Advances and Applications in Mathematical Sciences
Volume 21, Issue 4, February 2022, Pages 1835-1841
© 2022 Mili Publications, India

DECOMPOSITION OF GRID GRAPHS $P_4 \square P_m$ AND $P_5 \square P_m$ INTO STARS

M. SUBBULAKSHMI and I. VALLIAMMAL

Associate Professor of Mathematics
G.V.N. College, Kovilpatti, Thoothukudi
Tamil Nadu-628502, India

Part-time Research Scholar
Assistant Professor (T) of Mathematics
Manonmaniam Sundaranar University
Tirunelveli-12, Tamil Nadu, India
Research centre: G.V.N College, Kovilpatti
Affiliated to Manonmaniam Sundaranar
University, Tirunelveli-12, India
E-mail: valli.vasanthi@gmail.com

Abstract

Let $G = (V, E)$ be a simple and finite graph. A grid graph $P_n \square P_m$ is a graph with vertex sets and edge sets by $V(P_n \square P_m) = \{u_{i,j} / 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(P_n \square P_m) = \{e_{i,j} = u_{i,j} u_{i,(j+1)} / 1 \leq i \leq n, 1 \leq j \leq m-1\} \cup \{f_{i,j} = u_{i,j} u_{(i+1),j} / 1 \leq i \leq n-1, 1 \leq j \leq m\}$. The decomposition of the grid graph is denoted by $D(P_n \square P_m)$. In this paper, we discuss the decomposition of the grid graph $P_n \square P_m$ into stars when $n = 4$ and 5.

1. Introduction

The decomposition of grid graphs $P_2 \square P_m$ and $P_3 \square P_m$ have already been discussed in [11]. In this paper, we discuss the decomposition of grid graph $P_m \square P_m$ into stars when $n = 4$ and 5.

2020 Mathematics Subject Classification: 05C70.
Keywords: Grid graph, decomposition, Cartesian product, corona product, stars.
Received October 23, 2021; Accepted November 28, 2021

2. Decomposition of the grid graph $P_4 \square P_m$ into stars

In this section, we investigate the decomposition of the grid graphs $P_4 \square P_m$ into stars.

Theorem 2.1. *The grid graph* $P_4 \square P_m$, $m \geq 3$ *can be decomposed into the following ways*:

$$D(P_4 \square P_m) = \begin{cases} (8d-3)S_3,\ 2dS_1\ and\ (d-1)S_2, & n = 4d-1,\ d = 1, 2, 3, \ldots \\ (8d-1)S_3,\ 2(d+1)S_1\ and\ dS_2, & n = 4d,\ d = 1, 2, 3, \ldots \\ 8dS_3\ and\ (2d+1)S_1\ and\ (d+1)S_2, & n = 4d+1,\ d = 1, 2, 3, \ldots \\ (8d+1)S_3,\ (2d+1)S_1\ and\ (d+3)S_2, & n = 4d+2,\ d = 1, 2, 3, \ldots \end{cases}$$

Proof. Let $V(P_n \square P_m) = \{u_{i,\,j}/1 \leq i \leq 4,\ 1 \leq j \leq m\}$ and $E(P_n \square P_m)$

$= \{e_{i,\,j}/1 \leq i \leq 4,\ 1 \leq j \leq m-1\} \cup \{f_{i,\,j} = u_{i,\,j}u_{(i+1),\,j}/\ 1 \leq i \leq 3,\ 1 \leq j \leq m\}$.

Case 1. If $n = 4d-1$, $d = 1, 2, 3, \ldots$

Claim. The grid graph $P_4 \square P_m$ is decomposable into $(8d-3)S_3$, $2dS_1$ and $(d-1)S_2$, $d = 1, 2, 3, \ldots$

By applying Lemma 4.1 [11], the grid graph $P_3 \square P_3$ is decomposed into $4S_3$. By applying Lemma 4.3 [11], the graph $P_3 \odot 2K_1$ is decomposed into $2S_3$ and S_2. By applying Lemma 4.2 [11], the comb graph $P_m \odot K_1$ is decomposed into $(2d-1)S_3$ and $2dS_1$, where $= 4d-1$, $d = 1, 2, 3, \ldots$.

Then, the grid graph $P_4 \square P_m$ consists of d copies of disjoint grid graph $P_3 \square P_3$, $(d-1)$ copies of disjoint graph $P_3 \odot 2K_1$ and a comb graph $P_m \odot K_1$. Therefore, the grid graph $P_4 \square P_m$ decomposes into d copies of $4S_3$, $(d-1)$ copies of $[2S_3$ and $S_2]$, and a copy of $[(2d-1)S_3$ and $2dS_1]$ where $m = 4d-1$, $d = 1, 2, 3, \ldots$. That is, the grid graph $P_4 \square P_m$ decomposes into $4dS_3 + (d-1)2S_3 + (d-1)S_2 + (2d-1)S_3 + 2dS_1$. Hence, the grid graph $P_4 \square P_m$ can be decomposed into $(8d-3)S_3$, $2dS_1$, and $(d-1)S_2$.

