Advances and Applications in Mathematical Sciences
Volume 18, Issue 10, August 2019, Pages 1173-1184

A STUDY ON FUZZY TRANSPORTATION PROBLEM USING HEXAGONAL FUZZY NUMBER

R. PAVITHRA and G. M. ROSARIO

Department of Mathematics
JA College, Periyakulam, India

Abstract

In this paper fuzzy transportation problem in which the values of transportation costs are represented as Hexagonal fuzzy number is considered. It has different optional ways to transport the goods. The aim of this study is to find the suitable defuzzification method to find the minimum transportation cost.

1. Introduction

In mathematics and economics, transportation theory is a name given to the study of optimal transportation and allocation of resources. The problem was formalized by the French mathematician Gaspard Monge in 1781. Unfortunately, understanding the process and interpreting the results are not easy tasks. The method is very complex.

Transportation problem deals with the problem of how to plan production and transportation in such as industry given several plans at different location and large number of customers of their product. The transportation problem deals with distribution of goods from several points, such as factories of ten known as sources, to a number of points of demand, such as warehouses of ten known as destination.

Each source is able to supply a fixed number of units of products, usually called the capacity or availability and each destination has a fixed demand usually known as requirements.

2010 Mathematics Subject Classification: 90B06.

Keywords: fuzzy transportation problem, hexagonal fuzzy number, defuzzification, transportation cost.

Received July 19, 2019; Accepted September 7, 2019

Because of its major application in solving problems which involving several product sources and several destination of products, this type of problem is frequently called "The Transportation Problem".

In recent times the demand and supply of any commodity cannot be fixed and due to various reasons it has been changing from time to time. This lead to variation in the transportation cost also. So this study concentrates on transportation problem where costs are given as fuzzy numbers. Fuzzy set is a mathematical model of vague qualitative or quantitative data, frequently generated by means of the natural language. The model is based on the generalization of the classical concepts of set and its characteristic function.

In particular fuzzy transportation problem in which the values of transportation costs are represented as Hexagonal fuzzy number is considered. It has different optional ways to transport the goods. The aim of this study is to find the suitable defuzzification method to find the minimum transportation cost.

Section 2 explains the basic definitions and derivation of different defuzzification methods which are relevant to this study. Hexagonal Fuzzy Transportation Problem is solved using various defuzzification techniques in Section 3. Conclusion of this study is given in Section 4.

2. Basic Definitions

2.1. In this section we give some basic definitions which are relevant to this study.

Definition 2.1. Let X be a non empty set. Then a fuzzy set A in X (i.e., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0, 1]$. Such a function μ_A is called the membership function and for each $x \in X$, $\mu_A(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A. In other words, A fuzzy set $A = \{(x, \mu_A(x)) | x \in X\}$ where $\mu_A : X \to [0, 1]$. The characteristic function $\mu_A(x)$ has only values 0(false) and 1(true). Such sets are crisp sets.

Definition 2.2. Let M be a fuzzy subset of set of real numbers $\mathbb{R}$. Then M is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \phi$ such that

$$\mu_M(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty) \end{cases}$$

where $l(x)$ is a function from $(-\infty, a)$ to $[0, 1]$ that is monotonic increasing, continuous from the right and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; $r(x)$ is a function from (b, ∞) to $[0, 1]$ that is monotonic decreasing, continuous from the left and such thatr $r(x) = 0$ for $x \in (\omega_2, \infty)$.

Definition 2.3 [10]**.** A fuzzy number A_H is a Hexagonal fuzzy number denoted by $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function $\mu_{A_H}(x)$ is given below.

$$\mu_{A_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right) & \text{for } a_5 \leq x \leq a_6 \\ 0 & \text{otherwise} \end{cases}$$

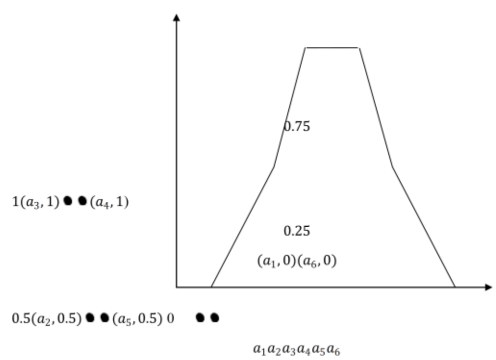

Figure 1. Graphical representation of a normal hexagonal fuzzy number for $x \in [0, 1]$.

