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A NOVEL WAY FOR SOLVING FRACTIONAL TRANSPORTATION PROBLEM

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Abstract

A novel way, which uses "Ghadle-Munot algorithm" to solve fractional transportation problem, is described. In fractional transportation problem, as a ratio of two objective functions is considered it becomes more perfect to apply in real life situations. The proposed algorithm is coded in MATLAB. For illustrative purpose, proposed algorithm is examined through real life examples. Decision maker can use this algorithm with ease due to coding.

Introduction

Transportation Problem (TP) is well known application of Linear Programming Problem (LPP) studied in OR to minimize the cost of transporting goods from one place to other or to maximize the profit. Fractional programming is the generalization of linear programming used for optimization in which ratio of physical or economic quantities such as output to employee, inventory to sale, profit to time, profit to cost, return to cost, actual cost to standard cost are considered. The TP in which objective function consist of two functions in numerator and denominator part respectively such that denominator function is non zero for given constraint

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region is called Fractional Transportation Problem (FrTP). It has received interest of many researchers as ratio of two functions points to efficiency of Transport System. Considering benefits and cost of transportation leads to FrTP which minimizes cost of transportation and maximizes profit of transportation together in a single objective function. While solving these type of problems it's not essential that the optimal solution for objective in the numerator will also be optimal for the objective in the denominator in such cases a settlement solution is obtained which will be optimal for both the objectives i.e. for given FrTP. In the literature, several methods are reported to solve FrTP which is initiated by Swarup [10] in 1966 it works on reducing cost and increasing service of supply chain. Later many researchers have worked on different algorithm to solve FrTP. Recently Liu [8] worked on bounds of TP with varying demand and supply same idea is extended by Joshi et al. [7], Gupta et al. [5]. Jain et al. [6] introduced inverse TP with linear fractional objective function. Raina et al. [9] solved FrTP with nonlinear discount cost using 'Karush-Kuhn-Tucker algorithm'. Gomathi et al. [4] solved FrTP by decomposition restriction method. Veeramani et al. [11] worked on multi objective FrTP by Neutrosophic Goal Programming. Many recent methods to solve TP are listed by Ghadle et al. [1].

In continuation to our research work on solving TP using modular arithmetic [2, 3] here, we developed a novel way for solving FrTP.

Mathematical Formulation

FrTP is the part of fractional programing problem which has objective function in the form of ratio of two objective functions such that denominator function is nonzero for the constraint region. FrTP is formulated as:

$$Max\,Z(x) = \frac{N(x)}{D(x)} = \frac{Max\,Z_1(x)}{Min\,Z_2(x)} = \frac{\sum_{a=1}^{m}\sum_{b=1}^{n} p_{ab}x_{ab}}{\sum_{a=1}^{m}\sum_{b=1}^{n} c_{ab}x_{ab}},$$

or

$$Min\,Z(x) = \frac{N(x)}{D(x)} = \frac{Min\,Z_1(x)}{Max\,Z_2(x)} = \frac{\sum_{a=1}^{m}\sum_{b=1}^{n} p_{ab}x_{ab}}{\sum_{a=1}^{m}\sum_{b=1}^{n} c_{ab}x_{ab}},$$

with,

$$\sum_{a=1}^{m} x_{ab} = s_a, \text{ for } a = 1, 2, \ldots, m \text{ (i)}$$

$$\sum_{b=1}^{m} x_{ab} = t_b, \text{ for } b = 1, 2, \ldots, n \text{ (ii)}$$

and $x_{ab} \geq 0$, $\forall a = 1, 2, \ldots, m$ and $b = 1, 2, \ldots, n$ (iii)

Where,

s_a -Number of product available for supply from origin point a

t_b -Number of product demanded at destination point b

x_{ab} -Number of product reached to destination point b from origin point a

p_{ab} -per unit profit cost of transport from origin point a to destination point b

c_{ab} -per unit cost of transport from origin point a to destination point b

$s_a > 0$ and $t_b > 0$ $\forall a = 1, 2, \ldots, m$ and $b = 1, 2, \ldots, n$

Here, we want to get optimal solution of objective function $Z(x)$, satisfying (i) to (iii).

Proposed Ghadle-Munot algorithm to solve Fractional Transportation Problem:

I. Let $Z(x)$ be given FrTP. Consider two separate TP, $N(x)$ and $D(x)$ from given Maximization $Z(x)$ [for Minimization $Z(x)$] where

$$N(x) = Max\, Z_1(x) = \sum_{a=1}^{m} \sum_{b=1}^{n} p_{ab} x_{ab},$$

[for minimization $N(x) = Min\, Z_1(x) = \sum_{a=1}^{m} \sum_{b=1}^{n} p_{ab} x_{ab}$]

Subject to (i) to (iii).

And

$$D(x) = Min\, Z_2(x) = \sum_{a=1}^{m} \sum_{b=1}^{n} c_{ab} x_{ab},$$

[for minimization $D(x) = Max\, Z_2(x) = \sum_{a=1}^{m} \sum_{b=1}^{n} c_{ab} x_{ab}$]

Subject to (i) to (iii).

II. Obtain feasible solution for $N(x)$ by Ghadle-Munot algorithm to solve TP which uses modular arithmetic, then use these same allocations obtained in the case of maximizing profit in minimizing cost $D(x)$ and get solution of $D(x)$ as well. Take the ratio $R_1 = \frac{N(x)}{D(x)}$.

