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THE *b*-COLOURING OF EXTENDED DUPLICATE GRAPH OF STAR GRAPH FAMILIES

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Abstract

In this paper we investigate b-chromatic number of Middle graph of Extended Duplicate graph of Star graph $(M[EDG(K_{1,n})])$ and Total graph of Extended Duplicate graph of Star graph $(T[EDG(K_{1,n})])$. We prove that the b-chromatic number of the Middle graph of Extended Duplicate graph of Star graph is $\varphi(M(EDG(K_{1,n}))) = n+1, n \geq 2$. Also we show that the Total graph of Extended Duplicate graph of Star graph is $\varphi(T(EDG(K_{1,n}))) = n+1, n \geq 2$.

Introduction

A proper colouring of a graph G is the colouring of the vertices of G such that no two neighbors in G are assigned the same colour. Throughout this paper, by a graph, we mean a finite, undirected, simple graph and the term colouring will be used to define vertex colouring of graphs.

The b-colouring of a graph G is a colouring of the vertices of G such that each colourclass contains at least one vertex that has a neighbour in all other colour classes. The b-chromatic number of a graph G, denoted by $\varphi(G)$, is the largest integer k such that G admits the b-colouring with k colours. The concept of b-chromatic number was introduced in 1999 by Irving and

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Manlove [3], who proved that the determining $\varphi(G)$ is NP-hard for general graphs but polynomial-time solvable for trees.

In 1975, J. Akiyama, T. Hamada, I. Yoshimura introduced the Middle graph of a graph G, denoted by $M(G)$, is defined as follows [1]. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Any two vertices x, y in $M(G)$ are adjacent in $M(G)$ if one of the following case holds.

(i) x, y are in $E(G)$ and x, y are adjacent in G

(ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

The Total graph [2] graph of G, denoted by $T(G)$, is defined as follows. The vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y are adjacent in $T(G)$ if one of the following cases holds.

(i) x, y are in $V(G)$ and x is adjacent to y in G

(ii) x, y are in $E(G)$ and x, y are adjacent in G

(iii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

The b-colouring of some class of graphs have been studied in the literature [4, 8].

In 2011, the concept of extended duplicate graph of G, denoted by $DG = (V_1, E_1)$, is defined as follows [6]. The vertex set $V = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is the edge v_1v_2 is in E if and only if both v_1v_2' and $v_1'v_2$ are edges in E_1. The extended duplicate graph of DG, denoted by EDG, is defined as, add an edge between any two vertex from V to any other vertex in V', except the terminal vertices of V and V'. For convenience, we take $v_2' \in V$ and $v_2' \in V'$ and thus the edge v_1v_2' is formed [7].

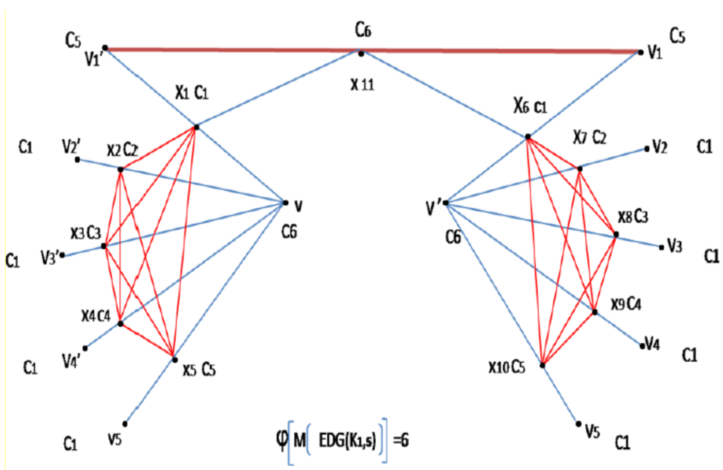
A complete bipartite graph $K_{1,n}$ is a *star graph* with $(n+1)$ vertices and n edges. The Extended Duplicate graph of star graph [5], denoted by

$EDG(K_{1,n})$ is obtained from the duplicate graph of star by joining the v_1 vertices and v_1'.

b-Chromatic number of Total graph of Extended Duplicate graph of Star graph $(T[EDG(K_{1,n})])$

Theorem 1. *The b-chromatic number of Middle graph of Extended Duplicate graph of Star graph $K_{1,n}$ is given by* $\varphi[M(EDG(k_{1,n}))] = n+1, n \geq 2$.

Proof. Let $V(EDG(K_{1,n})) = \{v, v_1, v_2, \cdots, v_n\} U \{v', v_1', v_2', \cdots, v_n'\}$. By the definition of middle graph, each edge vv_i', $v'v_i$ for $(1 \leq i \leq n)$ and $v_1'v_1$ of $EDG(K_{1,n})$ is Subdivided by the vertex x_i and $x_j (n+1 \leq j \leq 2n)$ respectively. In $M(EDG(K_{1,n}))$ the vertices $v, x_1, x_2 \cdots x_n$ and $v', x_{n+1}, x_{n+2} \cdots x_{2n}$ induce a clique of order $(n+1)$. That is $V[M(EDG(K_{1,n}))] = \{v, v'\} U \{v_i, v_i', 1 \leq i \leq n\} U \{x_k, 1 \leq K \leq 2n+1\}$. Therefore $\varphi[M(EDG(K_{1,n}))] \geq n+1$.