2.2. Arithmetic operations on hexagonal fuzzy numbers [13]

Let $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $B_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers which performs the following three operations:

- Addition: $A_H + B_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
- Subtraction: $A_H - B_H = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$
- Multiplication: $A_H * B_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

2.3. Mathematical formulation of fuzzy transformation problem [8]

The fuzzy transportation problems, in which a decision maker is uncertain about precise value of transportation cost, availability and demand, can be formulated as follows Minimize $\widetilde{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij} \widetilde{x}_{ij}$. Subject to

$$\sum_{j=1}^{n} \widetilde{X}_{ij} \approx \widetilde{a}_i, \; i = 1, 2, 3, \dots m$$

$$\sum_{i=1}^{m} \widetilde{X}_{ij} \approx \widetilde{b}_j, \; j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^{m} \widetilde{a}_i \approx \sum_{j=1}^{n} \widetilde{b}_j, \; i = 1, 2, 3, \dots, m, \; j = 1, 2, 3, \dots, n \text{ and } \widetilde{x}_{ij} \geq 0,$$

where

m = total number of sources

n = total number of destinations

$\widetilde{a}_i$ = the fuzzy availability of the product at i^{th} source

$\widetilde{a}_i$ = the fuzzy demand of the product at j^{th} destination

$\widetilde{c}_{ij}$ = the fuzzy transportation cost for unit quantity of the product from i^{th} and j^{th} destination

$\widetilde{X}_{ij}$ = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m} \widetilde{a}_i$ = total fuzzy availability of the product

$\sum_{j=1}^{n} \widetilde{b}_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij} \widetilde{x}_{ij}$ = total fuzzy transportation cost

If $\sum_{i=1}^{m} \widetilde{a}_i \approx \sum_{j=1}^{n} \widetilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

2.4. There are many methods namely, North West Corner Method, Least Cost Method, Best Candidate Method, MODI Method, Matrix Minima Method, Zero Point Method and Vogel Approximation Method to solve Transportation Problem. Among these methods Vogel Approximation Method is considered in this study to find the minimum transportation cost.

2.5. Defuzzification Methods. Different Defuzzification methods are proposed. They are

- Total Integral Value Method (TI)
- Graded Mean Integration Representation Method (GMI)
- Ranking technique based on Centroid Method (CM)
- Ranking technique based on Centroid of Centroid Method (CC)

2.5.1. Total integral value of a hexagonal fuzzy number (TI). We define the Hexagonal Fuzzy Number,

$$\begin{Bmatrix} f_A^{L_1}(x) \\ f_A^{L_2}(x) \\ f_{A_H} = 1 \\ f_A^{R_1}(x) \\ f_A^{R_2}(x) \end{Bmatrix},$$

where

$$f_A^{L_1}(x) = \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right),\ f_A^{L_2}(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right),$$

$$f_A^{R_1}(x) = 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right),\ f_A^{R_2}(x) = \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right).$$

The inverse functions of $f_A^{L_1}(x), f_A^{R_1}(x), f_A^{L_2}(x), f_A^{R_2}(x)$ are $g_A^{L_1}(x), g_A^{L_2}(x), g_A^{R_1}(x), g_A^{R_2}(x)$ and its total integral value is defined as

$$R(TI_H) = \int_0^1 (g_A^{L_2}(y) + g_A^{L_2}(y) + g_A^{R_1}(y) + g_A^{R_2}(y))dy,$$

where

$$g_A^{L_1}(y) = 2y(a_2 - a_1) + a_1;\ g_A^{L_2}(y) = (2y - 1)(a_3 - a_1) + a_2;$$

$$g_A^{R_1}(y) = a_4 - (2y - 2)(a_5 - a_4);\ g_A^{R_2}(y) = a_6 - 2y(a_6 - a_5).$$

