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SOME PROPERTIES ON STRONG AND WEAK DOMINATION IN PICTURE FUZZY GRAPHS

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Abstract

The strong and weak dominating set of the picture fuzzy graph are introduced in this paper. The strong and weak domination number $\gamma_{spf}(G)$ and $\gamma_{wpf}(G)$ for some picture fuzzy graphs are obtained. The strong and weak independent picture fuzzy dominating set are defined. Some properties and bounds related to these parameters are discussed.

1. Introduction

The strong and weak domination [11] in graph theory was introduced by E. Sampathkumar and Pushpalatha in 1996. A. Somasundaram and S. Somasundaram [12] presented more concepts of independent domination, connected domination in fuzzy graphs. A. Nagoorgani and Basheer Ahamed [7] studied strong and weak domination in fuzzy graphs. A. Nagoorgani and

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Chandrasekaran [8] introduced domination in fuzzy graphs using strong edges. Intuitionistic fuzzy graph theory was introduced by Krassimir T. Atanassov in [1]. In [9], authors introduced intuitionistic fuzzy graph as a special case of Atanassov IFG. R. Parvathi and Thamizhendhi [10] introduced dominating set, domination number, independent set, total dominating and total domination number in intuitionistic fuzzy graphs.

Cuong and Kreinovich [3] introduced the picture fuzzy set which is an advanced version of fuzzy set and intuitionistic fuzzy set. It is a useful model with uncertain real-life problems, in which intuitionistic fuzzy set may decline to reveal satisfactory results. Tahir et al. [13], introduced the new concept of picture fuzzy graph. It allows the positive membership degree, neutral membership degree and negative membership degree of each vertex. The degree of neutrality can be seen in circumstances in which we face individual opinions involving more answers of type: yes, abstain, no, refusal. Cen Zuo [2] introduced several types of Picture Fuzzy Graphs.

This paper is organized as follows. Section 2 contains preliminaries of picture fuzzy graphs and in section 3, the strong and weak dominating set of picture fuzzy graphs are introduced. Strong and weak domination number in picture fuzzy graph are obtained. Some properties and bounds of these parameters are also discussed. In section 4, Independent strong and weak picture fuzzy dominating set and independent strong and weak picture fuzzy domination number are introduced. Some theorems and properties for these parameters are investigated.

2. Preliminaries

In this section, basic definitions which are used to construct theorems related to picture fuzzy graph are given.

Definition 2.1. A fuzzy graph $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \vee$ and μ is a symmetric fuzzy relation on σ.

Definition 2.2. An intuitionistic fuzzy graph is of the form $G = (V, E)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that the mapping $\mu_1 : V \to [0, 1]$ is the degree of membership and the mapping $\gamma_1 : V \to [0, 1]$ is the degree of non-membership of the element $v_i \in V$ respectively and $O \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V\ (i = 1, 2, \ldots, n)$.

(ii) $E \subseteq VXV$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min\ [\mu_1(v_i), \mu_1(v_j)]\, \gamma_2(v_i, v_j) \leq \max\ [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1, \forall (v_i, v_j) \varepsilon \in (i, j = 1, 2, \ldots, n)$.

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non-membership of the Vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

In an intuitionistic fuzzy graph G, when $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition 2.3. A pair $G = (V, E)$ is known as picture fuzzy graph (PFG) if

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0, 1]\, \eta_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ degree of Positive membership, neutral membership and negative membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \eta_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V, i = 1, 2, \ldots, n$.

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$, $\eta_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min\ (\mu_1(v_i), \mu_1(v_j))$ $\eta_2(v_i, v_j) \leq \min\ (\eta_1(v_i), \eta_1(v_j))\, \gamma_2(v_i, v_j) \leq \max\ (\gamma_1(v_i), \gamma_2(v_j))$ where $0 \leq \mu_2(v_i, v_j) + \eta_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E, i, j = 1, 2, \ldots, n$. Here the 4-tuple $(v_i, \mu_{ii}, \eta_{ii}, \gamma_{ii})$ denotes the degree of positive membership, neutral membership and negative membership of the vertex V_i and the 4-tuple $(e_{ij}, \mu_{2ij}, \eta_{2ij}, \gamma_{2ij})$ denotes the degree of positive membership, neutral membership and negative membership of the edge relation $e_{ij} = (v_i, v_j)$.