Case 2. If $n = 4d$, $d = 1, 2, 3, \ldots$.

Claim. The grid graph $P_4 \square P_m$ is decomposable into $(8d-2)S_3$, $2(d+1)S_1$ and dS_2, $d = 1, 2, 3, \ldots$.

By applying Lemma 4.1 [11], the grid graph $P_3 \square P_3$ is decomposed into $4S_3$. By applying Lemma 4.3 [11], the graph $P_3 \odot 2K_1$ is decomposed into $2S_3$ and S_2. By applying Lemma 4.2 [11], the comb graph $P_3 \odot K_1$ is decomposed into S_3 and $2dS_1$. By applying Lemma 4.2 [11], the comb graph $P_m \odot K_1$ is decomposed into $(2d-1)S_3$, $2dS_1$ and S_2, where $= 4d$, $d = 1, 2, 3, \ldots$.

Then, the grid graph $P_4 \square P_m$ consists of d copies of disjoint grid graph $P_3 \square P_3$, $(d-1)$ copies of disjoint graph $P_3 \odot 2K_1$, a comb graph $P_3 \odot K_1$, and a comb graph $P_3 \odot K_1$. Therefore, the grid graph $P_4 \square P_m$ decomposes into d copies of $4S_3$, $(d-1)$ copies of $[S_3$ and $S_2]$, a copy of $[S_3$ and $2dS_1]$, and a copy of $[(2d-1)S_3$, $2dS_1$ and $S_2]$ where $m = 4d$, $d = 1, 2, 3, \ldots$. That is, the grid graph $P_4 \square P_m$ decomposes into $4dS_3 + (d-1)2S_3 + (d-1)S_2 + S_3 + S_1 + (2d-1)S_3 + 2dS_1 + S_2$. Hence, the grid graph $P_4 \square P_m$ can be decomposed into $(8d-2)S_3$, $2(d+1)S_1$ and $2S_2$.

Case 3. If $n = 4d+1$, $d = 1, 2, 3, \ldots$.

Claim. The grid graph $P_4 \square P_m$ is decomposable into $8dS_3$, $(2d+1)S_1$ and $(d+1)S_2$, $d = 1, 2, 3, \ldots$.

By applying Lemma 4.1 [11], the grid graph $P_3 \square P_3$ is decomposed into $4S_3$. By applying Lemma 4.3 [11], the graph $P_3 \odot 2K_1$ is decomposed into $2S_3$ and S_2. By applying Lemma 4.2 [11], the comb graph $P_m \odot K_1$ is decomposed into $2dS_3$ and $(2d+1)S_1$ where $= 4d+2$, $d = 1, 2, 3, \ldots$.

Then, the grid graph $P_4 \square P_m$ consists of d copies of disjoint grid graph $P_3 \square P_3$, d copies of disjoint graph $P_3 \odot 2K_1$, a comb graph $P_m \odot K_1$, and a star S_2. Therefore, the grid graph $P_4 \square P_m$ decomposes into d copies of $4S_3$, d copies of $[2S_3$ and $S_2]$, a copy of $[2dS_3$, and $(2d+1)S_1]$ and a copy of star S_2, where $m = 4d+1$, $d = 1, 2, 3, \ldots$. That is, the grid graph $P_4 \square P_m$

decomposes into $4dS_3 + 2dS_3 + dS_2 + 2dS_3 + 2dS_1 + S_1 + S_2$. Hence, the grid graph $P_4 \square P_m$ can be decomposed into $8dS_3$, $(2d+1)S_1$, and $(d+3)S_2$.

Case 4. If $n = 4d + 2$, $d = 1, 2, 3, \ldots$.

Claim. The grid graph $P_4 \square P_m$ is decomposable into $(8d+1)S_3$, $(2d+1)S_1$ and $(d+3)S_2$, $d = 1, 2, 3, \ldots$.

By applying Lemma 4.1 [11], the grid graph $P_3 \square P_3$, is decomposed into $4S_3$. By applying Lemma 4.3 [11], the graph $P_3 \odot 2K_1$ is decomposed into $2S_3$ and S_2. By applying Lemma 4.2 [11], the comb graph $P_m \odot K_1$ is decomposed into $2dS_3$, $(2d+1)S_1$ and S_2 where $= 4d + 2$, $d = 1, 2, 3, \ldots$. By applying Theorem 3.1 [11], the ladder graph L_3 is decomposed into S_3, and $2S_2$.

