



# NANO STRONGLY $\alpha^*$ AS-CONTINUOUS MAPS AND NANO PERFECTLY $\alpha^*$ AS-CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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## Abstract

Lellis Thivagar introduced nano topological spaces and studied some of their properties. nano  $\alpha^*$ AS introduced by I. Sahaya Dani and P. Anbarasi Rodrigo in nano topological spaces. The purpose of this paper is to introduce and investigate the notion of Strongly nano  $\alpha^*$ AS Continuous and Perfectly Nano  $\alpha^*$ AS Continuous. We also examine some of the relations and properties of such functions.

## 1. Introduction

Levine introduced and investigated the concept of strong continuity in topological spaces. Sundaram introduced strongly  $g$ -continuous maps and

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perfectly  $g$ -continuous maps in topological spaces. One of the main concepts of topology is continuous functions. Different types of generalizations of continuous functions were introduced and studied by various authors in the recent development of topology. The concepts of nano topology were introduced by Lellis Thivagar, which was defined in terms approximations and boundary region of a subset of a universe using an equivalence relation on it. He has also defined a nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphisms and their representations in terms of nano closure and nano interior. I. Sahaya Dani and P. Anbarasi Rodrigo, was introduced and studied the nano  $\alpha^*AS$ -closed sets in nano topological spaces. In this paper is to introduce and investigate the notion of strongly nano  $\alpha^*AS$  Continuous and Perfectly nano  $\alpha^*AS$  Continuous. We also examine some of the relations and properties of such functions.

## 2. Preliminaries

The following are the necessary concepts and definitions that are used in this work.

**Definition 2.1.** Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ . Then,

1. The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $L_R(X)$ .

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ , where  $R(x)$  denotes the equivalence class determined by  $X \in U$ .

2. The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $U_R(X)$ .

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}.$$

3. The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither as  $X$  nor as not- $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ .

$$B_R(X) = U_R(X) - L_R(X).$$

**Definition 2.2.** Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \emptyset, U_R(X), L_R(X), B_R(X)\}$  where  $X \subseteq U$ .  $R(X)$  satisfies the following axioms:

1.  $U$  and  $\emptyset \in \tau_R(X)$ ,
2. The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ ,
3. The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

That is,  $\tau_R(X)$  is a topology on  $U$  called the nano topology on  $U$  with respect to  $X$ . We call  $(U, \tau_R(X))$  as the nano topological space. The elements of  $\tau_R(X)$  are called as nano open sets. The complement of nano-open sets is called nano closed sets.

**Remark 2.3.** If  $\tau_R(X)$  is the nano topology on  $U$  with respect to  $X$ , then the set  $B = \{U, \emptyset, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.4.** A subset  $A$  of a nano topological space  $(U, \tau_R(X))$  is called nano  $\alpha^*$ AS (briefly  $N\alpha^*$ AS) closed sets if  $N\alpha cl(A) \subseteq N\text{int}(V)$  whenever  $A \subseteq V$  and  $V$  is nano open.

**Definition 2.5.** Let  $(U, \tau_R(X))$  and  $(V, \sigma_R(Y))$  be a nano topological spaces. Then the function  $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is said to be nano continuous on  $U$  if the inverse image of every nano open set in  $V$  is nano open in  $U$ .

**Definition 2.6.** A function  $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano  $\alpha^*$ AS-continuous (briefly  $N\alpha^*$ AS-continuous) if the inverse image of every nano closed set in  $(V, \sigma_R(Y))$  is  $N\alpha^*$ AS-closed in  $(U, \tau_R(X))$ .

**Definition 2.7.** A function  $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano contra continuous if the inverse image of every nano open set in  $(V, \sigma_R(Y))$  is nano closed set in  $(U, \tau_R(X))$ .

**Definition 2.8.** A function  $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano strongly continuous if  $f^{-1}(V)$  is nano clopen in  $U$  for every subset  $v$  in  $V$ .

**Definition 2.9.** A function  $f : (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano perfectly continuous if  $f^{-1}(V)$  is nano clopen in  $U$  for every nano open set  $v$  in  $V$ .

### 3. Nano Strongly $\alpha^*$ AS- Continuous Maps in Topological Spaces

In this section, we introduce the concept of nano strongly  $\alpha^*$ AS-continuous maps in topological spaces and study some of its properties.

**Definition 3.1.** A map  $f : X \rightarrow Y$  is said to be nano strongly  $\alpha^*$ AS-continuous if the inverse image of every  $N\alpha^*$ AS-closed set in  $Y$  is nano closed in  $X$ .

