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SOME ARITHMETIC OPERATIONS ON REARWARD AND MUTATED FUZZY NUMBERS

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Abstract

This paper forwards a new concept of fuzzy numbers with membership function in a reverse direction called Rearward membership function. Extending the idea of Rearward fuzzy numbers to Mutated fuzzy numbers, in particular, triangular rearward fuzzy numbers to mutated triangular fuzzy numbers and trapezoidal rearward fuzzy numbers to mutated trapezoidal fuzzy numbers with their graphical representations. It also explains the addition and scalar multiplication of Rearward and Mutated Fuzzy Numbers.

1. Introduction

Lotfi. A. Zadeh was first introduced the concept of fuzzy sets. This set theory intended to develop an idea for mathematical uncertainty and vagueness for dealing with more problems regarding this. Fuzzy numbers are an extension to real numbers (real intervals) with membership function. Triangular and Trapezoidal fuzzy numbers are the most commonly used fuzzy numbers. Fuzzy numbers are convex normalized fuzzy sets. This paper introduces a new concept of reverse membership function called rearward membership function. This concept is mainly deals with rearward triangular and rearward trapezoidal fuzzy numbers. Again extend the concept of Rearward membership function together with Membership function gives a new fuzzy number called Mutated fuzzy number.

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2. Preliminaries

Definition 2.1 (Fuzzy Sets)**.** Fuzzy sets are uncertain sets whose elements have degrees of membership between 0 and 1. A fuzzy set is defined in terms of a membership function.

Definition 2.2 (Fuzzy Numbers)**.** A fuzzy number is a normalized convex fuzzy set on the real line, which is upper semi continuous.

Definition 2.3 (Triangular Fuzzy Numbers)**.** A fuzzy number is a triangular fuzzy number denoted by $(a, b, c), a, b, c \in R$ if its membership function is

$$\mu_A(x) - \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ \dfrac{c-x}{c-b}, & \text{for } b \le x \le c \\ 0, & \text{for } x > c \end{cases}$$

Definition 2.4 (Trapezoidal Fuzzy Numbers)**.** A fuzzy number is a trapezoidal fuzzy number denoted by $(a, b, c, d), a, b, c, d \in R$ if its membership function

$$\mu_A(x) - \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ 1, & \text{for } b \le x \le c \\ \dfrac{d-x}{d-c}, & \text{for } c \le x \le d \\ 0, & \text{for } x > d \end{cases}$$

3. Main Results

Definition 3.1 (Fuzzy sets with Rearward membership function)**.** Fuzzy Sets whose elements have degrees of membership function $\mu \in [-1, 0]$ are called fuzzy sets with Rearward membership function. That is, a Rearward membership function of a fuzzy set A is a function $\mu_A : R \to [-1, 0]$. The absolute value of μ_A will be taken for deciding the belongingness of the elements to A.

Definition 3.2 (Rearward Fuzzy Numbers-RFN)**.** A Rearward Fuzzy Number A is a fuzzy set with Rearward membership function $\mu_A : R \to [-1, 0]$ having the following properties:

- A is a convex fuzzy set
- A is normalized in the sense that $|\mu_A(x)| = 1$, for at least one $x \in A$
- μ_A is piecewise continuous

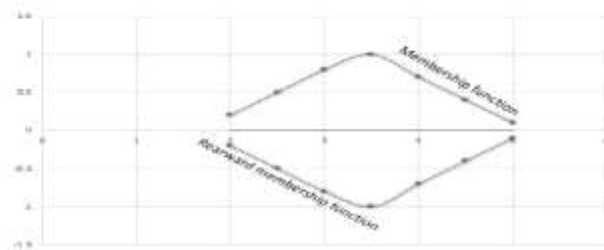

Figure 3.1. Membership function and rearward membership function.