Example 2.4

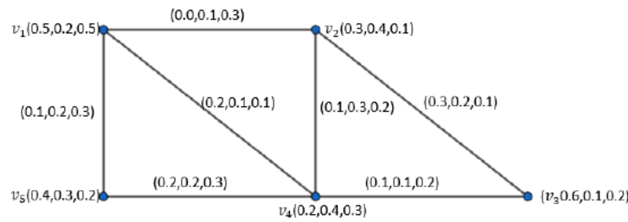

Figure 1.

Remark 2.5. There is no edge between V_i and V_j when $\mu_2(v_i, v_j) = \eta_2(v_i, v_j) = \gamma_2(v_i, v_j) = 0$ for some i and j. Otherwise there exists an edge between v_i and v_j.

Definition 2.6. Let $G = (V, E)$ be the PFG. Then the cardinality of G is defined to be

$$|G| = \sum_{v_j \in V} \frac{1 + \mu_1(v_i) - \eta_1(v_i) - \gamma_1(v_i)}{2} + \sum_{(v_i v_j) \in E} \frac{1 + \mu_2(v_i, v_j) - \eta_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}.$$

Definition 2.7. Let $G = (V, E)$ be the PFG. Then the vertex cardinality of V is defined by

$$|V| \sum_{v_i \in V} \frac{1 + \mu_1(v_i, v_j) - \eta_1(v_i, v_j) - \gamma_1(v_i)}{2}$$

for all $v_i \in V$. It is also called the order of a PFG and it is denoted by p.

Definition 2.8. Let $G = (V, E)$ be the PFG. Then the edge cardinality of E defined by

$$|E| = \sum_{(v_i, v_j) \in E} \frac{1 + \mu_2(v_i, v_j) - \eta_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}$$

for all $(v_i, v_j) \in E$. It is also called the size of a PFG and it is denoted by q.

Definition 2.9. An arc (v_i, v_j) is called a strong edge, if $\mu_2(v_i, v_j) \geq \mu_2'^{\infty}(v_i, v_j) \geq \eta_2(v_i, v_j) \geq \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) \leq \gamma_2'^{\infty}(v_i, v_j)$. For every $v_i, v_j \in V$. Where $\mu_2'^{\infty}(v_i, v_j) \geq \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2'^{\infty}(v_i, v_j)$ is the strength of the connectedness between v_i and v_j in the picture fuzzy graph obtained from G by deleting the arc (v_i, v_j).

Definition 2.10. The strong degree of a vertex v_i in the Picture fuzzy graph $G = (V, E)$ is defined to be the addition of the weights of the strong arcs incident at v_i. It is denoted by $d_S(v_i)$.

The minimum strong degree of PFG G is $\delta_s(G) = \min \{d_s(v_i)/v_i \in V\}$

The maximum strong degree of PFG G is $\Delta_s(G) = \max \{d_s(v_i)/v_i \in V\}$.

Definition 2.11. Two vertices v_i and v_j are said to be neighbors in PFG if either one of the following conditions hold.

(i) $\mu_2(v_i, v_j) > 0,\ \eta_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) > 0$

(ii) $\mu_2(v_i, v_j) = 0,\ \eta_2(v_i, v_j) \geq 0,\ \gamma_2(v_i, v_j) > 0$

(iii) $\mu_2(v_i, v_j) > 0,\ \eta_2(v_i, v_j) = 0,\ \gamma_2(v_i, v_j) > 0$

(iv) $\mu_2(v_i, v_j) \geq 0,\ \eta_2(v_i, v_j) > 0,\ \gamma_2(v_i, v_j) > 0,\ \forall v_i, v_j \in V$.

