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# ISOMORPHIC PROPERTIES OF FUZZY SEMIGRAPHS

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## Abstract

In this paper, isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced and some of their properties are studied. End vertex isomorphism (ev-isomorphism), edge isomorphism (e-isomorphism) and adjacency isomorphism (a-isomorphism) of fuzzy semigraphs are defined. Properties of effective edges and effective fuzzy semigraphs under isomorphism are studied. Also, it is proved that isomorphism is an equivalence relation and week isomorphism is a partial order relation.

## 1. Introduction

In the year 1975 Rosenfeld introduced the theory of fuzzy graphs. Characteristics of fuzzy graphs were dealt by him. Some wonderful results and remarks on fuzzy graphs were contributed by Bhattacharya. Some operations on fuzzy graphs were defined by Modeson and J. N. Peng. A. Nagoor Gani and K. Radha studied the regularity properties of fuzzy graphs. The concept of semigraph was introduced by E. Sampath Kumar. The

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Concept of fuzzy semigraphs was introduced by K. Radha and P. Renganathan, K. Radha and P. Renganathan defined effective fuzzy semigraph and studied some properties of it. The order, size and the degree of isomorphic fuzzy graphs were studied by A. Nagoor Gani and J. Malarvizhi. The degree sequence and the degree set of fuzzy graphs and their properties were studied by K. Radha and A. Rosemine. Fuzzy K. Radhaand P. Renganathan semigraphs have wide range of applications in Railway network, Road network, telecommunication system, etc. In this paper isomorphism, weak isomorphism and co-weak isomorphism of fuzzy semigraphs are introduced. Also, some isomorphic properties of fuzzy semigraphs are discussed.

## 2. Preliminaries

**Definition 2.1.** A graph $G$ is a pair $(V, E)$, where $V$ is a non-empty set of points which are called the vertices and $E$ is a set of ordered pairs of elements of $V$ which are called edges of $G$.

**Definition 2.2.** A fuzzy graph $G : (\sigma, \mu)$ on $G : (V, E)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(uv) \le \sigma(u) \wedge \sigma(v)$, $\forall\, uv \in E$.

**Definition 2.3.** $G$ is an effective fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$ and $G$ is a complete fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

**Definition 2.4** [3]**.** A semigraph is a pair $(V, X)$, where $V$ is a non-empty set of elements called vertices and $X$ is a set of $n$-tuples called edges of distinct vertices for various $n \ge 2$ satsfying the conditions

(1) Any two edges have at most one vertex in common

(2) Two edges $x_1 = (u_1, u_2, \ldots, u_n)$ and $x_2 = (v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if (a) $m = n$ (b) either $u_i = v_i$ for $i = 1, 2, \ldots, n$ or $u_i = v_{n-i+1}$ for $i = 1, 2, \ldots, n$.

In the edge $x = (u_1, u_2, \ldots, u_n)$, $u_1$ and $u_n$ are called the end vertices and all the vertices in between them are called middle vertices ($m$-vertices). If a middle vertex is an end vertex of some other edge, then it is called a middle end vertex.

**Definition 2.5** [3]. A *subedge* (*fs-edge*) of an edge $x = (v_1, v_2, v_3, \ldots, v_n)$ is a $k$-tuple $x' = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$ where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \ldots < i_1 \leq n$ and a *partial edge* (*fp-edge*) of an edge $x = (v_1, v_2, v_3, \ldots, v_n)$ is a $(j - i + 1)$-tuple $x'' = (v_i, v_{i+1}, \ldots, v_j)$ where $1 \leq i \leq n$.

If $E \subseteq V \times V$ is taken as the set of all $uv$ which is a partial edge of some edge $x \in X$, then a semigraph can be taken as a triple $(V, E, X)$.