Solving the above integration, using MATLAB Program we get the Ranking of Hexagonal fuzzy number using Total Integral Value Method is

$$R(TI_H) = 2(a_2 + a_5). \tag{2.1}$$

2.5.2. Graded Mean Integration Ranking Method (GMI). Graded Mean Integration Representation Method is another method used to defuzzify Hexagonal fuzzy numbers. The defuzzified value $R(GMI_H)$ of the Graded Mean Integration Representation method is given by the following formula:

$$R(GMI_H) = \int_0^w y\left[\frac{g_A^L(y) + g_A^R(y)}{2}\right]dy \Big/ \int_0^w ydy$$

where

$$0 < h \leq w \text{ and } 0 < w \leq 1.$$

The inverse function of Hexagonal fuzzy number is $g_A^{L_1}(y)$, $g_A^{L_2}(y)$, $g_A^{R_1}(y)$, $g_A^{R_2}(y)$ and the value of these defined as above. If $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a HFN, then the Graded Mean Integration of A_H from the above formula is calculated as follows:

$$R(GMI_H) = \int_0^1 y\left[\frac{g_A^{L_1}(y) + g_A^{L_2}(y) + g_A^{R_1}(y) + g_A^{R_2}(y)}{2}\right]dy \Big/ \int_0^1 ydy.$$

Solving the above integration, using MATLAB Program we get the Ranking of Hexagonal fuzzy number using Graded Mean Integration Method

$$R(GMI_H) = \frac{7a_2 - 2a_1 + a_3 + a_4 + 6a_5 - a_6}{6}. \quad \text{(II)}$$

2.5.3. Ranking techniques based on centroid method (CM) [11]**.** The magnitude of a $HFN\ A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ using Ranking techniques based on CM is defined as

$$R(CM_H) = \left(\frac{7a_2 - 2a_1 + a_3 + a_4 6a_5 - a_6}{6}\right). \quad \text{(III)}$$

2.5.4. Ranking technique based on centroid of centroid method (CC) [6, 5]**.** The magnitude of a $HFN\ A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ using Ranking technique based on CC Method is defined as

$$R(CC_H) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 3a_5 + 2a_6}{18}, \frac{5}{18}\right). \quad \text{(IV)}$$

3. Numerical Example

In this Section, Fuzzy Transportation Problems (FTP) is solved using various defuzzification techniques illustrated in Section-2.

Example 1 [1]**.** Consider the following Fuzzy Transportation problem where supply and demand are given as Hexagonal Fuzzy Numbers:

	d_1	d_2	d_3	d_4	supply
o_1	(3,7,11,15,19,24)	(13,18,23,28,33,40)	(6,1320,28,36,45)	(15,20,25,31,38,45)	(6,8,11,14,19,25)
o_2	(16,19,24,29,34,39)	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(20,23,26,30,35)	(9,11,13,15,18,20)
o_3	(11,14,17,21, 25,30)	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14, 17)	(7,9,11,13,16,20
Demand	(3,4,5,6,8, 10)	(3,5,7,9,12,15)	(6,7,9,11,13, 16)	(10,12,14,16,20,24)	Balanced

(i) Total Integral Method:

We know $R(TI_H) = 2(a_2 + a_5)$.

By defuzzification, the given fuzzy transportation table (FTT) is reduced to crisp transportation table as follows:

	d_1	d_2	d_3	d_4	supply
o_1	52	102	98	116	54
o_2	106	30	48	116	58
o_3	78	54	20	42	50
demand	24	34	40	64	162

By applying Vogal Approximation Method,

	d_1	d_2	d_3	d_4	supply
o_1	52.86	234.97	38.5	29.33	120.69
	119.78		223.06	261.56	
o_2	244.14	75.78	54.39	261.56	130.17
		71.5	108.47	113.06	
o_3	177.22	119.47	46.44	60.91	113.06
demand	52.86	75.78	92.89	142.39	

Transportation cost =
$(20 \times 52) + (4 \times 106) + (34 \times 30) + (14 \times 116) + (50 \times 42) = 8128$ Units

(ii) Graded Mean Integration Method: We know $R(GMI_H) = \dfrac{7a_2 - 2a_1 + a_3 + a_4 + a_6}{6}$.