Definition 2.12. Let v_i be a vertex in a Picture fuzzy graph $G = (V, E)$ then $N_s(v_i) = \{v_j \in V : (v_i, v_j)$ is a strong$\}$ is called strong neighborhood of v_i. $N_s[v_i] = N_s(v_i) \cup \{v_i\}$ is called the closed strong neighborhood of v_i.

Definition 2.13. A Picture fuzzy graph $G = (V, E)$ is said to be complete, if $\mu_{2ij} = \min(\mu_1(v_i), \mu_1(v_j)),\ \mu_{2ij} = \min(\eta_1(v_i), \eta_1(v_j))$ and $\gamma_{2ij} = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $v_i, v_j \in V$.

Definition 2.14. Let $G = (V, E)$ be a PFG. If $d_\mu(v_i), = c_i,\ d_\eta(v_i) = c_j$ and $d_\gamma(v_i) = c_k$ for all $v_i, v_j \in V$. Then the graph is called as $(c_i, c_j, c_k) - PFG$ or constant PFG of degree (c_i, c_j, c_k).

Example 2.15. Consider the PFG, $G = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$. In figure 2, the degree of v_1, v_2, v_3, v_4 is (0.2, 0.3, 0.5)

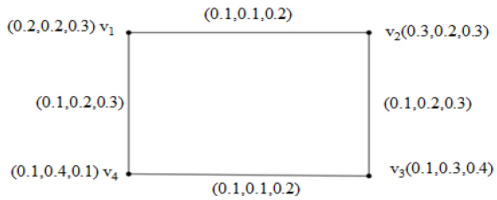

Figure 2.

Definition 2.16. A vertex $v_i \in V$ of the PFG $G = (V, E)$ is said to be an isolated vertex if $\mu_2(v_i, v_j) = 0$, $\eta_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for all $v_i \in V$ i.e.) $N(u) = \varphi$. Thus, an isolated vertex does not dominate any other vertex in G.

3. Strong and Weak Domination in Picture Fuzzy Graphs

In this section, the strong and weak domination and its number are defined. Some properties related to these parameters are stated and proved.

Definition 3.1. Let $G = (V, E)$ be a PFG. Let $v_i, v_j \in V$. Then v_i dominates v_j in G if there exists a strong arc between them.

Definition 3.2. A dominating set D of the PFG G is said to be minimal picture fuzzy dominating set if there is no proper subset of D is a picture fuzzy dominating set.

Definition 3.3. A dominating set D of the PFG is said to be minimal picture fuzzy dominating set if there is no proper subset of D is a picture fuzzy dominating set.

Definition 3.4. The minimum cardinality among all picture fuzzy dominating set is called domination number or lower domination number of G and it is denoted by $\gamma_{pf}(G)$.

Definition 3.5. Any two vertices in a PFG $G = (V, E)$ is called an independent vertices if there is no strong edge between these two vertices.

Definition 3.6. A subset D of V is said to be an independent set of G if $\mu_2(v_i, v_j) < \mu_2'^{\infty}(v_i, v_j)$, $\eta_2(v_i, v_j) < \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) > \gamma_2'^{\infty}(v_i, v_j)$ for all $v_i, v_j \in D$.

Definition 3.7. An independent set D of G in a Picture fuzzy graph $G = (V, E)$ is said to be maximal independent, if for every vertex $v_i \in V - D$, the set $D \cup \{v_i\}$ is not independent.

Definition 3.8. The minimum cardinality among all maximal independent set is called lower independence number of G and it is denoted by $i_{pf}(G)$.

Definition 3.9. Let v_i and v_j be any two vertices in a picture fuzzy graph. $G = (V, E)$. Then v_i strongly dominates v_j (i) (v_i, v_j) is a strong arc (ii) $d_s(v_i) \geq d_s(v_j)$. Otherwise v_i weakly dominates v_j.