**Example 3.2.** Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Then nano closed sets are  $= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ . Let  $\sigma = \{Y, \emptyset, \{c\}\}$ . Then  $N\alpha^*$ AS- closed sets are  $= \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Let  $f : X \rightarrow Y$  be an identity map. Hence the inverse image of every  $N\alpha^*$ AS-closed set in  $Y$  is nano closed in  $X$ . Thus,  $f$  is nano strongly  $\alpha^*$ AS-continuous.

**Theorem 3.3.** *If a map  $f : X \rightarrow Y$  is nano strongly  $\alpha^*$ AS-continuous then it is nano continuous but not conversely.*

**Proof.** Assume that  $f$  is nano strongly  $\alpha^*$ AS-continuous. Let  $S$  be any closed set in  $Y$ . Since every closed set is  $N\alpha^*$ AS-closed,  $S$  is  $N\alpha^*$ AS-closed in

$Y$ . Since  $f$  is nano strongly  $\alpha^*$ AS-continuous  $f^{-1}(S)$  is nano closed in  $X$ . Therefore,  $f$  is nano continuous.

**Remark 3.4.** The converse need not be true as seen from the following example.

**Example 3.5.** Let  $X = Y = \{a, b, c, d\}$  and  $\tau = \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ ,  $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Let  $f : X \rightarrow Y$  defined by  $f(a) = c, f(b) = a, f(c) = b, f(d) = d$  then  $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a, f^{-1}(d) = d$ . Then  $f$  is nano continuous but not nano strongly  $\alpha^*$ AS-continuous since  $\{a, b\}$  is  $N\alpha^*$ AS-closed in  $Y$  but  $f^{-1}(\{a, b\}) = \{b, c\}$  is not a nano closed set of  $X$ .

**Theorem 3.6.** *A map  $f : X \rightarrow Y$  is nano strongly  $\alpha^*$ AS-continuous if and only if the inverse image of every  $N\alpha^*$ AS-open set in  $Y$  is nano open in  $X$ .*

**Proof.** Assume that  $f$  is nano strongly  $\alpha^*$ AS-continuous. Let  $S$  be any  $N\alpha^*$ AS-open set in  $Y$ . Then  $S^c$  is  $N\alpha^*$ AS-closed set in  $Y$ . Since  $f$  is nano strongly  $\alpha^*$ AS-continuous,  $f^{-1}(S^c)$  is nano closed in  $X$ . But  $f^{-1}(S^c) = X - f^{-1}S$  and so  $f^{-1}S$  is nano open in  $X$ . Conversely assume that the inverse image of every  $N\alpha^*$ AS-open set in  $Y$  is nano open in  $X$ . Let  $S$  be any  $N\alpha^*$ AS-closed set in  $Y$ . Then  $S^c$  is  $N\alpha^*$ AS-open set in  $Y$ . By assumption,  $f^{-1}(S^c)$  is nano open in  $X$ . But  $f^{-1}(S^c) = X - f^{-1}S$  and so  $f^{-1}S$  is nano closed in  $X$ . Therefore,  $f$  is nano strongly  $\alpha^*$ AS-continuous.

**Theorem 3.7.** *If a map  $f : X \rightarrow Y$  is nano strongly continuous then it is nano strongly  $\alpha^*$ AS-continuous, but not conversely.*

**Proof.** Assume that  $f$  is nano strongly continuous. Let  $S$  be any  $N\alpha^*$ AS-closed set in  $Y$ . Since  $f$  is  $N\alpha^*$ AS-continuous,  $f^{-1}(S)$  is nano closed in  $X$ . Therefore,  $f$  is nano strongly  $\alpha^*$ AS-continuous.

**Remark 3.8.** The converse need not be true as seen from the following example.

**Example 3.9.** Let  $X = \{a, b, c, d\}$  and  $Y = \{a, b, c\}$ . Let  $\tau = \{X, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ ,  $\sigma = \{Y, \emptyset, \{a, c\}\}$ . Let  $f : X \rightarrow Y$  defined by  $f(a) = f(d) = c, f(b) = b, f(c) = a$  then  $f^{-1}(a) = c, f^{-1}(b) = b, f^{-1}(c) = ad$ . Hence the inverse image of every  $N\alpha^*AS$ -closed set in  $Y$  is nano closed in  $X$ . Thus,  $f$  is nano strongly  $\alpha^*AS$ -continuous but not nano strongly continuous since  $f^{-1}(b) = b$  is nano closed in  $X$  but not nano open in  $X$ .

