 K \frac{d}{nq}$$

$$+\left(d\left(c_{re}\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)+c_{mn}\left(\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)\right)\right)+\left(\frac{dF_t}{t_c}\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)\right)$$

$$+c_{ghg}d((c_r-b_rV+a_rV^2)\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)$$

$$+(c_p-b_p\delta+a_p\delta^2)\left(\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)+\frac{g_Te_T}{t_c}\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)\Bigg)\Bigg). \tag{3}$$

### D. Collector inventory cost function

Figure 4 shows the on-hand stock level collector's used item inventory. Annual inventory cost to collector consists of holding cost, collection cost, inspection cost and waste disposal cost and is given by

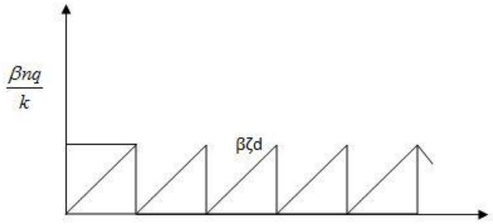

**Figure 4.** On-hand stock level at collector used item inventory.

$$TC_c = H_3\frac{nq}{2K}\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)+d(P_a+c_{ic}+c_x)$$

$$\left(\left(1-\frac{1-\beta}{1-\beta^{(\eta+1)}}\right)+\rho\right). \tag{4}$$

Finally, we get the function of annual joint total cost for the three parties and is given by

$$EJTC(m,n,q,k,\eta)=TC_s+TC_r+TC_m+TC_c. \tag{5}$$

## IV. Solution Procedure

To solve this problem, taking the first order partial derivatives of $EJTC(m, n, q, k, \eta)$ with respect to $m, n, q$ and $K$ respectively, we obtain

$$\frac{\partial EJTC(m, n, k, \eta)}{\partial q} = \frac{H_4 n}{2}\left(1 - \frac{1-\beta}{1-\beta^{\eta+1}}\right)^2 + \frac{H_5 n}{2}\left(\frac{1-\beta}{1-\beta^{\eta+1}}\right)^2$$

$$- \frac{O_2 d}{nq^2}\left(1 - \frac{1-\beta}{1-\beta^{\eta+1}}\right) - \frac{A_2 d}{nq^2}\left(\frac{1-\beta}{1-\beta^{\eta+1}}\right) - \frac{O_1 Kd}{nq^2}$$

$$+ [[\frac{m}{2d} + \frac{mk_m\sigma\sqrt{L_m}}{d} + \frac{l}{2d} + \frac{nK_n\sigma\sqrt{L_n}}{d}]H_1/T] + \frac{H_3 n}{2K}\left(\frac{1-\beta}{1-\beta^{\eta+1}}\right), \tag{6}$$

$$\frac{\partial EJTC(m, q, k, \eta)}{\partial n} = \frac{O_2}{T} + \left[\left(\frac{q}{2} + (k_n\sigma\sqrt{L_n})\frac{q}{d}\right)\right]\frac{\sigma\sqrt{L_n(\sqrt{1+k^2}-k)\pi}}{2T}$$

$$- \frac{2H_6 d^2 nT}{2Pn^2}\left(1 - \frac{1}{l}\right) - \frac{Pd}{n^2}, \tag{7}$$

$$\frac{\partial EJTC(n, q, k, \eta)}{\partial m} = \frac{A_4}{T} + \left[\left(\frac{q}{2} + (k_m\sigma\sqrt{L_m})\frac{q}{d}\right)\right] + \frac{\sqrt{L_m(\sqrt{1+k^2}-k)\pi}}{2T}, \tag{8}$$

$$\frac{\partial EJTC(m, n, q, \eta)}{\partial K} = \frac{H_3 nq}{2K^2}\left(1 - \frac{1-\beta}{1-\beta^{\eta+1}}\right) + \frac{O_1 d}{nq}, \tag{9}$$

$$\frac{\partial^2 EJTC(m, n, k, \eta)}{\partial q^2} = \frac{O_2 d}{nq^3}\left(1 - \frac{1-\beta}{1-\beta^{\eta+1}}\right) + \frac{A_2 d}{nq^3}\left(\frac{1-\beta}{1-\beta^{\eta+1}}\right)$$

$$+ \frac{O_1 Kd}{nq^3} + \left[\left[\frac{m}{2d} + \frac{mk_m\sigma\sqrt{L_m}}{d} + \frac{l}{2d} + \frac{nK_n\sigma\sqrt{L_n}}{d}\right]H_1/T\right] \geq 0.$$

$$\frac{\partial EJTC(m, q, k, \eta)}{\partial m} = \frac{2H_6 d^2 nT}{2Pn^3}\left(1 - \frac{1}{l}\right) + \frac{Pd}{n^3} \geq 0.$$

$$\frac{\partial^2 EJTC(n, q, k, \eta)}{\partial m^2} \geq 0.$$

$$\frac{\partial EJTC(m, n, q, \eta)}{\partial K} = \frac{H_3 nq}{2K^3}\left(1 - \frac{1-\beta}{1-\beta^{\eta+1}}\right) \geq 0.$$

Here, all the principal minors of the Hessian matrix of the cost function are positive, i.e. $|H_1| > 0, |H_2| > 0, |H_3| > 0, |H_4| > 0$. Hence, it is shown that the cost function s is a convex function of $(m, n, k, q, \eta)$ in (5).

