**Advances and Applications in Mathematical Sciences**
Volume 22, Issue 1, November 2022, Pages 43-56


# EXTENDED FIFTH ORDER METHODS FOR EQUATIONS UNDER THE SAME CONDITIONS

**IOANNIS K. ARGYROS, SANTHOSH GEORGE and CHRISTOPHER ARGYROS**

Department of Mathematical Sciences
Cameron University
Lawton, OK 73505, USA
E-mail: iargyros@cameron.edu

Department of Mathematical
and Computational Sciences
National Institute of Technology
Karnataka, India-575 025
E-mail: sgeorge@nitk.edu.in

Department of Computing Science
University of Oklahoma
Norman, OK 73071, USA
E-mail: Christopher.I.Argyros-1@ou.edu

## Abstract

In this paper we consider fifth order methods for solving nonlinear equations in Banach spaces under the same conditions. We use only the first derivatives to extend the usage of these methods, where earlier papers require up to the sixth derivative for convergence. Numerical examples complete the article.

## 1. Introduction

Let $F : D \subset X \to Y$ is continuously Frechet differentiable, $X, Y$ are Banach spaces and $D$ is a nonempty convex set. Consider the problem of solving equation

2020 Mathematics Subject Classification: 65H10, 41A25, 65J10.
Keywords: Frechet derivative, Banach space, Convergence order.
Received June 14, 2021; Accepted October 28, 2021

$$F(x) = 0. \tag{1.1}$$

Iterative methods are used to approximate solution of equations of the type (1.1), since finding an exact solution is possible only in rare cases.

In this paper we study the convergence of two fifth order methods under the same conditions.

The methods we are interested are:

By J. R. Sharma et al. [14]

$$\begin{aligned} y_n &= x_n - \frac{1}{2} F'(x_n)^{-1} F(x_n) \\ z_n &= x_n - F'(y_n)^{-1} F(x_n) \\ x_{n+1} &= z_n - (2F'(y_n)^{-1} - F'(x_n)^{-1}) F(z_n) \end{aligned} \tag{1.2}$$

By A. Cordero et al. [6]

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1} F(x_n) \\ z_n &= x_n - 2A_n^{-1} F(x_n) \\ x_{n+1} &= z_n - F'(y_n)^{-1} F(z_n) \end{aligned} \tag{1.3}$$

where $A_n = F'(y_n) + F'(x_n)$.

The efficiency and convergence order was given in [6] (see also [14]) using conditions up to the sixth derivative, restricting the applicability of these algorithms.

For example: Let $X = Y = R$, $D = \left[-\frac{1}{2}, \frac{3}{2}\right]$. Define $f$ on $D$ by

$$f(t) = \begin{cases} t^3 \log t^2 + t^5 - t^4 & \text{if } t \neq 0 \\ 0 & \text{if } t = 0. \end{cases}$$

Then, we have $t_* = 1$, and

$$f'''(t) = 6 \log t^2 + 60t^2 - 24t + 22.$$

Obviously $f'''(t)$ is not bounded on $D$. So, the convergence of these methods is not guaranteed by the analysis in earlier papers [1-15]. Our convergence analysis is based on the first Frechet derivative that only appears on the method.

We also provide a computable radius of convergence also not given in [6, 14]. This way we locate a set of initial points for the convergence of the method. The numerical examples are chosen to show how the radii theoretically predicted are computed. In particular, the last example shows that earlier results cannot be used to show convergence of the method. Our results significantly extends the applicability of these methods and provide a new way of looking at iterative methods.

The article contains local convergence analysis in Section 2 and the numerical examples in Section 3.

## 2. Local Convergence

In this section the convergence of methods (1.2) and (1.3) is given. Set $G = [0, \infty)$.

Suppose function:

(a)

$$\xi_0(t) - 1$$

has a least zero $r_0 \in G - \{0\}$ for some function $\xi_0 : G \to G$ nondecreasing and continuous. Set $G_0 = [0, r_0)$.

(b)

$$\zeta_1(t) - 1$$

has a least zero $\gamma_1 \in G_0 - \{0\}$ for some function $\xi_1 : G_0 \to G$ which is nondecreasing and continuous and function $\zeta_1 : G_0 \to G$ defined by

$$\zeta_1(t) = \frac{\int_0^1 \xi_0((1-\theta)t)d\theta + \frac{1}{2}\int_0^1 \xi_1(\theta t)d\theta}{1 - \xi_0(t)}.$$

(c)

$$p(t)-1$$

has a least zero $r_1 \in G_0 - \{0\}$, where $p(t) = \frac{1}{2}(\xi_0(t) + \xi_0(\zeta_1(t)t))$. Set $r_2 = \min\{r_0, r_1\}$ and $G_1 = [0, r_2)$.