**Theorem 3.10.** *If a map  $f : X \rightarrow Y$  is nano strongly  $\alpha^*AS$ -continuous and a map  $g : Y \rightarrow Z$  is  $N\alpha^*AS$ -continuous, then the composition  $g \circ f : X \rightarrow Z$  is nano continuous.*

**Proof.** Let  $S$  be any nano closed set in  $Z$ . Since  $g$  is  $N\alpha^*AS$ -continuous  $g^{-1}(S)$  is  $N\alpha^*AS$ -closed in  $Y$ . Since  $f$  is nano strongly  $\alpha^*AS$ -continuous,  $f^{-1}(g^{-1}(S))$  is nano closed in  $X$ . But  $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$ . Therefore,  $g \circ f$  is nano continuous.

**Theorem 3.11.** *If a map  $f : X \rightarrow Y$  is nano strongly  $\alpha^*AS$ -continuous and a map  $g : Y \rightarrow Z$  is  $N\alpha^*AS$ -irresolute, then  $g \circ f : X \rightarrow Z$  is nano strongly  $\alpha^*AS$ -continuous.*

**Proof.** Let  $S$  be any  $N\alpha^*AS$ -closed set in  $Z$ . Since  $g$  is  $N\alpha^*AS$ -irresolute,  $g^{-1}(S)$  is  $N\alpha^*AS$ -closed in  $Y$ . Also,  $f$  is nano strongly  $\alpha^*AS$ -continuous  $f^{-1}(g^{-1}(S))$  is nano closed in  $X$ . But  $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$  is nano closed in  $X$ . Hence,  $g \circ f : X \rightarrow Z$  is nano strongly  $\alpha^*AS$ -continuous.

**Theorem 3.12.** *If a map  $f : X \rightarrow Y$  is  $N\alpha^*AS$ -continuous and a map  $g : Y \rightarrow Z$  is nano strongly  $\alpha^*AS$ -continuous then  $g \circ f : X \rightarrow Z$  is  $N\alpha^*AS$ -irresolute.*

**Proof.** Let  $S$  be any  $N\alpha^*$ AS- closed set in  $Z$ . Since  $g$  is strongly  $N\alpha^*$ AS- continuous,  $f^{-1}(S)$  is nano closed in  $Y$ . Also,  $f$  is  $N\alpha^*$ AS- continuous,  $f^{-1}(g^{-1}(S))$  is  $N\alpha^*$ AS- closed in  $X$ . But  $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$ . Hence  $g \circ f : X \rightarrow Z$  is  $N\alpha^*$ AS- irresolute.

**Theorem 3.13.** *The composition of two nano strongly  $\alpha^*$ AS- continuous maps is nano strongly  $\alpha^*$ AS- continuous.*

**Proof.** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two nano strongly  $\alpha^*$ AS- continuous maps. Let  $S$  be a  $N\alpha^*$ AS- closed set in  $Z$ . Since  $g$  is nano strongly  $\alpha^*$ AS- continuous, we get  $g^{-1}(S)$  is nano closed in  $Y$ . Then  $g^{-1}(S)$  is  $N\alpha^*$ AS- closed in  $Y$ . As  $f$  is also nano strongly  $\alpha^*$ AS- continuous,  $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$  is nano closed in  $X$ . Hence,  $(g \circ f)$  is nano strongly  $\alpha^*$ AS- continuous.

**Theorem 3.14.** *If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be any two maps. Then their composition  $g \circ f : X \rightarrow Z$  is nano strongly  $\alpha^*$ AS- continuous if  $g$  is nano strongly  $\alpha^*$ AS- continuous and  $f$  is nano continuous.*

**Proof.** Let  $S$  be a  $N\alpha^*$ AS- closed in  $Z$ . Since  $g$  is nano strongly  $\alpha^*$ AS- continuous,  $g^{-1}(S)$  is nano closed in  $Y$ . Since  $f$  is nano continuous,  $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$  is nano closed in  $X$ . Hence,  $(g \circ f)$  is nano strongly  $\alpha^*$ AS- continuous.

#### 4. Nano Perfectly $\alpha^*$ AS- Continuous Maps in Topological Spaces

**Definition 4.1.** A map  $f : X \rightarrow Y$  is said to be nano perfectly  $\alpha^*$ AS- continuous if the inverse image of every  $N\alpha^*$ AS- closed set in  $Y$  is both nano open and nano closed in  $X$ .

**Example 4.2.** Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Then nano closed sets are  $= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ . Let  $\sigma = \{Y, \emptyset, \{c\}\}$ . Then  $N\alpha^*AS$ -closed sets are  $= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . Let  $f : X \rightarrow Y$  be an identity map. Hence the inverse image of every  $N\alpha^*AS$ -closed set in  $Y$  is both nano open and nano closed in  $X$ . Thus,  $f$  is perfectly  $N\alpha^*AS$ -continuous.