Then, the grid graph $P_4 \square P_m$ consists of d copies of disjoint grid graph copies of disjoint graph $P_3 \odot 2K_1$, a comb graph $P_m \odot K_1$, and a ladder graph L_3. Therefore, the grid graph $P_4 \square P_m$ decomposes into d copies of $4S_3$, d copies of $[2S_3$ and $S_2]$, a copy of $[2dS_3, (2d+1)S_1$, and $S_2]$ and a copy of $[S_3$ and $2S_2]$, where $m = 4d + 2$, $d = 1, 2, 3, \ldots$. That is, the grid graph $P_4 \square P_m$ decomposes into $4dS_3 + 2dS_3 + dS_2 + 2dS_3 + 2dS_1 + S_1$ $+S_2 + S_3 + 2S_2$. Hence, the grid graph $P_4 \square P_m$ can be decomposed into $(8d+1)S_3$, $(2d+1)S_1$, and $(d+3)S_2$.

Illustration. Decomposition of a grid graph $P_4 \square P_{11}$ on case 1 is explained through the following Figure 1.

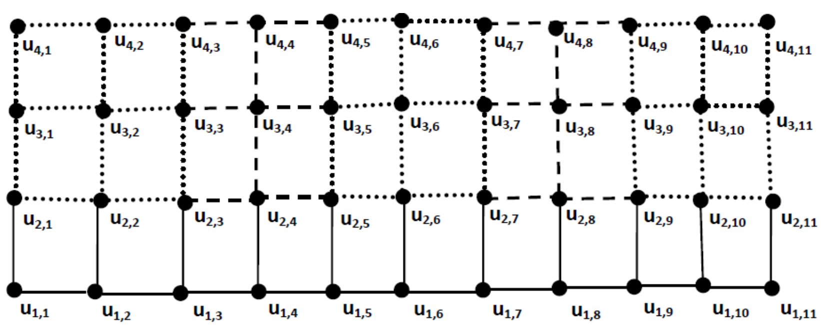

Figure 1. $P_4 \square P_{11}$.

Figure 1 represents the decomposition of the grid graph $P_4 \square P_{12}$ into 3 copies of disjoint grid graph $P_3 \square P_3$, 2 copies of disjoint graph $P_3 \odot 2K_1$ and a copy of comb graph $P_{11} \odot K_1$.

All edges of the grid graph $P_3 \square P_3$, the graph $P_3 \odot 2K_1$ and the comb graph $P_{11} \odot K_1$ are differentiated in Figure 1.

Illustration. Decomposition of a grid graph $P_4 \square P_{12}$ on case 2 is explained through the following Figure 2.

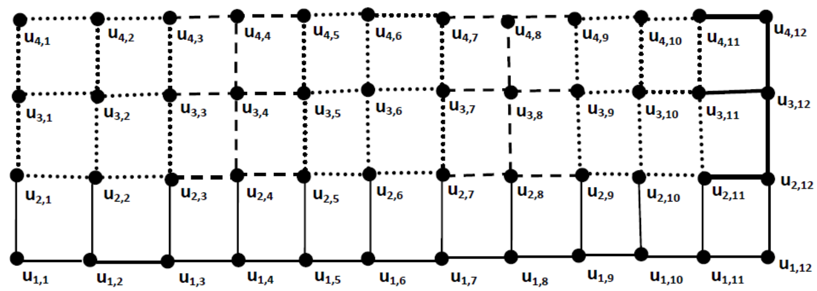

Figure 2. $P_4 \square P_{12}$.

Figure 2 represents the decomposition of the grid graph $P_4 \square P_{12}$ into 3 copies of disjoint grid graph $P_3 \square P_3$, 2 copies of disjoint graph $P_3 \odot 2K_1$, a copy of comb graph $P_3 \odot K_1$ and a copy of comb graph $P_{12} \odot K_1$.

All edges of the grid graph $P_3 \square P_3$, the graph $P_3 \odot 2K_1$, the comb graph $P_3 \odot K_1$, and the comb graph $P_{12} \odot K_1$ are differentiated in Figure 2.

3. Decomposition of the Grid Graph $P_5 \square P_m$ into Stars

In this section, we investigate the decomposition of the grid graph $P_5 \square P_m$ into stars.

Theorem 3.1. *The grid graph* $P_5 \square P_m$, $m \geq 3$ *can be decomposed into* $(3m - 5)$ *copies of disjoint stars* S_3 *and* 5 *copies of disjoint stars* S_2.

Proof. Let $V(P_n \square P_m) = \{u_{i,j} / 1 \leq i \leq 5, 1 \leq j \leq m\}$ and $E(P_n \square P_m)$

$= \{e_{i,\,j} / 1 \leq i \leq 5,\, 1 \leq j \leq m-1\} \cup \{f_{i,\,j} = u_{i,\,j} u_{(i+1),\,j} /\, 1 \leq i \leq 4,\, 1 \leq j \leq m\}$.