(d)

$$\zeta_2(t)-1$$

has a least zero $\gamma_2 \in G_0 - \{0\}$, for some function $\xi : G_1 \to G$ nondecreasing and continuous, and function $\zeta_2 \in G_1 \to G$ defined as

$$\zeta_2(t) = \frac{\int_0^1 \xi((1-\theta)t)\,d\theta}{1-\xi_0(0)} + \frac{(\xi_0(t) + \xi_0(\zeta_1(t)t))\int_0^1 \xi_1(\theta t)\,d\theta}{2(1-\xi_0(t))(1-p(t))}.$$

(e)

$$\xi_0(\zeta_2(t)t)-1$$

has a least zero $r_3 \in G_1 - \{0\}$. Set $r = \min\{r_2, r_3\}$ and $G_2 = [0, r)$.

$$\zeta_3(t(-1))$$

has a least zero $\gamma_3 \in G_2 = \{0\}$, where $\zeta_3 : G_2 \to G$ is defined as

$$\zeta_3(t) = \left[\frac{\int_0^1 \xi((1-\theta)\zeta_2(t)t)\,d\theta}{1-\xi_0(\zeta_2(t)t)} + \frac{(\xi_0(\zeta_1(t)t) + \xi_0(\zeta_2(t)t))\int_0^1 \xi_1(\theta\zeta_2(t)t)d\theta}{(1-\xi_0(\zeta_1(t)t))(1-\xi_0(\zeta_2(t)t))}\right]\zeta_2(t).$$

Set

$$\gamma = \min\{\gamma_i\},\ i = 1, 2, 3. \tag{2.1}$$

It shall be proven that $\gamma$ defined by (2.23) is a convergence radius for method (1.2).

By $\overline{B}(x, \delta)$ we denote the closure of the open ball $B(x, \delta)$ with center $x \in X$ and of radius $\delta > 0$.

The conditions (C) are needed provided that $x_*$ is a simple solution of equation (1.1), and functions "ξ" are as previously defined. Suppose:

(c1) For each $x \in D$,

$$\| F'(x_*)^{-1}(F'(x) - F'(x_*)) \| \leq \xi_0(\| x - x_* \|).$$

Set $D_1 = D \cap B(x_*, r_*)$.

(c2) For each $x, y \in D_0$.

$$\| F'(x_*)^{-1}(F'(x) - F'(y)) \| \leq \xi(\| x - y \|)$$

and

$$\| F'(x_*)^{-1} F'(x) \| \leq \xi_1(\| x - x_* \|).$$

(c3) $\overline{B}(x_*, \delta) \subset D,$ for some $\delta > 0$ to be given later.

(c4) There exists $\gamma_* \geq \gamma$ satisfying $\int_0^1 \xi_0(\theta\gamma_*)\, d\theta < 1$.

Set $D_1 = D \cap B(x_*, \gamma_*)$.

Next, condition (C) are used to show the local convergence result for method (1.2).

**Theorem 2.1.** *Suppose conditions* (C) *hold with* $\delta = \gamma$. *Then, if* $x_0 \in B(x_*, \bar{\gamma}) - \{x_*\}$, $\lim_{n \to \infty} x_n = x_*$, *which is the only solution in the region* $D_1$ *of equation* $F(x) = 0$.

**Proof.** Let $d_n = d_n$. We based our proof on the verification of items

$$\| y_n - x_* \| \leq \zeta_1(d_n) d_n \leq d_n < \gamma, \tag{2.2}$$

$$\| z_n - x_* \| \leq \zeta_2(d_n) d_n \leq d_n, \tag{2.3}$$

and

$$d_{n+1} \leq \zeta_3(d_n) d_n \leq d_n, \tag{2.4}$$

to be shown using mathematical induction. Set $G_3 = [0, \gamma)$. The definition of implies that for all $t \in G_3$

$$0 \le \xi_0(t) < 1, \tag{2.5}$$

$$0 \le p(t) < 1, \tag{2.6}$$

$$d_{n+1} \le \zeta_3(d_n) d_n \le d_n, \tag{2.7}$$

and

$$0 \le \zeta_i(t) < 1. \tag{2.8}$$

Pick $u \in \overline{B}(x_*, \gamma) - \{x_*\}$. Then, by (2.23), (2.5) and (c1), we have