**Theorem 4.3.** *If a map  $f : X \rightarrow Y$  is nano perfectly  $\alpha^*AS$ -continuous then it is nano continuous but not conversely.*

**Proof.** Assume that  $f$  is nano perfectly  $\alpha^*AS$ -continuous. Let  $S$  be any nano closed set in  $Y$ . Since every nano closed set is  $N\alpha^*AS$ -closed,  $S$  is  $N\alpha^*AS$ -closed in  $Y$ . Since  $f$  is nano strongly  $\alpha^*AS$ -continuous  $f^{-1}(S)$  is both nano open and nano closed in  $X$ . Therefore,  $f$  is nano continuous.

**Remark 4.4.** The converse of the above theorem need not be true as seen from the following example.

**Example 4.5.** Let  $X = Y = \{a, b, c, d\}$  and  $\tau = \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ ,  $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Let  $f : X \rightarrow Y$  defined by  $f(a) = c$ ,  $f(b) = a$ ,  $f(c) = b$ ,  $f(d) = d$  then  $f^{-1}(a) = b$ ,  $f^{-1}(b) = c$ ,  $f^{-1}(c) = a$ ,  $f^{-1}(d) = d$ . Then  $f$  is nano continuous but not nano perfectly  $\alpha^*AS$ -continuous since  $\{a, b\}$  is  $N\alpha^*AS$ -closed in  $Y$  but  $f^{-1}(\{a, b\}) = \{b, c\}$  is neither nano open nor nano closed set of  $X$ .

**Theorem 4.6.** *If a map  $f : X \rightarrow Y$  is nano perfectly  $\alpha^*AS$ -continuous then it is nano strongly  $\alpha^*AS$ -continuous.*

**Proof.** Assume that  $f$  is nano perfectly  $\alpha^*AS$ -continuous. Let  $S$  be any  $N\alpha^*AS$ -closed set in  $Y$ . Since  $f$  is nano perfectly  $\alpha^*AS$ -continuous,  $f^{-1}(S)$  is nano closed in  $X$ . Therefore,  $f$  is nano strongly  $\alpha^*AS$ -continuous.

**Remark 4.7.** The converse of the above theorem need not be true.



**Example 4.8.** Let  $X = \{a, b, c, d\}$  and  $Y = \{a, b, c\}$ . Let  $\tau = \{X, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ ,  $\sigma = \{Y, \emptyset, \{a, c\}\}$ . Let  $f : X \rightarrow Y$  defined by  $f(a) = f(d) = c, f(b) = b, f(c) = a$  then  $f^{-1}(a) = c, f^{-1}(b) = b, f^{-1}(c) = \{a, d\}$ . Hence the inverse image of every  $N\alpha^*$ AS-closed set in  $Y$  is nano closed in  $X$ . Thus,  $f$  is nano strongly  $\alpha^*$ AS-continuous but not nano perfectly  $\alpha^*$ AS-continuous since  $f^{-1}(b) = b, f^{-1}(a, b) = \{b, c\}$  and  $f^{-1}(b, c) = \{a, b, d\}$  are nano closed in  $X$  but not nano open in  $X$ .

**Theorem 4.9.** *If a map  $f : X \rightarrow Y$  is nano perfectly  $\alpha^*$ AS-continuous then it is nano perfectly continuous.*

**Proof.** Assume that  $f$  is nano perfectly  $\alpha^*$ AS-continuous. Let  $S$  be any nano closed set in  $Y$ . Since every nano closed set is  $N\alpha^*$ AS-closed,  $S$  is  $N\alpha^*$ AS-closed set in  $Y$ . Since  $f$  is nano Strongly  $\alpha^*$ AS-Continuous Maps and nano Perfectly  $\alpha^*$ AS-Continuous Maps in Topological Spaces perfectly  $\alpha^*$ AS-continuous,  $f^{-1}(S)$  is both nano open and nano closed in  $X$ . Therefore,  $f$  is nano perfectly continuous.

**Remark 4.10.** The converse of the above theorem need not be true.

**Example 4.11.** Let  $X = \{a, b, c, d\}$  Let  $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \tau^c = \{X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}\}$ ,  $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ ,  $\sigma^c = \{Y, \emptyset, \{b\}, \{b, c\}, \{a, b, d\}\}$ . Also,  $N\alpha^*$ AS-closed set in  $Y = \{Y, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}$ . Let  $f : X \rightarrow Y$  defined be identity map. Hence the inverse image of every nano closed set in  $Y$  is both nano closed and nano open in  $X$ . Thus,  $f$  is nano perfectly continuous but not nano perfectly  $\alpha^*$ AS-continuous since  $f^{-1}(a, b, c) = \{a, b, c\}$  is nano open but not nano closed in  $X$ . Hence,  $f$  is not nano perfectly  $\alpha^*$ AS-continuous.