**Definition 2.6** [5]. A fuzzy semigraph on $G^* : (V, E, X)$ is defined as $G : (\sigma, \mu. \eta)$ where $\sigma : V \to [0, 1], \mu : V \times V \to [0, 1], \eta : X \to [0, 1]$ are such that

(i) $\mu(uv) \leq \sigma(u) \wedge \sigma(v), \forall uv \in E$

(ii) $\eta(x) = \mu(u_1u_2) \wedge \ldots \wedge \mu(u_{n-1}u_n) \leq \sigma(u_1) \wedge \sigma(u_n), \forall x = (u_1, u_2, \ldots, u_n) \in X, n \geq 2$.

**Definition 2.7** [6]. An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called an *effective* edge, if $\eta(x) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n)$ and $\mu(u_iu_{i+1}) = \sigma(u_i) \wedge \sigma(u_{i+1})$ for all $i$.

**Definition 2.8** [6]. A fuzzy semigraph $G : (\sigma, \mu. \eta)$ is said to be an effective fuzzy semigraph if all the edges of $G$ are effective edges.

**Definition 2.9** [4]. A homomorphism of fuzzy graphs $f : G \to G'$ is a map $f : V \to V'$ such that $\sigma(u) = \sigma'(f(u))$ for all $u \in V, \mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

**Definition 2.10** [4]. An isomorphism of fuzzy graphs $f : G \to G'$ is a bijective map $f : V \to V'$ such that $\sigma(u) = \sigma'(f(u)), \forall u \in V, \mu(uv) = \mu'(f(u)f(v)), u, v \in V$.

**Definition 2.11** [4]. A weak isomorphism of fuzzy graphs $f : G \to G'$ is a map $f : V \to V'$ which is bijective and satisfies $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

**Definition 2.12** [4]. A co-weak isomorphism of fuzzy graphs $f : G \to G'$ is a map $f : V \to V'$ which is bijective and satisfies $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ and $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$.

**Definition 2.13** [3]. Let $G_1 : (V_1, X_1)$ and $G_2 : (V_2, X_2)$ be two semigraphs and $f$ is a bijection from $V_1$ to $V_2$. Let $x = (v_1, v_2, \ldots, v_n)$ be an edge in $G_1$, $f$ *is an isomorphism* if $(v_1, v_2, \ldots, v_n)$ is an edge in $G_1$ then $(f(v_1), f(v_2), \ldots, f(v_n))$ is an edge in $G_2$, $f$ *is an end vertex isomorphism* (*ev-isomorphism*) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G_2$ with end vertices $f(v_1)$ and $f(v_n)$. $f$ is an *edge isomorphism* (*e-isomorphism*) if the set $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G_2$ and $f$ is an adjacency isomorphism (a-isomorphism) if the adjacent vertices in $G_1$ are mapped onto adjacent vertices in $G_2$.

## 3. Isomorphisms on Fuzzy Semigraphs

**Definition 3.1.** Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A homomorphism of fuzzy semigraphs $f : G \to G'$ is a map denoted by $f : V \to V'$ which satisfies $\sigma(u) \leq \sigma'(f(u))$ for all $u \in V$, $\mu(uv) \leq \mu'(f(u)f(v))$ for all $u, v \in V$ and $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

**Definition 3.2.** Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. An *isomorphism of fuzzy semigraphs* $f : G \to G'$ is a bijective map denoted by $f : V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $(f(v_1), f(v_2), \ldots, f(v_n))$ is an edge in $G'$ (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$

(3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and 4. $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

**Definition 3.3.** An *end vertex isomorphism* (*ev-isomorphism*) of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ with end vertices $f(v_1)$ and $f(v_n)$. (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$, and (4). $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

**Definition 3.4.** An *edge isomorphism* (*e-isomorphism*) of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$ such that 1. If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$. (2) $\sigma(u) = \sigma'(f(u)), \forall u \in V$, 3. $\mu(uv) \leq \mu'(f(u)f(v)), uv \in E$, 4. $\eta(x) \leq \eta'(f(x)), x \in X$.