Definition 3.10. Let $G = (V, E)$ be a PFG. Then $D \subset V$ is said to be strong picture fuzzy dominating set of G if every vertex $v_j \in V - D$ is strongly dominated by some vertex $v_i \in D$.

Definition 3.11. Let $G = (V, E)$ be a PFG. Then $D \subset V$ is said to be weak picture fuzzy dominating set of G if every vertex $v_j \in V - D$ is weakly dominated by some vertex $v_i \in D$.

Definition 3.12. The minimum scalar cardinality of a strong picture fuzzy dominating set is called the strong picture fuzzy domination number of G and it is denoted by $\gamma_{spf}(G)$.

Definition 3.13. The minimum scalar cardinality of a strong picture fuzzy dominating set is called the strong picture fuzzy domination number of G and it is denoted by $\gamma_{wpf}(G)$.

Example 3.14. For the picture fuzzy graph $G = (V, E)$ in Figure 3 $\gamma_{spf}(G) = 0.4$ and $\gamma_{wpf}(G) = 1.8$, since $\{v_2\}$ and $\{v_1, v_3, , v_4, v_5\}$ are the minimal strong picture fuzzy dominating set and minimal weak picture fuzzy dominating set respectively.

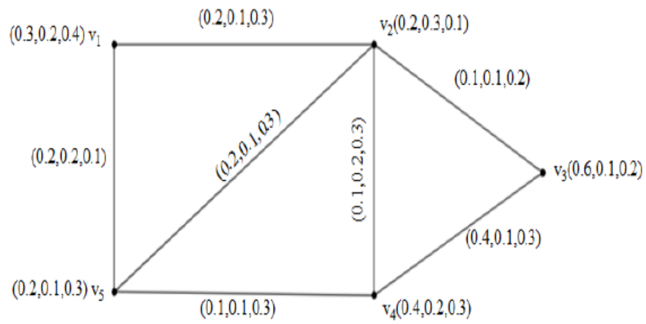

Figure 3.

Remark 3.15. If D is a minimal strong picture fuzzy dominating set, then $V - D$ need not be a weak picture fuzzy dominating set.

For example, consider the picture fuzzy graph G in Figure 4, Here vertices v_1 and v_3 form an strong picture fuzzy dominating set where as $V - D = \{v_2, v_4\}$ does not form a weak picture fuzzy dominating set of G.

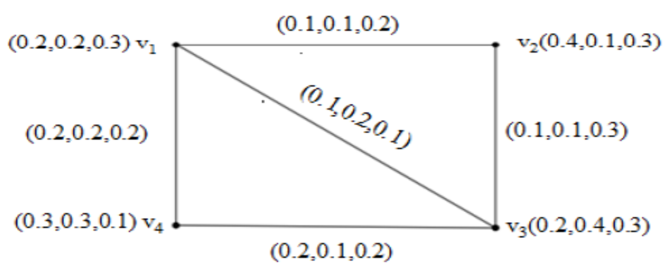

Figure 4.

Theorem 3.16. *A strong picture fuzzy dominating set D of* $G = (V, E)$ *is a minimal dominating set if and only if* $\forall$ *vertex* $v_i \in D$ *one of the following conditions holds.*

(i) $N(v_i) \cap D = \varphi$

(ii) $\exists$ *vertex* $v_j \in V - D$ *such that* $N(v_j) \cap D - \{v_i\}$.

Proposition 3.17. *Let D be a minimal strong or weak picture fuzzy dominating set. Then, for each* $v_i \in D$, *one of the following conditions holds*:

(i) *No vertex in D strongly or weakly dominates* v_j.

(ii) *There exists a vertex* $v_j \in V - D$ *such that* v_i *is the only vertex in D which strongly* (*weakly*) *dominates* v_j.

Proof. Let D be a minimal strong picture fuzzy dominating set of the PFG $G = (V, E)$.

Any vertex $v_i \in D$, then $D - \{v_i\}$ is not a picture fuzzy dominating set in G.