Definition 3.3 (Triangular Rearward Fuzzy Numbers -Triangular RFN)**.** A Rearward fuzzy number $A = (a\ b\ c)$ is said to be Triangular Rearward Fuzzy Number(Triangular RFN) if its rearward membership function is

$$\mu_A(x) = \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ \dfrac{c-x}{c-b}, & \text{for } b \le x \le c \\ 0, & \text{for } x > c \end{cases}$$

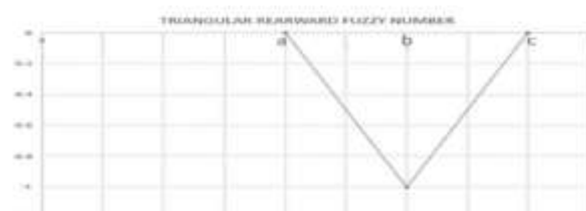

Figure 3.2. Graphical Representation of Triangular RFN.

Definition 3.4 (Trapezoidal Rearward Fuzzy Numbers-Trapezoidal RFN)**.** A Rearward Fuzzy Number $A = (a\ b\ c\ d)$ is said to be Trapezoidal Rearward Fuzzy Number if its rearward membership function is

$$\mu_A(x) = \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ 1, & \text{for } b \le x \le c \\ \dfrac{d-x}{d-c}, & \text{for } c \le x \le d \\ 0, & \text{for } x > d \end{cases}$$

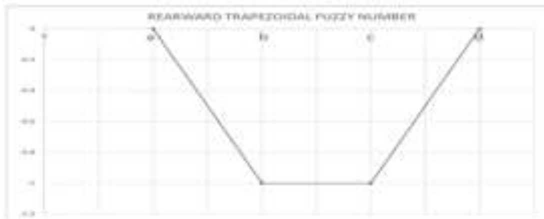

Figure 3.3. Graphical Representation of Trapezoidal RFN.

Remarks. In Triangular Rearward Fuzzy Numbers μ_A is first monotonically decreasing from zero to -1 and then monotonically increasing to zero; In trapezoidal rearward fuzzy numbers μ_A is first monotonically decreasing from zero to -1 and then constantly moving horizontally in -1 up to certain distance and then monotonically increasing to zero.

Definition 3.5 (Mutated Fuzzy Numbers-MFN)**.** A Mutated Fuzzy Number A is a fuzzy set having two subclasses A_1 and A_2 with the first subclass A_1 is a fuzzy number and the second subclass A_2 is a rearward fuzzy number.

Definition 3.6 (Mutated Triangular Fuzzy Numbers)**.** A Mutated Triangular Fuzzy Number is a Mutated Fuzzy Number $A = (a\ b\ c\ d\ e)$ having subclasses $A_1 = [a, b, c]$ and $A_2 = [c, d, e]$ with

$$\mu_{A_1}(x) = \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ \dfrac{c-x}{c-b}, & \text{for } b \le x \le c \\ 0, & \text{for } x = c \end{cases}$$

and

$$\mu_{A_2}(x) - \begin{cases} 0, & \text{for } x = c \\ \dfrac{x-c}{c-d}, & \text{for } c \le x \le d \\ \dfrac{x-e}{e-d}, & \text{for } d \le x \le e \\ 0, & \text{for } x > e \end{cases}$$

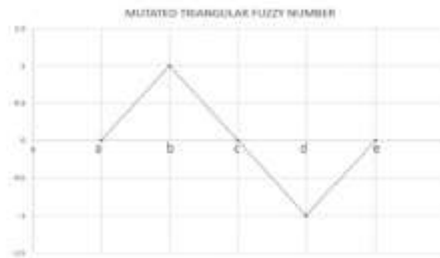

Figure 3.4. Graphical Representation Mutated Triangular Fuzzy Numbers.

Remarks. Here μ_A is first monotonically increasing from zero to 1 and then monotonically decreasing to -1, again increasing to zero.