Claim. The grid graph $P_5 \square P_m$ is decomposable into $(3m-5)S_3$ and $5S_2$.

By applying Theorem 3.1 [11], the ladder graph L_m is decomposed into $(m-2)S_3$, and $2S_2$. By applying Lemma 4.3 [11], the graph $P_m \odot 2K_1$ is decomposed into $(m-1)S_3$ and S_2.

Then, the grid graph $P_5 \square P_m$ consists of 2 copies of disjoint ladder graph L_m and a copy of the graph $P_m \odot 2K_1$. Therefore, the grid graph $P_5 \square P_m$ decomposes into 2 copies of $[(m-2)S_3$, and $2S_2]$ a copy of $[(m-1)S_3$ and $S_2]$. That is, the grid graph $P_5 \square P_m$ decomposes into $2(m-2)S_3 + 4S_2$ $+(m-1)S_3 + S_2$. Hence, the grid graph $P_5 \square P_m$ can be decomposed into $(3m-5)S_3$ and $5S_2$.

Illustration. Decomposition of a grid graph $P_5 \square P_{11}$ is explained through the following Figure 3.

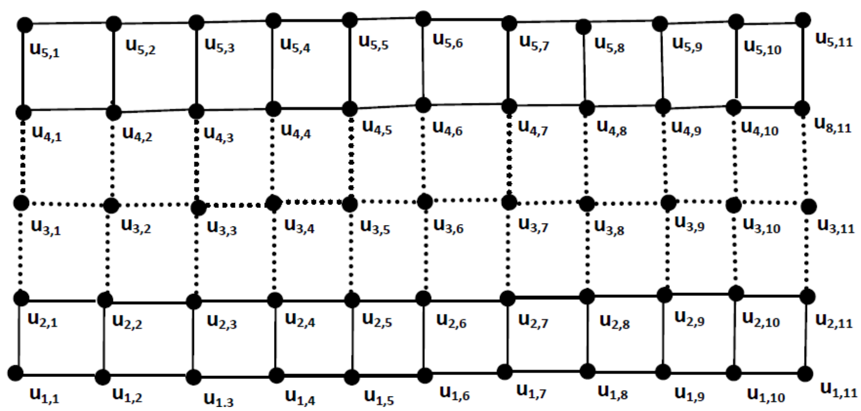

Figure 3. $P_5 \square P_{11}$.

Figure 3 represents the decomposition of the grid graph $P_5 \square P_{11}$ into 2 copies of disjoint ladder graph L_{11} and a graph $P_3 \odot 2K_1$.

All edges of ladder graph L_{11} and a graph $P_3 \odot 2K_1$ are differentiated in Figure 3.

References

[1] Tay-Woei Shyu, Decomposition of Complete Graphs into Paths and Cycles, ARS Combinatoria 97 (2010), 257-270.

[2] Tay-Woei Shyu, Decomposition of Complete Graphs into Paths and Stars, Discrete Mathematics 310 (2010), 2164-2169.

[3] S. Arumugam, I. Sahul Hamid and V. M. Abraham, Decomposition of graphs into Paths and Cycles, Journal of Discrete Mathematics 721051 (2013), 6.

[4] Tay-Woei Shyu, Decomposition of Complete Graphs into Cycles and Stars, in Graphs and Combinatorics 29 (2013), 301-313.

[5] L. T. Cherin Monish Femila and S. Asha, Hamiltonian decomposition of wheel related graphs, International Journal of Scientific Research and Review 7(11) (2018), 338-345.

[6] M. Sujitha, M. Sasikala and R. Mathivanan, A study on decomposition of graphs, Emerging Trends in Pure and Applied Mathematics (ETPAM-2018)-March, 2018.

[7] Hendy, A. N. Mudholifah and K. A. Sugeng, Martin Baca and Andrea semanicova fenovcikova, On H-antimagic decomposition of toroidal grids and triangulations, AKCE International Journal of Graphs and Combinatorics 01 June 2020.

[8] Jim Geelen, Bert Gerards and Geoff Whittle, Tangles, tree-decompositions and grids in matroids, Journal of Combinatorial Theory 8 April 2009.

[9] J. A. Bondy and U. S. R. Murty, Graph theory with applications, The Macmillan Press Ltd, New Yark, (1976).

[10] West, D.: Introduction to Graph Theory, 3rd edn. Prentice Hall, Saddle River, (2007).

[11] M. Subbulakshmi and I. Valliammal, Decomposition of Grid Graphs into stars, Proceedings of Icammct-2021, 30 June 2021.