Therefore the vertex in D does not strongly dominate V.

Let $v_j \in V - D$. Assume that $v_i, v_k \in D$ strongly dominates $v_j \in V - D$.

Then $D - \{v_k\}$ is a strong picture fuzzy graph dominating set. Therefore D is not a minimal strong picture fuzzy dominating set which contradicts to our assumption. Hence we get $v_j \in V - D$ such that v_i is the only vertex in D which strongly dominates v_j.

Theorem 3.18. *Let* $G = (V, E)$ *be a constant Picture fuzzy graph of degree* (c_i, c_j, c_k) *without isolated vertices and D is a minimal strong picture fuzzy dominating set. Then* $V - D$ *is a picture fuzzy dominating set of G.*

Proof. Let D be a minimal strong picture fuzzy dominating set. Consider v_i be any vertex in D, there exists a vertex $v_j \in N(v_i)$, v_i must be dominated by exactly one vertex in $D - \{v_i\}$ which is also a picture fuzzy dominating set.

Since G has no isolated vertices, every vertex in D is strongly dominated by at least one vertex in $V - D$ and hence $V - D$ is a picture fuzzy dominating set.

Theorem 3.19. *Let D be a strong picture fuzzy dominating set in a constant picture fuzzy graph of degree* (c_i, c_j, c_k) *iff* $\Delta_s(G) = \delta_s(G) = c$ *where* $c = c_i + c_j + c_k$.

Example. In figure 5, the strong picture fuzzy dominating sets are $\{v_1, v_4\}, \{v_2, v_5\}, \{v_3, v_6\}$, and $\Delta_s(G) = \delta_s(G) = 1$

$\therefore\ 0.3 + 0.2 + 0.5 = 1$

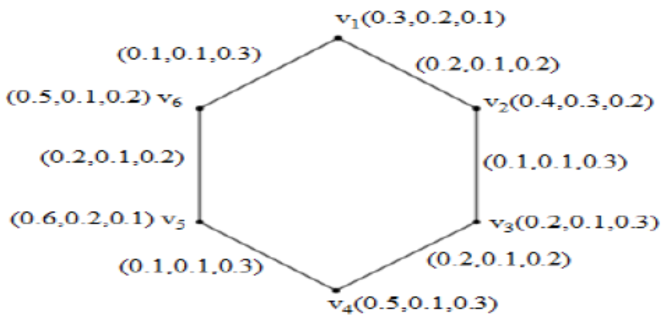

Figure 5.

Proposition 3.20. *For a picture fuzzy graph* $G = (V, E)$ *of order* p, $\gamma_{pf} \leq \gamma_{spf} \leq q - \Delta_s(G)$ *and* $\gamma_{pf} \leq \gamma_{wpf} \leq q - \delta_s(G)$.

Proof. Let D be the strong (Weak) picture fuzzy dominating set. Since every strong (Weak) Picture fuzzy dominating set, we have $\gamma_{pf} \leq \gamma_{spf}$ and $\gamma_{pf} \leq \gamma_{wpf}$. (1)

For $v_i, v_j \in V$, if $d_s(v_i) = \Delta_s(G)$ and $d_s(v_j) = \delta_s(G)$ then clearly $V - N_s(v_i)$ is an strong picture fuzzy dominating set and $V - N_s(v_j)$ is a weak picture fuzzy dominating set D and q be the size of the picture fuzzy graph G, then

$$\left.\begin{aligned} \gamma_{spf} &\leq q - \Delta_s(G) \\ \gamma_{wpf} &\leq q - \delta_s(G) \end{aligned}\right\} \tag{2}$$

From (1) and (2)

$$\gamma_{pf} \leq \gamma_{spf} \leq q - \Delta_s(G)$$

$$\gamma_{pf} \leq \gamma_{wpf} \leq q - \delta_s(G).$$

Proposition 3.21. *For a constant PFG of degree* (c_i, c_j, c_k), *then* $\gamma_{spf}(G) \leq p \leq q$ *and* $\gamma_{wpf} \leq p \leq q$.