**Definition 3.5.** An *adjacency isomorphism* (*a-isomorphism*) of fuzzy semigraphs $f: G \to G'$ is a bijective map $f: V \to V'$ which satisfies (1). The adjacent vertices in $G$ are mapped onto adjacent vertices in $G'$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and (4). $\eta(x) \leq \eta'(f(x))$, for all $x \in X$.

**Theorem 3.6.** *Isomorphisms between fuzzy semigraphs is an equivalence relation.*

**Proof of Theorem 3.6.** Let $G: (\sigma, \mu, \eta), G': (\sigma', \mu', \eta')$ and $G'': (\sigma'', \mu'', \eta'')$ be fuzzy semigraphs with vertex sets $V, V'$ and $V''$ respectively.

Let $f: V \to V$ be such that $f(v) = v, \forall v \in V$. This mapping $f$ is a bijection. Also $\sigma(u) = \sigma(f(u))$ for all $u \in V, \mu(uv) = \mu(f(u), \mu(v))$ for all $uv \in E$ and $\eta(x) \leq \eta(f(x))$, for all $x \in X$. Thus *f* is an isomorphism from $G$ to itself.

Hence isomorphism is a *reflexive relation*.

Let $f: G \to G'$ be an isomorphism between the fuzzy semigraphs $G$ and $G'$ Then the mapping $f: V \to V'$ satisfies

$$\sigma(u) = \sigma'(f(u)) \text{ for all } u \in V \text{ and} \tag{1}$$

$\mu(uv) = \mu(f(u), \mu(v))$ for all $uv \in E$ (2)

$\eta(x) \leq \eta'(f(x))$, for all $x \in X$. (3)

Since $f$ is bijective, for $u'$ in $V'$, there exists $u$ in $V$ such that $f^{-1}(u') = u$

Hence by (1) $\sigma(f^{-1}(u')) = \sigma'(f(u)) = \sigma'(u')$ for all $u' \in V'$. From this it follows that $\eta(x') = \eta'(f^{-1}(x'))$, for all $x'$ in $X'$.

Also $\mu((f^{-1}(u')f^{-1}(v')) = (f(u)f(v)) = \mu'(u'v')$ for all $u'v' \in E'$ (4)

Hence we get a 1-1, onto map $f^{-1} : V' \to V$ which is an isomorphism.

Thus $G'$ is isomorphic to $G$. Hence isomorphism is the symmetric.

Let $f : V \to V'$ and $g : V' \to V''$ be isomorphisms from fuzzy semigraphs $G$ to $G'$ and from $G'$ to $G''$ respectively.

Then $g \circ f$ is 1-1 and onto map from $V \to V''$ where $(g \circ f)u = g(f(u))$, for all $u \in V$.

Then $\sigma(u) = \sigma'(f(u)) = \sigma''(g(f(u)))$ for all $u$ in $V$

$\mu(uv) = \mu'(f(u), \mu(v)) = \mu''(g(f(u)), g(\mu(v)))$, $\forall uv \in E$

$\eta(x) = \eta''(g(f(x)))$ for all $x \in X$.

Therefore $g \circ f$ is an isomorphism between $G$ and $G''$

Hence isomorphism is transitive and hence it is an equivalence relation.

**Theorem 3.7.** *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in $G$ is an effective edge if and only if the corresponding image edge in $G'$ is effective.*

**Proof of Theorem 3.7.** Let $f : G \to G'$ be an isomorphism between the fuzzy semigraphs $G$ and $G'$ with underlying sets $V$ and $V'$.

$x$ is an effective edge in $G$

$\Leftrightarrow \eta(x) = \mu(u_1u_2) \wedge \mu(u_2u_3) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n)$,

$x = (u_1, u_2, \ldots, u_n)$

$\Leftrightarrow \eta'(f(x)) = \mu'(f(u_1)f(u_2)) \wedge \ldots \wedge \mu'(f(u_{n-1})f(u_n)) = \sigma'(f(u_1)) \wedge \sigma'(f(u_n))$

$\Leftrightarrow f(x)$ is an effective edge in $G'$.