$$\| F'(x_*)^{-1}(F'(u) - F'(x_*)) \| \le \xi_0(\| u - x_* \|) \le \xi_0(\gamma) < 1, \tag{2.9}$$

so

$$\| F'(u)^{-1} F'(x_*) \| \le \frac{1}{1 - \xi_0(\| u - x_* \|)} \tag{2.10}$$

by a lemma due to Banach on invertible operators [2]. Notice also that iterate $y_0$ is well defined, and we can also write

$$y_0 - x_* = x_0 - x_* - F'(x_0)^{-1} F(x_0) + \frac{1}{2} F'(x_0)^{-1} F(x_0)$$

$$= (F'(x_0)^{-1} F'(x_*))$$

$$\times \int_0^1 F'(x_*)^{-1}(F'(x_* + \theta(x_0 - x_*)) - F'(x_0) d\theta)(x_0 - x_*) \tag{2.11}$$

$$+ \frac{1}{2}(F'(x_0)^{-1} F(x_*)) F'(x_*)^{-1} F(x_0) \tag{2.12}$$

By (2.23), (2.5), (2.10) (for $u = x_0$), (c2), (c3), (2.8) (for $i = 1$, and (2.12), we get in turn that

$$\| y_0 - x_* \| \le \frac{\int_0^1 \xi((1 - \theta) d_0) d\theta + \int_0^1 \xi_1(\theta d_0) d\theta}{1 - \xi_0(d_0)}$$

$$\le \zeta_1(d_0) d_0 \le d_0 < \gamma, \tag{2.13}$$

showing $y_0 \in \overline{B}(x_*, \gamma)$ and (2.2) for $n = 0$. Next, we show $A_0^{-1} \in L(Y, X)$. Indeed, by (2.23), (2.6), (2.14) and (C1), we have

$$\| (2F'(x_*))^{-1}(A_0 - 2F'(x_*)) \| \leq \frac{1}{2}\| F'(x_*)^{-1}(F'(y_0) - F'(x_*)) \| + \| F'(x_*)^{-1}(F'(x_0) - F'(x_*)) \| \leq \frac{1}{2}(\xi_0(\| y_0 - x_* \|) + \xi_0(d_0)) \leq p(d_0) \leq p(\gamma) < 1,$$

so

$$\| A_0^{-1}F'(x_*) \| \leq \frac{1}{2(1 - p(d_0))}. \tag{2.15}$$

Moreover, iterate $z_0$ is well defined by the second substep of method (1.2), and we can also write

$$\begin{aligned} z_0 - x_* &= x_0 - x_* - F'(x_0)^{-1}F(x_0) + (F'(x_0)^{-1} - 2A_0^{-1})F(x_0) \\ &= x_0 - x_* - F'(x_0)^{-1}F(x_0) + F'(x_0)^{-1}(A_0 - 2F'(x_0))A_0^{-1}F(x_0) \\ &= x_0 - x_* - F'(x_0)^{-1}F(x_0) \\ &\quad + F'(x_0)^{-1}(F'(y_0) - F'(x_0))F'(y_0)^{-1}F(x_0). \end{aligned} \tag{2.16}$$

Using (2.23), (2.8) (for $1 = 2$), (2.10) (for $u = x_0$), and (2.14)-(2.16), we get

$$\| z_0 - x_* \| \leq \left[ \frac{\int_0^1 \xi((1-\theta)d_\theta)d\theta}{1 - \xi_0(d_0)} + \frac{(\xi_0(d_0) + \xi_0(\| y_0 - x_* \|))\int_0^1 \xi_1(\theta\| x_0 - x_* \|)d\theta}{2(1 - p(d_0))(1 - \xi_0(d_0))} \right] d_0 \leq \zeta_2(d_0)d_0 \leq d_0, \tag{2.17}$$

showing (2.3) for $n = 0$ and $z_0 \in B(x_*, \gamma)$. Notice that iterate $x_1$ is well defined by the third substep of method (1.2), $F'(z_0)^{-1} \in L(Y, X)$ by (2.10) (for $u = z_0$), and we can write

$$\begin{aligned} x_1 - x_* &= z_0 - x_* - F'(z_0)^{-1} F(z_0) + (F'(z_0)^{-1} - F'(y_0)^{-1}) F(z_0) \\ &= z_0 - x_* - F'(z_0)^{-1} F(z_0) \\ &\quad + F'(z_0)^{-1}(F'(y_0) - F'(z_0)) F'(y_0)^{-1} F(z_0). \end{aligned} \tag{2.18}$$