By setting Equations 6, 7, 8, 9 equal to zero, we obtain

$$q = \sqrt{\frac{\begin{array}{l}2d[A_1 dK(\beta^{\eta+1} - 1) + O_1 Kd \\ + (\beta - 1) d (A_1 - A_2)]\end{array}}{H(n)}} \tag{10}$$

where

$$H(n) = n[d(\beta^{\eta+1} - 1)[(2KH_4 n + H_3 n)d$$

$$+ (H_4 n + H_5 n)K] - (H_4 n 2K + H_3 n)(\beta - 1)d] + H_1 TK(\beta^{\eta+1} - 1)$$

$$[(n + m) + 2mk_m \sigma\sqrt{L_m} + 2nk_n \sigma\sqrt{L_n}],$$

$$m = \frac{\Gamma}{2H_6 d^2 T^2} \tag{11}$$

where

$$\Gamma = P(m + n)\left(-2A_1 - \left(\frac{q}{2} + k_m \sigma\sqrt{L_m}\right)\frac{q}{d}\right)2H_1$$

$$- \sigma\sqrt{L_m}(\sqrt{1 + k^2} - K)\pi - 2TPd.$$

$$n = \sqrt{\frac{\begin{array}{l}T\left(A_1 d\left(1 - \dfrac{1 - \beta}{1 - \beta^{(\eta+1)}}\right) + A_2 d\right. \\ \left(\dfrac{1 - \beta}{1 - \beta^{(\eta+1)}}\right) + q(O_1 + lR)\end{array}}{\Theta}} \tag{12}$$

where

$$\Theta = q\left[\frac{H_1}{2d} + \frac{H_4 q}{2}\left(1 - \frac{1 - \beta}{1 - \beta^{(\eta+1)}}\right)^2 - \frac{H_5 q}{2}\left(\frac{1 - \beta}{1 - \beta^{(\eta+1)}}\right)^2\right.$$

$$+ [[\frac{q}{2} + k_n \sigma\sqrt{L_n})\frac{q}{2}]\frac{H_1}{T}] - \left[\frac{\sqrt{L_n}(\sqrt{1 + k^2} - k}{2T}\right]$$

$$+ \frac{H_3 q}{2K}\left(1 - \frac{1-\beta}{1-\beta^{(\eta+1)}}\right) - \frac{d^2 T H_6}{2P}\left(1 - \frac{1}{l}\right)\Bigg]$$

and

$$K = \sqrt{\frac{n^2 H_3 q^2 \beta}{2dO_1}\left(\frac{\beta - 1}{\beta^{(\eta-1)}}\right)} \tag{13}$$

According to the analysis procedure, the solution algorithm for the proposed model is summarized as follows.

**Algorithm**

1. $\eta = 1$.

2. Substitute $K(n, m, \eta, q)$ into Equation (8), by solving the equation we then obtain the function of $n(K, m, \eta, q)$.

3. Substitute $n(K, m, \eta, q)$ into Equation (7) and solve the equation. We then obtain the function of $m(K, n, \eta, q)$.

4. Substitute $n(K, m, \eta, q)$. into Equation (6) and solve the equation. We then obtain the function of $q(K, n, \eta, q)$.

5. Compute $m(K, m, \eta, q)$ with $q = q^*$.

6. Compute $n(K, m, \eta, q)$ with $q = q^*$ and $m = m^*$.

7. Compute $K(n, m, \eta, q)$ with $q = q^*$, $m = m^*$ and $n = n^*$.

8. Repeat steps 1 to 7 with $\eta = \eta + 1$.

9. Find the value of $\eta$ that gives the minimum value of $EJTC(m, n, q, k, \eta)$ and set as $\eta$.

By solving the optimization problem analytically, we guarantee a globally optimal solution to the problem.

## V. Numerical Analysis

To illustrate the model, we provide a numerical example using a set of data collected from previous studies Mitra [13], Bazan [2].