In this method we use VAM directly to FTP.

	d_1	d_2	d_3	d_4	supply	RP
O_1	(3,7,11,15,19,24)	(13,18,23,28,33,40)	(6,13,20,28,36,45)	(15,20,25,31,38,45)	(6,8,11,14,19,25)	(-18,-6,5,17,29,42)
O_2	(16,19,24,29,34,39)	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(20,23,26,30,35,40)	(9,11,13,15,18,20)	(-7,-3,1,6,12 ,18)
O_3	(11,14,17,21,25,30)	(7,9,11,14,18,22)	(2,3,4,6,7, 9)	(5,7,8,11,14,17)	(7,9,11,13,16,20)	(-4,0,2,7,11, 15)
demand	(3,4,5,6,8, 10)	(3,5,7,9,12,15)	(6,7,9,11,13,16)	(10,12,14,16,20,24)		
CP	(-13,-5,2,10,18,27)	(-5,-1,2,7,13,19)	(-4,0,4,9,14,19)	(3,9,15,22,28,35)		

$$x_{34} = (7,\ 9,\ 11,\ 13,\ 16,\ 20)$$

Similarly proceeding like this, $x_{22} = (3, 5, 7, 9, 12, 15)$, $x_{23} = (6, 7, 9, 11, 13, 16)$, $x_{14} = (-10, -4, 1, 5, 11, 17)$, $x_{13} = (-11, -6, 1, 7, 14, 22)$, $x_{11} = (3, 4, 5, 6, 8, 10)$.

Transportation Cost = (3, 4, 5, 6, 8, 10) (3, 7, 11, 15, 19, 24) + (– 11,–6, 1, 7, 14, 22) (6, 13, 20, 28, 36, 45) + (– 10, – 4, 1, 5, 11, 17) (15, 20, 25, 31, 38, 45) + (3, 5, 7, 9, 12, 15) (3, 5, 7, 9, 10, 12) + (6, 7, 9, 11, 13, 16) (5, 7, 10, 13, 17, 21) + (5, 7, 8, 11, 14, 17) (7, 9, 11, 13, 16, 20) = (9, 28, 55, 90, 152, 240) + (– 66,–78, 20, 196, 504, 990) + (– 150, – 80, 25, 155, 418, 765) + (9, 25, 49, 81, 120, 180) + (30, 49, 90, 143, 221, 336) + (35, 63, 88, 143, 224, 340) = (–133, 7, 327, 808, 1639, 2851) Units.

Using Graded Mean Integration Method, the above fuzzy transportation cost is defuzzified as

$$\frac{7(7) - 2(-133) + 327 + 808 + 6(1639) - 2851}{6} = 1405.5 \text{ Units}$$

$\therefore$ Transportation Cost =1405.5 Units

(iii) Ranking Technique based on Centroid Method: We know $R(TA)_H = \left(\frac{2a_1 + 4a_2 + 9a_3 + 9a_4 + 4a_5 + 2a_6}{6} \times \frac{11}{6}\right)$.

By defuzzification the given FTT is reduced to crisp transportation table as follows

	d_1	d_2	d_3	d_4	supply
o_1	119.78	234.97	223.06	261.56	120.69
o_2	244.14	71.5	108.47	261.56	130.17
o_3	177.22	119.47	46.44	60.91	113.06
demand	52.86	75.78	92.89	142.39	363.92

By applying Vogal Approximation Method

	d_1	d_2	d_3	d_4	supply
o_1	52.86	234.97	38.5	29.33	120.69
	119.78		223.06	261.56	
o_2	244.14	75.78	54.39	261.56	130.17
		71.5	108.47	113.06	
o_3	177.22	119.47	46.44	60.91	113.06
demand	52.86	75.78	92.89	142.39	

Transportation cost $= (52.86 \times 119.78) + (38.5 \times 223.06) + (29.33 \times 261.56)$

$+ (75.78 \times 71.5) + (54.39 \times 108.47) + (113.06 \times 60.91) = 40796.25$ Units.

(iv) Ranking of Centroid of Centroid Method: We know $R(CC_H) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 3a_4 + 3a_5 + 2a_6}{18} \times \frac{5}{18} \right)$.