In view of (2.23), (2.8) (for $i = 3$), (2.10) (for $u = y_0, z_0$), (2.14), (2.17) and (2.18), we get

$$\begin{aligned} d_1 &\le \Bigg[ \frac{\int_0^1 \xi((1-\theta)\| z_0 - x_* \|) d\theta}{1 - \xi_0(\| z_0 - x_* \|)} \\ &\quad + \frac{(\xi_0(\| y_0 - x_* \|) + \xi_0(\| z_0 - x_* \|)) \int_0^1 \xi_1(\theta \| z_0 - x_* \|) d\theta}{(1 - \xi_0(\| y_0 - x_* \|))(1 - \xi_0(\| z_0 - x_* \|))} \Bigg] \| z_0 - x_* \| \\ &\le \zeta_3(d_0) d_0 \le d_0, \end{aligned} \tag{2.19}$$

showing (2.4) for $n = 0$ and $x_1 \in B(x_*, \gamma)$. Exchanging $x_0, y_0, z_0, x_1$ by $x_i, y_i, z_i, x_{i+1}$ in the previous calculations to complete the induction for (2.2)-(2.4). It then follows from the estimation

$$d_{i+1} \le \beta d_i < \gamma, \tag{2.20}$$

where $\beta = \zeta_3(d_0) \in [0, 1)$, that $\lim_{i \to \infty} x_i = x_*$, and $x_{i+1} \in B(x_*, \gamma)$.

Consider $T = \int_0^1 F'(x_* + \theta(v - x_*)) d\theta$ for some $v \in \Omega_1$ with $F(v) = 0$. Then, using (c1) and (c4), we obtain

$$\| F'(x_*)^{-1}(T - F'(x_*)) \| \le \int_0^1 \xi_0(\theta \| v - x_* \|) d\theta \le \int_0^1 \xi_0(\theta \gamma_*) d\theta < 1,$$

so $v = x_*$ follows from the identity $0 = F(v) - F(x_*) = T(v - x_*)$ and the invert ability of $T$. □

**Remark 2.2.** 1. In view of (c2) and the estimate

$$\| F'(x^*)^{-1} F'(x) \| = \| F'(x^*)^{-1}(F'(x) - F'(x^*)) + I \|$$

$$\leq 1 + \| F'(x^*)^{-1}(F'(x) - F'(x^*)) \| \leq 1 + \xi_0(\| x - x^* \|)$$

the second condition in (c3) can be dropped and $\omega_1$ can be replaced by

$$\xi_1(t) = 1 + \xi_0(t)$$

or

$$\xi_1(t) = 1 + \xi_0(\gamma), \text{ or } \xi_1(t) = 2,$$

since $t \in [0, r_0)$.

2. The results obtained here can be used for operators $F$ satisfying autonomous differential equations [2] of the form

$$F'(x) = P(F(x))$$

where $P$ is a continuous operator. Then, since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing $x^*$. For example, let $F(x) = e^x - 1$. Then, we can choose: $P(x) = x + 1$.

3. Let $\xi_0(t) = L_0 t$, and $\xi(t) = Lt$. In [2, 3] we showed that $r_A = \frac{2}{2L_0 + L}$ is the convergence radius of Newton's method:

$$x_{n+1} = x_n - F'(x_n)^{-1} F(x_n) \text{ for each } n = 0, 1, 2, \ldots \tag{2.21}$$

under the conditions (c1)-(c3). It follows from the definition of $r_A$, that the convergence radius $\rho$ of the method (1.2) cannot be larger than the convergence radius $r_A$ of the second order Newton's method (2.21). As already noted in [2, 3] $r_A$ is at least as large as the convergence radius given by Rheinboldt [10]

$$r_R = \frac{2}{3L}, \tag{2.22}$$

where $L_1$ is the Lipschitz constant on $D$. The same value for $r_R$ was given by

Traub [12]. In particular, for $L_0 < L_1$ we have that

$$r_R < r_A$$

and

$$\frac{r_R}{r_A} \to \frac{1}{3} \text{ as } \frac{L_0}{L_1} \to 0.$$

That is the radius of convergence $r_A$ is at most three times larger than Rheinboldt.

4. We can compute the computational order of convergence (COC) defined by

$$\xi = \ln\left(\frac{d_{n+1}}{d_n}\right) / \ln\left(\frac{d_n}{d_{n-1}}\right)$$

or the approximate computational order of convergence

$$\xi_1 = \ln\left(\frac{s_{n+1}}{s_n}\right) / \ln\left(\frac{s_n}{s_{n-1}}\right),$$

where $s_n = \| x_n - x_{n-1} \|$.