Also for any $e = uv$ in $E$,

$$\mu(uv) = \sigma(u) \wedge \wedge\sigma(v) \Leftrightarrow \mu'(f(u)f(v)) = \sigma'(f(u)) \wedge \sigma'(f(v)).$$

**Theorem 3.8.** *If G and $G'$ are isomorphic fuzzy semigraphs then G is an effective fuzzy semigraph if and only if $G'$ is also effective.*

**Proof of Theorem 3.8.** Since $G$ is isomorphic to $G'$, there is an isomorphism $f : G \to G'$. Therefore $G$ is an effective fuzzy semigraph $\Leftrightarrow$ Each edge in $G$ is effective $\Leftrightarrow$ The image of each edge in $G$ is effective $\Leftrightarrow$ Each edge in $G'$ is effective $\Leftrightarrow$ $G'$ is an effective fuzzy semigraph.

**Definition 3.9.** An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called an *e-effective* edge if

$$\eta(x) = \mu(u_1u_2) \wedge \ldots \wedge \mu(u_{n-1}u_n) = \sigma(u_1) \wedge \sigma(u_n) \text{ for } n > 2.$$

An edge $x = (u_1, u_2, \ldots, u_n)$ of a fuzzy semigraph is called a *semi-effective* edge if $\mu(u_iu_{i+1}) = \sigma(u_i) \wedge \sigma(u_{i+1})$ for all $i$.

**Definition 3.10.** A fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be an e-effective fuzzy semigraph if all the edges of $G$ are e-effective edges.

$G$ is said to be a semi-effective fuzzy semigraph if all the edges of G are semi-effective edges.

**Theorem 3.11.** *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in G is an e-effective edge if and only if the corresponding image edge in $G'$ is e-effective.*

**Theorem 3.12.** *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic fuzzy semigraphs. Then an edge in G is a semi-effective edge if and only if the corresponding image edge in $G'$ is semi-effective.*

**Theorem 3.13.** *Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two isomorphic* *fuzzy semigraphs. Then G is an e-effective fuzzy semigraph if and only if* $G'$ *is e-effective.*

**Theorem 3.14.** *Let* $G : (\sigma, \mu, \eta)$ *and* $G' : (\sigma', \mu', \eta')$ *be two isomorphic fuzzy semigraphs. Then G is a semi-effective fuzzy semigraph if and only* $G'$ *is semi-effective.*

## 4. Weak Isomorphisms

**Definition 4.1.** Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A *weak isomorphism of fuzzy semigraphs* $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$. (2). $\sigma(u) = \sigma'(f(u))$ for all vertices $u \in V$ (3). $\mu(uv) \le \mu'(f(u)f(v))$ for all $uv \in E$.

A *weak-end vertex isomorphism* (*weak-ev-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies, (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in $G$ then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ with end vertices $f(v_1)$ and $f(v_n)$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \le \mu'(f(u)f(v))$ for all $uv \in E$.

A *weak-edge isomorphism* (*weak-e isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map denoted by $f : V \to V'$ and which satisfies (1). If $e = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \le \mu'(f(u)f(v))$ for all $u, v \in V$.

A *weak-adjacency isomorphism* (*weak-a isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map denoted by $f : V \to V'$ and which satisfies (1). If the adjacent vertices in $G$ are mapped onto adjacent vertices in $G'$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \le \mu'(f(u)f(v))$ for all $uv \in E$.

**Theorem 4.2.** *Weak isomorphism between fuzzy semigraphs is a partial order relation.*

**Proof of Theorem 4.2.** Let $G:(\sigma, \mu, \eta), G':(\sigma', \mu', \eta')$ and $G'':(\sigma'', \mu'', \eta'')$ be fuzzy semigraphs with vertex sets $V, V'$ and $V''$ respectively.