$H_1 = \$50/\text{unit/year}, d = 10000/\text{unit/year}, \beta = 0.67,$

$H_4 = \$40/\text{unit/year}, H_5 = \$40/\text{unit/year}, A_1 = \$100/\text{batch},$

$A_2 = \$200/\text{batch}, O_1 = \$30/\text{order}, c_{re} = \$10/\text{unit},$

$c_{mn} = \$20/\text{unit}, F_t = \$100/\text{truck}, V = 100\,\text{unit/year}, \delta = 150$

$\text{unit/year}, t_c = 20\,\text{unit/year}, c_{ghg} = \$2/\text{ton}\ CO_2, c_r = 1.4$

$\text{ton/unit}, b_r = 0.0002\,\text{ton/year}\,2/\text{unit}\,3, a_r = 8.33 \times 10 - 8$

$\text{ton/year}\,2/\text{unit}\,3, c_p = 1.4\,\text{ton/year}, a_p = 3.10 \times 10 - 7$

$\text{ton/year}\,2/\text{unit}\,3, b_p = 0.0012\,\text{ton/year}\,2/\text{unit}\,3, g_T = 375\text{gallons/truck},$

$e_T = 0.1008414\,\text{ton/gallon}, l = 1, O_2 = \$100\,\text{perorder}, O_3 = \$75$

$\text{perorder}, T = 0.1129, k_m = 2.3679,$

$k_n = 2.4610, H_2 = \$40\,\text{perorderperyear}, \sigma = 100\,\text{unitperyear},$

$L_m = 0.012, L_n 0.01; H_3 = \$25/\text{unit/year}, c_{ic} = \$2/\text{unit},$

$c_x = \$/5/\text{unit}, \rho = 0.1, P_a = \$5/\text{unit}, , r = 20000\,\text{unitperyear},$

$\text{ton/year}\,2/\text{unit}\,3, c_p = 1.4\,\text{ton/year}, a_p = 3.10 \times 10 - 7$

$H_6 = \$1.2/\text{unit/year}, P = \$90, R = \$150.$

With the help of computer code using the software Matlab, we obtained the optimal results as follows: Number of shipments from manufacturer to retailer is 1, number of shipments of the remanufactured product from the manufacturer to the retailer is 4, number of shipments from collector to manufacturer is 4 and the expected joint total cost $EJTC(m, n, q, k, \eta) = 1259.5046$ with the help of computer code using the software Matlab, we obtained the optimal results as follows: Number of shipments from manufacturer to retailer is 1, number of shipments of the remanufactured product from the manufacturer to the retailer is 4, number of shipments from collector to manufacturer is 4 and the expected joint total cost $EJTC(m, n, q, k, \eta) = 1259.5046$.

## VI. Sensitivity Analysis

To assess the effects of the major parameters of the proposed model on the optimal results, sensitivity analysis is performed by changing the values of the key parameters one at a time; keeping the remaining parameters at their original levels. The results are reflected in Table 1 and graphically in Figures 5, 6, 7, 8 and 9, the following inferences are observed:

- From Figure 5 it is clear that $EJTC(m, n, q, k, \eta)$ increases with increase in $P$ and is negligible when $P$ is near the lowest possible value. It can also be seen that there is a fast increase and rate of increase in $EJTC(m, n, q, k, \eta)$ when $P$ is near its lowest possible value smaller in $EJTC$ for higher values of $P$.
- It is clear from Figures 6 and 7 that $EJTC(m, n, q, k, \eta)$ increases with increase in $L_m$ and $L_l$. This is due to increased costs associated with safety stock of new/remanufactured retailer's production and back line with increase in purchases the lead times are $L_m$ and $L_l$.
- From Figure 9 it is seen that $EJTC$ increases $\sigma$. This happens because the retailer's security-related costs increase stock of new/remanufactured product safety stock of remanufacturer Product and back order returns to retailer and remanufacturer with $\sigma$.
- From Figure 8 $EJTC(m, n, q, k, \eta)$ increases first and then decreases with the increase of $S_1$.