Next, we present the local convergence analysis of method (1.3) in an analogous way. But this time functions $\zeta_i$ are replaced $\bar{\zeta}_i$, respectively. And $\bar{\zeta}_i$ are defined by

$$\bar{\zeta}_1(t) = \frac{\int_0^1 \xi((1-\theta)t)d\theta}{1-\xi_0(t)},$$

$$\bar{\zeta}_2(t) = \frac{\int_0^1 \xi(1-\theta)td\theta}{1-\xi_0(t)} + \frac{(\xi_0(t) + \xi_0(\zeta_1(t)t))\int_0^1 \xi_1(\theta t)d\theta}{(1-\xi_0(t))(1-\xi_0(\zeta_1(t)t))},$$

$$\bar{\zeta}_3(t) = \left[ \frac{\int_0^1 \xi((1-\theta)\zeta_1(t)t)d\theta}{1-\xi_0(\zeta_1(t)t)} + \frac{(\xi_0(t) + \xi_0(\zeta_1(t)t))\int_0^1 \xi_1(\theta\zeta_2(t)t)d\theta}{(1-\xi_0(\zeta_1(t)t))(1-\xi_0(\zeta_2(t)t))} \right.$$

$$+\frac{(\xi_0(\zeta_2(t)t)+\xi_0(\zeta_2(t)t))\int_0^1\xi_1(\theta\zeta_2(t)t)d\theta}{(1-\xi_0(\zeta_1(t)t))(1-\xi_0(\zeta_2(t)t))}\Bigg]\zeta_2(t)$$

and

$$\bar{\gamma}=\min\{\bar{\gamma}_i\}, \tag{2.23}$$

where $\bar{\gamma}_i$ are the least positive zeros of functions $\bar{\zeta}_i(t)-1$ in $\Omega_0$ (provided they exist). These functions are motivated by the estimates (obtained under the conditions (C) for $\delta=\bar{\gamma}$):

$$\|y_n-x_*\|=\|x_n-x_*-F'(x_n)^{-1}F(x_n)\leq\frac{(\int_0^1\xi((1-\theta)d_n)d\theta}{1-\xi_0(d_n)}$$

$$\leq\bar{\zeta}_1(d_n)d_n\leq d_n<\bar{\gamma},$$

$$\|y_n-x_*\|=\|x_n-x_*-F'(x_n)^{-1}F(x_n)+F'(x_n)^{-1}(F'(y_n)$$

$$-F'(x_n))F'(y_n)^{-1}F(x_n)\|\leq\Bigg[\frac{\int_0^1\xi_0((1-\theta)d_n)d\theta}{1-\xi_0(d_n)}$$

$$+\frac{(\xi_0(d_n)+\xi_0(\|y_n-x_*\|))\int_0^1\xi_1(\theta d_n)d\theta}{(1-\xi_0(d_n))(1-\xi_0(\|y_n-x_*\|))}\Bigg]d_n\leq\bar{\zeta}_2(d_n)d_n\leq d_n,$$

and

$$d_{n+1}=\|z_n-x_*-F'(z_n)^{-1}F(z_n)+(F'(y_n)^{-1}-F'(x_n)^{-1})F(z_n)$$

$$+(F'(y_n)^{-1}-F'(z_n)^{-1})F(z_n)\|$$

$$\leq\Bigg[\frac{\int_0^1\zeta((1-\theta)\|z_n-x_*\|)d\theta}{1-\xi_0(\|z_n-x_*\|)}+\frac{(\xi_0(d_n)+\xi_0(\|y_n-x_*\|))\int_0^1\xi_1(\theta\|z_n-x_*\|)d\theta}{(1-\xi_0(\|y_n-x_*\|))(1-\xi_0(\|z_n-x_*\|))}$$

$$+ \frac{(\xi_0(\| y_n - x_* \|) + \xi_0(\| z_n - x_* \|))\int_0^1 \xi_1(\theta\| z_n - x_* \|)d\theta}{(1 - \xi_0(\| y_n - x_* \|))(1 - \xi_0(\| z_n - x_* \|))}\Bigg] \| z_n - x_* \|$$

$$\leq \bar{\zeta}_3(d_n)d_n \leq d_n.$$

Hence, we arrive at the corresponding local convergence analysis for method (1.3).