III. Get another feasible solution for $D(x)$ by Ghadle-Munot algorithm to solve TP which uses modular arithmetic, then use these same allocations obtained in the case of minimizing cost in maximizing profit $N(x)$ and get solution of $N(x)$ as well. Take the ratio $R_2 = \frac{N(x)}{D(x)}$.

IV. Compare the ratios R_1 and R_2. For maximization, if $R_1 > R_2$ then go to step V otherwise go to step VI. For minimization, if $R_1 < R_2$ then go to step VI otherwise go to step V, if $R_1 = R_2$ solution obtained is best.

V. Solution corresponding to R_1 is best settlement solution.

VI. Solution corresponding to R_2 is best settlement solution.

Now we will illustrate this algorithm by numerical examples.

Numerical Example

Example 1. A food and beverages production dealer sells its products from source *i* to destination *j*. Its inventory and sales per product are mentioned in upper right and lower left part of diagonal of each cell respectively in the following table. Solve the example to minimize the objective function.

Product \ Destination	D_1	D_2	D_3	D_4	Supply
P_1	4 1	6 3	5 4	2 6	15
P_2	2 4	5 3	1 6	4 2	25
P_3	2 5	1 3	4 3	3 2	20
Demand	14	18	12	16	

First, consider Inventory matrix and solve it by Ghadle-Munot Algorithm for Minimization,

Product \ Destination	D_1	D_2	D_3	D_4	Supply
P_1	4	6	5	2	15
P_2	2	5	1	4	25
P_3	2	1	4	3	20
Demand	14	18	12	16	

By Ghadle-Munot algorithm we get IBFS as $x_{14} = 15$, $x_{21} = 13$, $x_{23} = 12$, $x_{31} = 1$, $x_{32} = 18$ and $x_{34} = 1$ with minimum transportation cost 91.

Using these same allocations for sales matrix, we get sales cost 275.

Its ratio $R_1 = \frac{91}{275} = 0.3309$.

Now consider sales matrix and solve it by Ghadle-Munot algorithm for maximization.

Product \ Destination	D_1	D_2	D_3	D_4	Supply
P_1	1	3	4	6	15
P_2	4	3	6	2	25
P_3	5	3	3	2	20
Demand	14	18	12	16	

We get IBFS as $x_{14} = 15, x_{21} = 13, x_{23} = 12, x_{31} = 1, x_{32} = 18$ and $x_{34} = 1$ with maximum transportation cost of sales as 275.

Using these same allocations for inventory matrix, we get minimum cost 91.

Its ratio $R_2 = \frac{91}{275} = 0.3309.$

Here we have $R_1 = R_2 \Rightarrow$ the solution we have obtained is best and company is running well.

Example 2. A water purifier company has three plants P_1, P_2, P_3 with four different locations L_1, L_2, L_3, L_4 demanding the product. Profit and cost per product are mentioned in upper right and lower left part of diagonal of each cell respectively in the following table. Solve the example to maximize the objective function.

Plant \ Location	L_1	L_2	L_3	L_4	Supply
P_1	10 15	14 12	8 16	12 8	15
P_2	8 10	12 6	14 13	8 12	25

P_3	9 13	6 15	15 12	9 10	20
Demand	14	18	12	16	

First, consider profit matrix and solve it by Ghadle-Munot algorithm for maximization.

Plant \ Location	L_1	L_2	L_3	L_4	Supply
P_1	10	14	8	12	15
P_2	8	12	14	8	25
P_3	9	6	15	9	20
Demand	14	18	12	16	

We get IBFS as $x_{14} = 15, x_{22} = 13, x_{23} = 12, x_{31} = 14, x_{32} = 5$ and $x_{34} = 1$ with maximum profit 621.

Using these same allocations for cost matrix, we get minimum cost as 621.

Its ratio $R_1 = \frac{669}{621} = 1.07$

Now consider cost matrix and solve it by Ghadle-Munot algorithm for minimization.

Plant \ Location	L_1	L_2	L_3	L_4	Supply
P_1	15	12	16	8	15
P_2	10	6	13	12	25
P_3	13	15	12	10	20
Demand	14	18	12	16	

We get IBFS as $x_{14} = 15$, $x_{21} = 7$, $x_{22} = 18$, $x_{31} = 7$, $x_{33} = 12$ and $x_{34} = 1$ with minimum cost 543.

Using these same allocations for profit matrix, we get maximum cost as 704.

Its ratio $R_2 = \frac{704}{543} = 1.29$

Here $R_1 < R_2 \Rightarrow$ solution corresponding to R_2 is best settlement solution.

Result

Proficiency of the proposed algorithm is investigated through numerical examples. The optimal solution of the above examples are same as solution by Swarup method but proposed method is more simple because here given FrTP is converted to two different TP and solved. This approach minimizes complexity and time to solve the example; its MATLAB coding makes it more-handy for the user. This method can be extended to multi objective FrTP, Bottleneck FrTP, Inverse TP etc.

Conclusion

In conclusion, we have developed an efficient and novel way to solve FrTP which uses Ghadle-Munot algorithm and is extended to obtain best settlement solution of given FrTP. It will help decision maker to take appropriate decision about financial challenges as well as corporate planning which could increase the proficiency of the system.

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