Definition 3.7 (Mutated Trapezoidal fuzzy numbers)**.** A Mutated Trapezoidal Fuzzy Number is a Mutated Fuzzy Number having $A = (a\ b\ c\ d\ e\ f\ g)$ subclasses $A_1 = [a, b, c, d]$ and with $A_2 = [d, e, f, g]$

$$\mu_{A_1}(x) - \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-a}{b-a}, & \text{for } a \le x \le b \\ 1, & \text{for } b \le x \le c \\ \dfrac{d-x}{d-c}, & \text{for } c \le x \le d \\ 0, & \text{for } x = d \end{cases}$$

$$\mu_{A_2}(x) - \begin{cases} 0, & \text{for } x < a \\ \dfrac{x-d}{d-e}, & \text{for } d \le x \le e \\ -1, & \text{for } e \le x \le f \\ \dfrac{x-g}{g-f}, & \text{for } f \le x \le g \\ 0, & \text{for } x > g \end{cases}$$

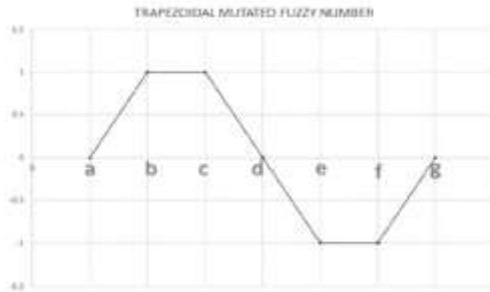

Figure 3.5. Graphical Representation Mutated Trapezoidal Fuzzy Numbers.

Remarks. Here μ_A is first monotonically increasing from zero to 1 after a horizontal movement in 1, then monotonically decreasing to -1, again after a horizontal movement in -1 increasing monotonically to zero.

3.8 Addition and Scalar Multiplication of Triangular Rearward fuzzy numbers

Addition

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two Triangular Rearward fuzzy Numbers. Then we can define the addition

$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ and its rearward membership function as

$$\mu_{A+B}(x) = \min(\mu_A(x), \mu_B(x))$$

Then $\mu_{A+B}(a_1 + b_1) = Min(\mu_A(a_1), \mu_B(b_1)) = 0$

$\mu_{A+B}(a_2 + b_2) = Min(\mu_A(a_2), \mu_B(b_2)) = -1$

$\mu_{A+B}(a_3 + b_3) = Min(\mu_A(a_3), \mu_B(b_3)) = 0$

Thus $A + B$ will also a Triangular Rearward fuzzy Number whose membership function can be defined as

$$\mu_{A+B}(x) - \begin{cases} 0, & \text{for } x < a_1 + b_1 \\ \dfrac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)}, & \text{for } a_1 + b_1 \le x \le a_2 + b_2 \\ \dfrac{x - (a_3 + b_3)}{(a_3 + b_3) - (a_2 + b_2)}, & \text{for } a_2 + b_2 \le x \le a_3 + b_3 \\ 0, & \text{for } x > a_3 + b_3 \end{cases}$$

Numerical Example

Let $A = (1\ 2\ 3)$ and $B = (4\ 5\ 6)$ are two Triangular Rearward fuzzy Numbers with $\mu_A(1) = \mu_A(3) = 0$ and $\mu_A(2) = -1$ and $\mu_A(4) = \mu_A(6) = 0$ and $\mu_A(5) = -1$

Then $A + B = (5\ 7\ 9)$

$$\mu_{A+B}(5) = \frac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)} = \frac{5 - 5}{5 - 7} = 0$$

$$\mu_{A+B}(7) = \frac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)} = \frac{7 - 5}{5 - 7} = -1 \text{ or}$$

$$\mu_{A+B}(7) = \frac{x - (a_3 + b_3)}{(a_3 + b_3) - (a_2 + b_2)} = \frac{9 - 7}{7 - 9} = -1$$

$$\mu_{A+B}(9) = \frac{x - (a_3 + b_3)}{(a_3 + b_3) - (a_2 + b_2)} = \frac{9 - 9}{9 - 7} = 0$$

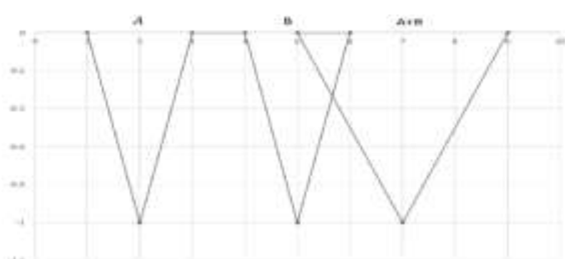

Figure 3.6.