Now, consider the Colour Class. $C = \{c_1, c_2, \cdots c_n, c_{n+1}\}$ and assign the b-colouring to $M(EDG(K_{1,n}))$ as follows. For every $1 \leq i \leq n$, assign the colour c_i to x_i and c_i to $x_j (n+1 \leq j \leq 2n)$. Assign the colour C_{n+1} to v, v'

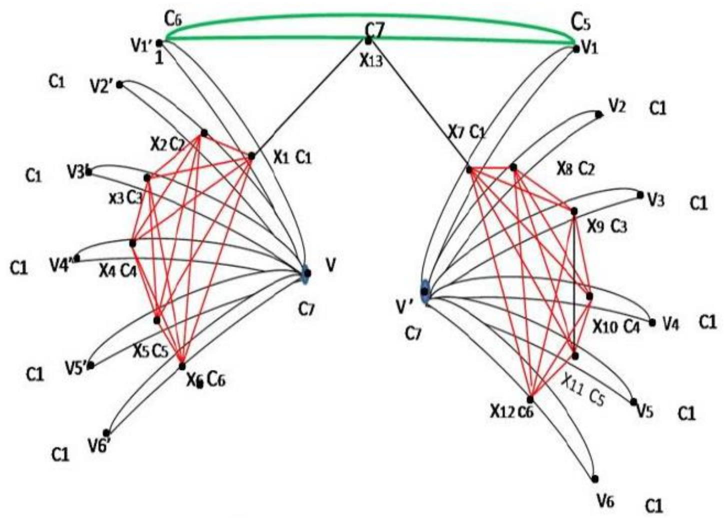
and x_{2n+1}. For every $2 \leq i \leq n$, assign the colour c_1 to v_i and v_i' also assign the colour c_n to v_1 and v_1'.

If $\varphi[M(EDG(K_{1,n}))] = n+2$, $n \geq 2$, there must be at least $n+2$ vertices of degree $n+1$ in $M(EDG(K_{1,n}))$, all with district colours, and each adjacent to vertices of all of other colours. Since the vertices $(x_1, x_2 \cdots x_n)$ and $\{x_{n+1}, x_{n+2} \cdots x_{2n+1}\}$ are only once with degree at least $n+1$ an $(n+2)$-colouring is impossible. Thus, we have $\varphi[M(EDG(K_{1,n}))] \leq n+1$. Hence, $\varphi[M(EDG(K_{1,n}))] = n+1, n \geq 2$.

b-Chromatic number of Total graph of Extended Duplicate graph of Star graph $(T[(EDG(K_{1,n})])$

Theorem 2. *The b-chromatic number of Total graph of Extended duplicate graph of star graph* $K_{1,n}$ *is given by* $\varphi[T(EDG(K_{1,n}))] = n+1, n \geq 2$.

Proof. Let $V(EDG(K_{1,n})) = \{v, v_1, v_2, \cdots, v_n\} U \{v', v_1', v_2', \cdots, v_n'\}$. By the definition of total graph, we have $V(T(EDG(K_{1,n}))) = \{v, v'\}\{v_i, v_i', 1 \leq i \leq n\}$ $U\{x_1, x_2 \cdots x_{2n+1}, 1 \leq i \leq 2n+1\}$. In $T(EDG(K_{1,n})$ the vertices $v, x_1, x_2 \cdots x_n$ and $v', x_{n+1}, x_{n+2} \cdots x_{2n}$ induce a clique of order $(n+1)$ Therefore $\varphi[T(EDG(K_{1,n}))] \geq n+1$, $n \geq 2$. Assign the following $n+1$-colouring to $T(EDG(K_{1,n}))$. For every $1 \leq i \leq n$ assign the colour c_i to x_i and c_i to $x_j (n+1 \leq j \leq 2n)$ and assign the colour c_{n+1} to v, v', x_{n+1}. For every $2 \leq i \leq n$ assign the colour c_1 to v_i, v_i' and assign the colour c_n, c_{n-1} to v_1' and v_1 respectively.



$T[EDG(K_{1,\,6})] = 7.$

If $\varphi[T(EDG(K_{1,n}))] = n+2, n \geq 2,$ there must be at least $n+2$ vertices of degree $n+1$ in $T(EDG(K_{1,\,n}))$ all with distinct colours, and each adjacent to vertices of all of other colours. Since the Vertices $\{x_1, x_2, \cdots x_n\}$ and $\{x_{n+1}, x_{n+2}, \cdots x_{2n+1}\}$ are only once with degree at least $n+1$. An $(n+2)$-colouring is impossible. Thus, we have $\varphi[T(EDG(K_{1,\,n}))] \leq n+1.$ Hence, $\varphi[T(EDG(K_{1,\,n}))] = n+1,\ n \geq 2.$

References

[1] J. Akiyama, T. Hamada and I. Yoshimura, On characterization of middle graphs, TRU Math. 11 (1975), 35-39.

[2] M. Behzad, A criterion for the planarity of the total graph of a graph, Proc. Cambridge Philos. Soc. 63 (1967), 679-681.

[3] R. W. Irving and D. F. Manlove, The b-chromatic number of a graph, Discrete Applied Mathematics 91(1-3) (1999), 127-141.

[4] Marko Jakovac and Sandi Klavzar, The b-chromatic number of cubic graphs, Graphs and Combinatorics 26 (2010), 107-118.

[5] C. Shobana Sarma and K. Thirusangu, Acyclic colouring of extended duplicate graph of star graph families, International Journal of Mathematics Trends and Technology (2018), 109-112.

[6] K. Thirusangu, P. P. Ulaganathan and B. Selvam, Cordial labeling in duplicate graphs, Int. J. Computer Math. Sci. Appl. 4(1-2) (2010), 179-186.

[7] P. P. Ulaganathan, K. Thirusangu and B. Selvam, Edge magic total labeling in extended duplicate graph of path, International Journal of Applied Engineering Research 6(10) (2011), 1211-1217.

[8] M. Venkatachalam and Vivin J. Vernold, The *b*-chromatic number of star graph families, Le Matematiche, LXV, (2010), Fasc. I, 119-125.