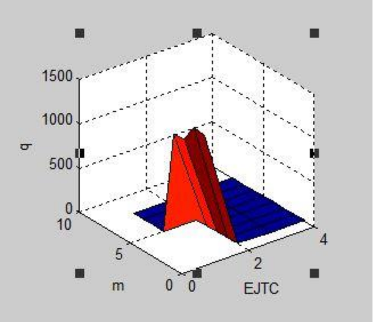

**Figure 5.** Effect of % changes in $P$.

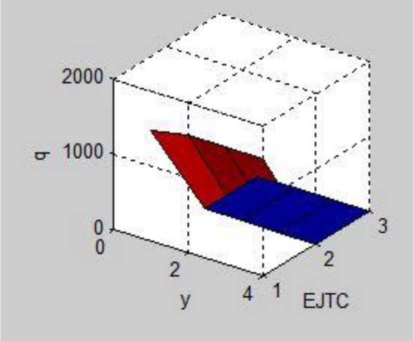

**Figure 6.** Effect of % changes in $L_m$.

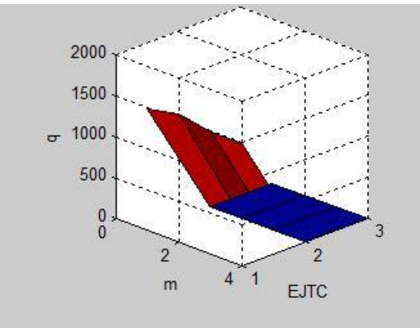

**Figure 7.** Effect of % changes in $L_n$.

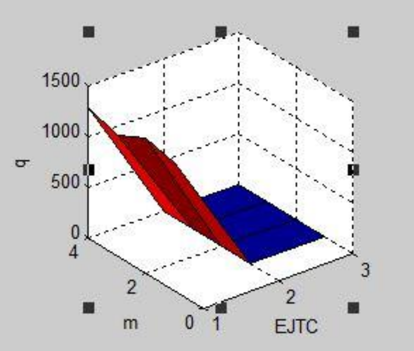

**Figure 8.** Effect of % changes in $S_1$.

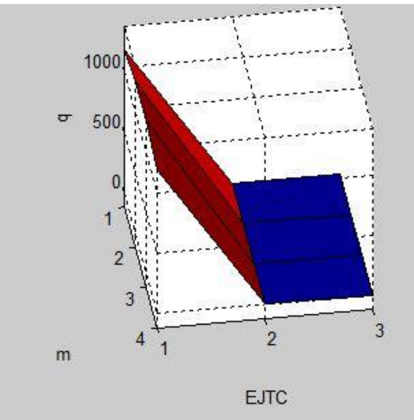

**Figure 9.** Effect of % changes in σ.

**Table I.** Sensitivity Analysis of Individual Parameters.

| Parameter | Changes | $m$ | $n$ | $K$ | $a$ | $EJTC(\cdot)$ |
|---|---|---|---|---|---|---|
| $P$ | 10000 | 10 | 3 | 3 | 0.7154 | 1360.039 |
| | 15000 | 5 | 3 | 2 | 0.5983 | 1384.93 |
| | 50000 | 3 | 1 | 2 | 1.7948 | 1182.69 |
| | 100000 | 3 | 1 | 3 | 2.1461 | 1158.12 |
| σ | 500 | 4 | 2 | 1 | 1.7948 | 1182.288 |
| | 200 | 4 | 1 | 1 | 1.342 | 1260.311 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 400 | 4 | 1 | 1 | 1.342 | 1261.76 |
| | 600 | 4 | 1 | 2 | 1.7948 | 1184.78 |
| $S_1$ | 10 | 5 | 1 | 2 | 1.2853 | 1253 |
| | 50 | 4 | 1 | 1 | 0.7893 | 1326.35 |
| | 70 | 4 | 2 | 1 | 3.5653 | 1185.99 |
| | 100 | 2 | 3 | 2 | 1.476 | 1281.2 |
| $L_m$ | 0.002 | 4 | 1 | 2 | 0.4487 | 1486.39 |
| | 0.042 | 4 | 1 | 1 | 0.3355 | 1564.21 |
| | 0.082 | 4 | 1 | 1 | 0.3355 | 1564.21 |
| | 0.091 | 4 | 1 | 1 | 0.3355 | 1564.21 |
| $L_n$ | 0.002 | 4 | 1 | 2 | 0.4487 | 1485.424 |
| | 0.042 | 4 | 1 | 1 | 0.3355 | 1567.32 |
| | 0.082 | 4 | 2 | 2 | 0.4487 | 1489.53 |
| | 0.091 | 4 | 2 | 2 | 0.4487 | 1489.78 |

## Conclusion

This study we developed a CLSC commodity model consisting of a supplier, manufacturer, retailer and collector. Previous works on this issue have not yet included a supplier and third party a collection dealer as an individual party in a reverse supply chain. Here, we limited the study number of regenerative generations and carbon emissions and energy effects component of inventory costs. The results illustrate that in order to obtain the minimum integral distribution in chain inventory costing, the system must design a product to be what it is reproduced in desired number before disposal. However, designing an object being highly reproducible is not necessarily a good strategy because it really gives away high cost of system. Future work on related issues should include additional recovery measures disassembly and reassembly of individual components and assemblies. Another one the challenge is to consider that not all collected materials are of the same quality. Since it comes from for different consumers, the residual quality of each product may not be uniformly distributed. As mentioned uncertainty in product acquisition can affect both production and remanufacturing strategies. It is explained that the rate of return on used material is actually restricted if the company makes a particular acquisition attempt.

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