Scalar Multiplication

Let $A = (a, b, c)$ be a Triangular Rearward fuzzy Number and α is a non-zero scalar

Now $\alpha A = (\alpha a, \alpha b, \alpha c)$ with membership function $\mu_{\alpha A}(x) = \mu_A(x)$.

Thus $\alpha A = (\alpha a, \alpha b, \alpha c)$ will also a Triangular Rearward fuzzy Number whose membership function can be defined as

$$\mu_{\alpha A}(x) - \begin{cases} 0, & \text{for } x < \alpha a \\ \dfrac{x - \alpha a}{\alpha a - \alpha b}, & \text{for } \alpha a \le x \le \alpha b \\ \dfrac{x - \alpha c}{\alpha c - \alpha b}, & \text{for } \alpha b \le x \le \alpha c \\ 0, & \text{for } x > \alpha c \end{cases}$$

Numerical Example

Let $A = (1\ 2\ 3)$ Triangular Rearward fuzzy Number with

$\mu_A(1) = \mu_A(3) = 0$ and $\mu_A(2) = -1$ and $\mu_A(4) = \mu_A(6) = 0$ and $\mu_A(5) = -1$ and take $\alpha = 2$

Then $\alpha A = (2\ 4\ 6)$ with $\mu_{\alpha A}(2) = \frac{2-2}{2-4} = 0$, $\mu_{\alpha A}(4) = \frac{4-6}{6-4} = -1$ and $\mu_{\alpha A}(6) = \frac{6-6}{6-4} = 0$.

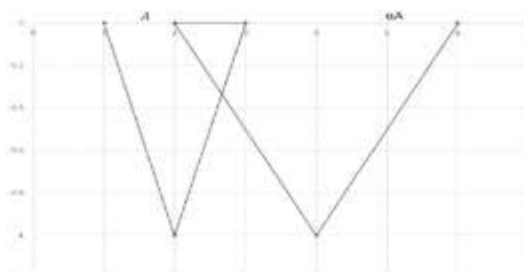

Figure 3.7.

3.9 Addition and Scalar Multiplication of Trapezoidal Rearward fuzzy numbers

Addition

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two Trapezoidal Rearward fuzzy Numbers. Then

$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$ and its rearward membership function as

$$\mu_{A+B}(x) = Min(\mu_A(x), \mu_B(x))$$

Then $\mu_{A+B}(a_1 + b_1) = Min(\mu_A(a_1), \mu_B(b_1)) = 0$

$\mu_{A+B}(a_2 + b_2) = Min(\mu_A(a_2), \mu_B(b_2)) = -1$

$\mu_{A+B}(a_3 + b_3) = Min(\mu_A(a_3), \mu_B(b_3)) = 0$

$\mu_{A+B}(a_4 + b_4) = Min(\mu_A(a_4), \mu_B(b_4)) = 0$

Thus $A + B$ will also a Trapezoidal Rearward fuzzy Number whose membership function can be defined as

$$\mu_{A+B}(x) = \begin{cases} 0, & \text{for } x < a_1 + b_1 \\ \dfrac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)}, & \text{for } a_1 + b_1 \le x \le a_2 + b_2 \\ -1, & \text{for } a_2 + b_2 \le x \le a_3 + b_3 \\ \dfrac{x - (a_4 + b_4)}{(a_4 + b_4) - (a_3 + b_3)}, & \text{for } a_3 + b_3 \le x \le a_4 + b_4 \\ 0, & \text{for } x > a_4 + b_4 \end{cases}$$

Scalar Multiplication

Let $A = (a_1, a_2, a_3, a_4)$ be a Trapezoidal Rearward fuzzy Number. and α is a non-zero scalar.

Now $\alpha A = (\alpha a, \alpha b, \alpha c, \alpha d)$ with membership function $\mu_{\alpha A}(x) = \mu_A(x)$.