**Theorem 4.12.** *A map  $f : X \rightarrow Y$  is nano perfectly  $\alpha^*$ AS-continuous if*

and only if  $f^{-1}(S)$  is both nano open and nano closed in  $(X, \tau)$  for every  $N\alpha^*AS$ -open set  $S$  in  $(Y, \sigma)$ .

**Proof.** Let  $S$  be any  $N\alpha^*AS$ -open set in  $Y$ . Then  $S^c$  is  $N\alpha^*AS$ -closed in  $Y$ . Since  $f$  is perfectly  $N\alpha^*AS$ -continuous,  $f^{-1}(S^c)$  is both nano open and nano closed in  $X$ . But  $f^{-1}(S^c) = X - f^{-1}(S)$  and so  $f^{-1}(S)$  is both nano open and nano closed in  $X$ . Conversely, assume that the inverse image of every  $N\alpha^*AS$ -closed set in  $Y$  is both nano open and nano closed in  $X$ . Let  $S$  be any  $N\alpha^*AS$ -open in  $Y$ . Then  $S^c$  is  $N\alpha^*AS$ -closed in  $Y$ . By assumption  $f^{-1}(S^c)$  is both nano open and nano closed in  $X$ . But  $f^{-1}(S^c) = X - f^{-1}(S)$  and so  $f^{-1}(S)$  is both nano open and nano closed in  $X$ . Therefore,  $f$  is nano perfectly  $\alpha^*AS$ -continuous.

**Theorem 4.13.** *If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are nano perfectly  $\alpha^*AS$ -continuous, then their composition  $g \circ f : X \rightarrow Z$  is also nano perfectly  $\alpha^*AS$ -continuous.*

**Proof.** Let  $S$  be a  $N\alpha^*AS$ -closed set in  $Z$ . Since  $g$  is nano perfectly  $\alpha^*AS$ -continuous, we get that  $g^{-1}(S)$  is both nano open and nano closed in  $Y$ . Then  $g^{-1}(S)$  is  $N\alpha^*AS$ -closed in  $Y$ . Since  $f$  is perfectly  $N\alpha^*AS$ -continuous,  $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$  is both nano open and nano closed in  $X$ . Hence,  $g \circ f$  is nano perfectly  $\alpha^*AS$ -continuous.

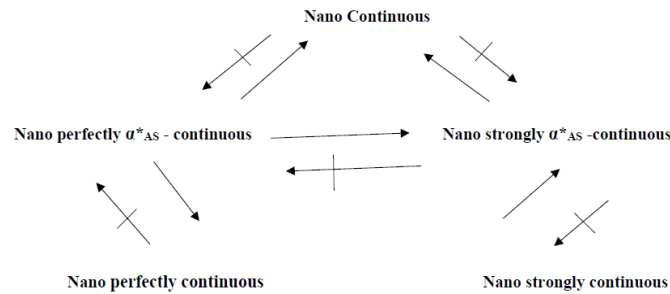
**Theorem 4.14.** *If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be any two maps. Then their composition is nano strongly  $\alpha^*AS$ -continuous if  $g$  is nano perfectly  $\alpha^*AS$ -continuous and  $f$  is nano continuous.*

**Proof.** Let  $S$  be any  $N\alpha^*AS$ -closed set in  $Z$ . Then,  $g^{-1}(S)$  is both nano open and nano closed in  $Y$ . Since  $f$  is nano continuous,  $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$  is nano closed in  $X$ . Hence,  $g \circ f$  is nano strongly  $\alpha^*AS$ -continuous.

**Theorem 4.15.** *If a map  $f : X \rightarrow Y$  is nano perfectly  $\alpha^*$ AS- continuous and a map  $g : Y \rightarrow Z$  is nano strongly  $\alpha^*$ AS- continuous then the composition  $g \circ f : X \rightarrow Z$  is nano perfectly  $\alpha^*$ AS- continuous.*

**Proof.** Let  $S$  be any  $N\alpha^*$ AS- closed set in  $Z$ . Then,  $g^{-1}(S)$  is nano closed in  $Y$ . Then  $g^{-1}(S)$  is  $N\alpha^*$ AS- closed in  $Y$ . By hypothesis,  $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$  is both nano open and nano closed in  $X$ . Therefore,  $g \circ f$  is nano perfectly  $\alpha^*$ AS- continuous.

**Remark 4.17.** From the above observations we have the following implications:



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