By defuzzification the given FTT is reduced to crisp transportation table as follows

	d_1	d_2	d_3	d_4	supply
o_1	3.64	7.14	6.80	8	3.75
o_2	7.42	2.14	3.33	8	3.97
o_3	5.42	3.69	1.42	2.82	3.47
demand	1.64	2.33	2.84	4.38	11.19

By applying Vogal Approximation Method

	d_1	d_2	d_3	d_4	supply
o_1	1.64	7.14	1.2	0.91	3.75
	3.64		2.82	8	3.97
o_2	7.42	2.33	1.64	8	
		2.14	3.33		
o_3	5.42	3.69	1.42	3.47	3.47
				2.82	
demand	1.64	2.33	2.84	4.38	

Transportation cost $= (1.64 \times 3.64) + (1.25 \times 6.8) + (0.91 \times 8) + (2.33 \times 2.14)$ $+(1.64 \times 3.33) + (3.47 \times 2.82) = 41.6424$ Units. Therefore, transportation cost is given by as follows

Table

Method	Transportation Cost
Total Integral Value	8128
Graded Mean	1405.5
Centroid Method	40796.25
Incenter of Centroid	41.6424

From the example in section 3, we find that defuzzification using Ranking technique based on centroid of centroid method is giving the minimum transportation cost. Next suitable method will be Graded Mean Integration Representation Method and Ranking technique based on Centroid Method is not suitable for defuzzification to find the minimum transportation cost for Hexagonal Transportation problem.

References

[1] M. S. Annie Christi and D. Malini, Solving transportation problems with hexagonal fuzzy numbers using best candidates method and different ranking techniques, International Journal of Engineering Research and Applications 6 (2016), 76-81.

[2] K. Dhurai and A. Karpagam, A new ranking technique of hexagonal fuzzy numbers, International Journal of Science and Research (IJSR) 5 (2016), 533-535.

[3] D. Dubois and H. Prade, Fuzzy Sets and System, Academic Press, New York, 1998.

[4] D. R. Fulkerson, Hitchchock Transportation Problem, RAND Corporation, 1956.

[5] K. Ghadle and P. Pathade, Optimal solution of balanced and unbalanced fuzzy transportation problem using hexagonal fuzzy numbers, International Journal of Mathematical Research 5(2) (2016), 131-137.

[6] P. Kirtiwant, Ghadle and A. Pathede Priyanka, Solving transportation problem with generalized hexagonal and generalized octagonal fuzzy numbers by ranking method, Global Journal of Pure and Applied Mathematics 13 (2017), 6367-6376.

[7] P. Malini and M. Ananthanarayanan, Solving fuzzy transportation problem using ranking of octagonal fuzzy numbers, International Journal of Pure Applied Mathematics 110 (2016), 275-282.

[8] M. K. Purushothkumar, M. Ananthanarayanan and S. Dhanasekar, Solution to transportation problem in fuzzy environment with new ranking technique, International Journal of Scientific Research and Reviews 7(3) (2018), 638-650.

[9] C. C. Ragin, Fuzzy Set Social Science, University of Chicago Press, 2000.

[10] P. Rajarajeswari, A. Sahaya Sudha and R. Karthika, A new operation on hexagonal fuzzy number, International Journal of Fuzzy Logic Systems (IJFLS) 3 (2013), 15-26.

[11] A. Sahaya Sudha and M. Revathy, A new ranking on hexagonal fuzzy numbers, International Journal of Fuzzy Logic Systems (IJFLS) 6 (2016), 1-8.

[12] S. Rao Singiresu, Engineering Optimization Theory and Practice (4th), John Wiley and Sons, p.221, 2009.

[13] A. Thamaraiselvi and R. Santhi, Optimal solution of fuzzy transportation problem using hexagonal fuzzy number, International Journal of Scientific & Engineering Research 6 (2015), 40-45.

[14] M. Thamaraiselvi and B. Srividhya, A new method for solving fuzzy transportation problem using dodecagonal fuzzy numbers, International Journal for Modern Trends in Science and Technology 03 (2017), 155-158.

[15] L. A. Zadeh, Fuzzy Sets Information and Control 8(3) (1965), 338-353.