Let $f: V \to V$ such that $f(v) = v, \forall v \in V$. Then $f$ is a weak isomorphism from $G$ to itself. Thus $G$ is weak isomorphic to itself. Hence weak isomorphism satisfies *reflexive relation*.

Let $f: V \to V'$ and $g: V' \to V$ be weak isomorphisms from fuzzy semigraphs $G$ to $G'$ and from $G'$ to $G$ respectively

Then $f$ and $g$ satisfies $\mu(uv) \leq \mu'(f(u)f(v)) \forall uv \in E$.

And $\mu'(u'v') \leq \mu(g(f(u))g(f(v)))$, for all $u'v' \in E'$.

This can happen only when $G$ and $G'$ have the same number of edges and the corresponding membership values of the edges are equal. Hence $G$ and $G'$ are identical (apart from the naming of the vertices). Thus Weak isomorphism between fuzzy semigraphs is *anti symmetric*.

Let $f: V \to V'$ and $g: V' \to V''$ be weak isomorphisms on fuzzy semigraphs $G$ to $G'$ and $G'$ to $G''$ respectively.

Then $\sigma(u) = \sigma'(f(u)) = \sigma''(g(f(u)))$ for all $u$ in $V$

$$\mu(uv) \leq \mu'(f(u)f(v)) \leq \mu''(g(f(u))g(f(v))), \forall u, v \in V$$

Thus $g \circ f$ is a weak isomorphism between $G$ and $G''$ and therefore weak isomorphism between fuzzy semigraphs is *transitive*. Hence the weak isomorphism between fuzzy semigraphs is a *partial order relation*.

**Remark 4.3.** When $G:(\sigma, \mu, \eta)$ is weak isomorphic to $G'(\sigma', \mu', \eta')$, then the effectiveness of one fuzzy semigraph need not imply the effectiveness of the other. The Fuzzy semigraph $G$ in Figure 4.1 and the Fuzzy semigraph $G'$ in Figure 4.2 are weak isomorphic, $G$ is effective but $G'$ is not effective. Also the Fuzzy semigraph $G$ in Figure 4.3 and the Fuzzy semigraph $G'$ in Figure 4.4 are weak isomorphic, $G'$ is effective but $G$ is not effective.

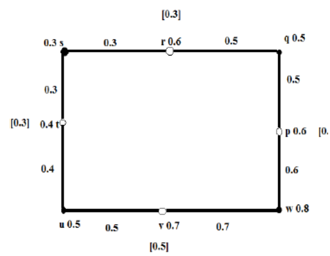


**Figure 4.1.** $G : (\sigma, \mu, \eta)$.

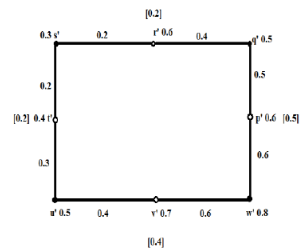


**Figure 4.2.** $G' : (\sigma', \mu', \eta')$.

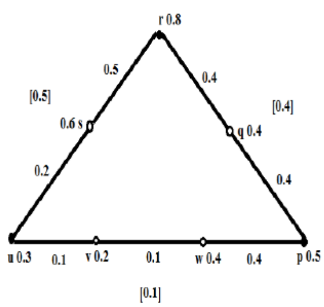


**Figure 4.3.** $G : (\sigma, \mu, \eta)$.

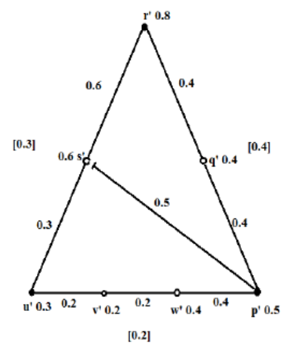


**Figure 4.4.** $G' : (\sigma', \mu', \eta')$.

**Remark 4.4.** Weak isomorphism need not preserve the e-effective and the semi-effective properties of the edges.

## 5. Co-Weak Isomorphisms

**Definition 5.1.** Let $G : (\sigma, \mu, \eta)$ and $G' : (\sigma', \mu', \eta')$ be two fuzzy semigraphs with underlying semigraphs $G^* : (V, E, X)$ and $G'^* : (V', E', X')$. A *co-weak isomorphism of fuzzy semigraphs* $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$ (3). $\mu(uv) = \mu'(f(u)f(v)), \forall\, uv \in E, \eta(x) = \eta'(f(x)), \forall\, x \in X$.