Proof. Let P be the order of the PFG $G = (V, E)$. It should not be a minimum value of a strong picture fuzzy domination number of a constant PFG.

Let γ_{spf} be the strong picture fuzzy domination number of a constant PFG G.

$$\therefore \gamma_{spf} \leq p. \tag{1}$$

Let q be the size of the picture fuzzy graph. Then it should not be a minimum of vertex cardinality p, and hence

$$p \leq q \tag{2}$$

from (1) and (2), $\gamma_{spf} \leq p \leq q$.

Similarly it is proved that $\gamma_{wpf} \leq p \leq q$.

4. Independent Strong (Weak) Domination in Picture Fuzzy Graphs

In this section, the independent strong and weak domination and its number are defined. Some properties related to these parameters are given.

Definition 4.1. The set of all vertices with minimum degree and maximum degree are denoted as $m(v)$ and $M(v)$ respectively and are defined as

$$m(v) = \{v_i \in V / d_s(v_i) = \delta_s(G)\} \text{ and } M(v) = \{v_i \in V / d_s(v_i) = \Delta_s(G)\}.$$

Definition 4.2. A strong (weak) picture fuzzy dominating set S of a fuzzy graph G is said to be an independent strong (weak) picture fuzzy dominating set of G, if it is independent.

The minimum scalar cardinality of an independent strong (weak) picture fuzzy dominating set is called the independent strong (weak) picture fuzzy domination number and it is denoted by $i_{spf}(G)(i_{wpf}(G))$.

Proposition 4.3. *Let* $G = (V, E)$ *be a picture fuzzy graph. If* S *is an strong independent picture fuzzy dominating set of* G*, then* $S \cap m(v) \neq \varphi$.

Proof. Let $v_i \in m(v)$, since S is an strong independent picture fuzzy dominating set of G, $v_i \in S$ or there exists a vertex $v_j \in S \ni (v_i, v_j)$ is a strong arc and $d_s(v_i) \geq d_s(v_j)$. If $v_i \in S$, then clearly $S \cap m(v) \neq \varphi$. On the other hand, $d_s(v_i) = d_s(v_j)$. Since $d_s(v_i) = \delta_s(G) \Rightarrow d_s(v_j) = \delta_s(G)$. This implies that $v_j \in m(v_i)$. Hence $S \cap m(v) \neq \varphi$.

Proposition 4.4. *Let* $G = (V, E)$ *be a picture fuzzy graph. If S is a weak independent picture fuzzy dominating set of G, then* $S \cap m(v) \neq \varphi$.

Proposition 4.5. *Let* $G = (V, E)$ *be a picture fuzzy graph. If S is a weak independent picture fuzzy dominating set of G, then* $i_{wpf}(G) \leq p - \delta_s(G)$.

Proof. Let S be an independent weak picture fuzzy dominating set of G. Then by proposition 4.3. $S \cap m(v) \neq \varphi$. Let $v_i \in S \cap m(v)$. Since S is an independent picture fuzzy dominating set,

$$S \cap N(v_i) = \varphi \Rightarrow S \cap V - N(v_i) = \varphi.$$

$$\Rightarrow S \subseteq V - N(v_i)$$

$$\Rightarrow |S|_{pf} \leq p - \delta_s(G)$$

$$\Rightarrow i_{wpf}(G) \leq |S|_{pf} \leq p - \delta_s(G).$$

Hence $i_{wpf}(G) \leq p - \delta_s(G)$.

Proposition 4.6. *Let* $G = (V, E)$ *be a picture fuzzy graph. If S is an independent strong picture fuzzy dominating set of G, then* $i_{spf}(G) \leq p - \Delta_s(G)$.

Conclusion

In this paper, we have defined and discussed strong and weak domination using strong arcs in picture fuzzy graphs. Some definitions, examples related to independent strong and weak picture fuzzy dominating set have been introduced. Some theorems and properties of these parameters have been proved.

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