**Theorem 2.3.** *Suppose that the conditions* (*C*) *hold with* $\delta = \bar{\gamma}$. *Then, the canclutions of Theorem* 2.1 *hold for method* (1.3) *with* $\bar{\gamma}, \bar{\zeta}_i$ *replacing* $\gamma$ *and* $\zeta_1$, *respectively*.

## 3. Numerical Examples

**Example 3.1.** Consider the kinematic system

$$F_1'(x) = e^x,\ F_2'(y) = (e-1)y + 1,\ F_3'(z) = 1$$

with $F_1(0) = F_2(0) = F_3(0) = 0$. Let $F = (F_1, F_2, F_3)$. Let $X = Y = \mathbb{R}^2$, $D = \bar{B}(0, 1)$, $x_* = (0, 0, 0)^t$. Define function $F$ on $D$ for $w = (x, y, z)t$ by

$$F(w) = \left(e^x - 1, \frac{e-1}{2} y^2 + y, z\right)^t.$$

Then, we get

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

so $\xi_0(t) = (e-1)t$, $\xi(t) = e^{\frac{1}{e-1}}t$, $\xi_1(t) = \frac{e-1}{2}$. Then, the radii are

$$r_1 = 0.221318,\ r_2 = 0.207758,\ r_3 = 0.193106.$$

$$\bar{r}_1 = 0.382692,\ \bar{r}_2 = 0.165361,\ \bar{r}_3 = 0.164905.$$

## 4. Conclusions

In this paper, we have considered two fifth order algorithms for solving systems of nonlinear equations. A comparison between the ball of convergence is provided using conditions on the derivative. Earlier studies have used hypotheses up to the sixth derivative. We also provide error estimates and uniqueness results not given before [6, 14], our idea can extend the usage of other methods too [1, 1-10, 12-15].

## References

[1] S. Amat, M. A. Hernandez and N. Romero, Semilocal convergence of a sixth order iterative method for quadratic equations, Applied Numerical Mathematics 62 (2012), 833-841.

[2] I. K. Argyros, Computational Theory of Iterative Methods, Series: Studies in Computational Mathematics, 15, Editors: Chui C. K. and Wuytack L. Elsevier Publ. Company, New York, (2007).

[3] I. K. Argyros and A. A. Magrenan, Iterative method and their dynamics with applications, CRC Press, New York, USA, 2017.

[4] R. Behl, A. Cordero, S. S. Motsa and J. R. Torregrosa, Stable high order iterative methods for solving nonlinear models, Appl. Math. Comput. 303 (2017), 70-88.

[5] A. Cordero, J. L. Hueso, E. Martinez and J. R. Torregrosa, A modified Newton-Jarratt's composition, Numer. Algor. 55 (2010), 87-99.

[6] A. Cordero, J. L. Hueso, E. Martinez and J. R. Torregrosa, Increasing the convergence order of an iterative method for nonlinear systems, Appl. Math. Lett. 25 (2012), 2369-2374.

[7] M. T. Darvishi and A. Barati, A fourth order method from quadrature formulae for solving systems of nonlinear equations, Appl. Math. Comput. 188 (2007), 257-261.

[8] A. A. Magrenan, Different anomalies in a jarratt family of iterative root finding methods, Appl. Math. Comput. 233 (2014), 29-38.

[9] M. A. Noor and M. Wassem, Some iterative methods for solving a system of nonlinear equations, Appl. Math. Comput. 57 (2009), 101-106.

[10] W. C. Rheinboldt, An adaptive continuation process for solving systems of nonlinear equations, In: Mathematical Models and Numerical methods (A. N. Tikhonov et al. eds.) pub. Banach Center, Warsaw Poland 3 (1977), 129-142.

[11] J. R. Sharma and S. Kumar, A class of computationally efficient new tonlike methods with frozen operator for nonlinear systems, International Journal of Nonlinear Sciences and Numerical Simulation.

[12] J. F. Traub, Iterative Methods for the Solution of Equations, Prentice-Hall, Englewood Cliffs, (1964).

[13] J. R. Sharma and H. Arora, Improved Newton-like methods for solving systems of nonlinear equations, SeMA 74 (2017), 147-163.

[14] J. R. Sharma and P. Gupta, An efficient fifth order method for solving systems of nonlinear equations, Computer and Mathematics with Applications 67 (2014), 591-601.

[15] S. Weerakoon and T. G. I. Fernando, A variant of Newton's method with accelerated third-orde rconvergence, Appl. Math. Lett. 13 (2000), 87-93.