A *co-weak end vertex isomorphism* (*co-weak ev-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ and which satisfies, (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$ then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ with end vertices $f(v_1)$ and $f(v_n)$, (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$, (4). $\eta(x) = \eta'(f(x))$, for all $x \in X$.

A *co-weak edge isomorphism* (*co-weak e-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ and which satisfies (1). If $x = (v_1, v_2, \ldots, v_n)$ is an edge in $G$, then $\{f(v_1), f(v_2), \ldots, f(v_n)\}$ forms an edge in $G'$ (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and 4. $\eta(x) = \eta'(f(x))$, for all $x \in X$.

A *co-weak adjacency isomorphism* (*co-weak a-isomorphism*) of fuzzy semigraphs $f : G \to G'$ is a bijective map $f : V \to V'$ which satisfies, (1). If the adjacent vertices in $G$ are mapped onto adjacent vertices in $G'$ (2). $\sigma(u) = \sigma'(f(u))$ for all $u \in V$, (3). $\mu(uv) \leq \mu'(f(u)f(v))$ for all $uv \in E$ and (4). $\eta(x) = \eta'(f(x))$, for all $x \in X$.

**Theorem 5.2.** *If $f : G \to G'$ is a co-weak isomorphism on fuzzy semigraphs G and $G'$ and if $G'$ is a semi-effective fuzzy semigraph, then G is also a semi-effective fuzzy semigrap*h.

**Proof of Theorem 5.2.** Since $G'$ is an effective fuzzy semigraph and $f$ is a co-weak isomorphism,

Now $\mu(uv) \leq \mu'(f(u)f(v)) = \sigma'(f(u)) \wedge \sigma'(f(v)) \geq \sigma(u) \wedge \sigma(v)$

But $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. Hence $\mu(uv) = \sigma(u) \wedge \sigma(v)$, for all $uv \in E$

Thus $G$ is a semi-effective fuzzy semigraph.

**Remark 5.3.** The semi-effectiveness of $G$ need not imply the semi-effectiveness of $G'$ when $G$ is co-week isomorphic to $G'$.

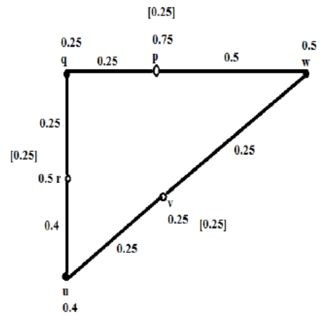


**Figure 5.1.** $G : (\sigma, \mu, \eta)$.

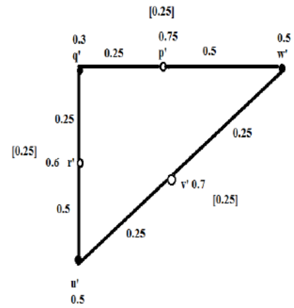


**Figure 5.2.** $G' : (\sigma', \mu', \eta')$.

Here $G$ is co-weak isomorphic to $G'$. $G$ is a semi-effective fuzzy semigraph but $G'$ is not semi-effective.

## 6. Conclusion

In this work, isomorphism, weak isomorphism and co-weak isomorphism of a fuzzy semigraph are introduced and their properties are discussed. Transport networks and telecommunication networks can be modeled as fuzzy semigraphs. Hence our findings may be useful for future Studies and Research.

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