Then $\alpha A = (\alpha a, \alpha b, \alpha c, \alpha d)$ will also a Trapezoidal Rearward fuzzy Number whose membership function can be defined as

$$\mu_{\alpha A}(x) = \begin{cases} 0, & \text{for } x < \alpha a \\ \dfrac{x - \alpha a}{\alpha a - \alpha b}, & \text{for } \alpha a \le x \le \alpha b \\ -1, & \text{for } \alpha b \le x \le \alpha c \\ \dfrac{\alpha d - x}{\alpha c - \alpha d}, & \text{for } \alpha c \le x \le \alpha d \\ 0, & \text{for } x > \alpha d \end{cases}$$

Numerical Example

Addition

Let $A = (1\ 2\ 3\ 4)$ and $B = (6\ 7\ 8\ 9)$ are two Triangular Rearward fuzzy

Numbers with $\mu_A(1) = \mu_A(3) = 0$ and $\mu_A(2) = -1$

$\mu_A(4) = \mu_A(6) = 0$ and $\mu_A(5) = -1$

Then $A + B = (7\ 9\ 11\ 13)$

$$\mu_{A+B}(7) = \frac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)} = \frac{7-7}{7-9} = 0$$

$$\mu_{A+B}(9) = \frac{x - (a_1 + b_1)}{(a_1 + b_1) - (a_2 + b_2)} = \frac{9-7}{7-9} = -1 \text{ or}$$

$$\mu_{A+B}(9) = \frac{x - (a_4 + b_4)}{(a_4 + b_4) - (a_3 + b_3)} = \frac{9-11}{11-9} = -1$$

$$\mu_{A+B}(11) = \frac{x - (a_4 + b_4)}{(a_4 + b_4) - (a_3 + b_3)} = \frac{11-13}{13-11} = -1$$

$$\mu_{A+B}(13) = \frac{x - (a_4 + b_4)}{(a_4 + b_4) - (a_3 + b_3)} = \frac{13-13}{13-11} = 0.$$

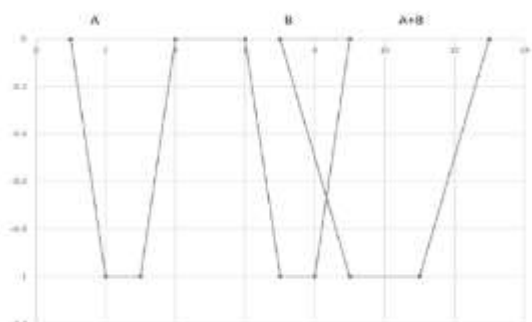

Figure 3.8.

Scalar Multiplication

Let $A = (1\ 2\ 3\ 4\ 5)$ be a Trapezoidal Rearward fuzzy Number. and $\alpha = 3$ is a non-zero scalar

Then $\alpha A = (3\ 6\ 8\ 10)$ is a Trapezoidal Rearward fuzzy Number.

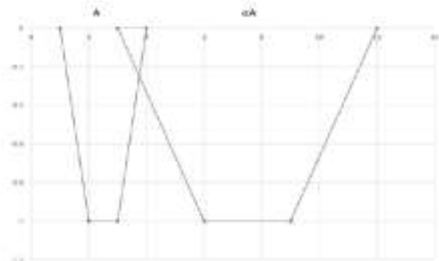

Figure 3.9.

3.10 Addition and Scalar Multiplication of Triangular and Trapezoidal Mutated fuzzy numbers

Since the Mutated Triangular (and Trapezoidal) fuzzy number gives two separation Triangular (and Trapezoidal) fuzzy number and Rearward Triangular (and Trapezoidal) fuzzy number, they are closed under addition and non-zero scalar multiplication.

4. Conclusion

A new concept of degrees of membership function of fuzzy numbers-Rearward membership function-has been defined and extended this idea to Rearward fuzzy numbers and then Mutated fuzzy numbers, in particular, Triangular rearward fuzzy numbers, Mutated triangular fuzzy numbers, Trapezoidal rearward fuzzy numbers and Mutated trapezoidal fuzzy numbers with their graphical representations. Addition and scalar multiplication of Triangular-Trapezoidal Rearward and Mutated fuzzy numbers are explained with numerical examples.

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