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EQUITABLE EDGE DETOUR DOMINATION OF BRICK PRODUCT GRAPHS

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Abstract

Let G be a connected graph with at least two vertices. An edge detour dominating set S of V is called an equitable edge detour dominating set of a graph if for every $u \in V - S$, there exist a vertex $v \in S$ such $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such a dominating set is called an equitable edge detour domination number of G and is denoted by $\gamma_{ed}^{e}(G)$. In this paper we determined the equitable edge detour domination of brick product graphs $C(2n, m, r)$ and brick product of shadow graphs $D_2\{C(2n, m, r)\}$.

I. Introduction

The concept of domination was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [5] is currently receiving much attention in literature.

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For basic definition and terminology we refer to Buckley and Harary. [1] A set $S \subseteq V(G)$ is called a detour set if every vertex v in G lie on a detour joining a pair of vertices of S. The detour number of a G is the minimum order of a detour set and any detour set of order is called a minimum detour set of G. This concept was studied by Chartrand et al. [2]. For a connected graph G, A set $S \subseteq V(G)$ is called a dominating set of G if every vertices in $V(G) - S$ is adjacent to some vertices in S.

The domination number $\gamma(G)$ is the minimum order of its dominating and any dominating set of order $\gamma(G)$ is called $\gamma(G)$. A set $S \subseteq V(G)$ is called a detour dominating set of G if S is both a detour and a dominating set of G. It is denoted by $\gamma_d(G)$.

II. Equitable Edge Detour Domination of Brick Product Graphs

Definition 2.1. A connected graph $G = (V, E)$, an edge detour dominating set S of G is called an equitable edge detour dominating set if for every $u \in V - S$, there exist a vertex $v \in S$ such $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$.

Definition 2.2. let m, n and r be a positive integers. Let $C_{2n} = a_0, a_1, a_2, \ldots, a_{2n-1}, a_0$ denote a cycle order $2n$. The (m, r)-brick product of C_{2n}, denoted by $C(2n, m, r)$, is defined in two cases as follows

(1) For $m = 1$, we require that r be odd and greater than 1. Then $C(2n, m, r)$ is obtained from C_{2n} by adding chords $a_{2k}a_{2k+r}$, $k = 1, 2, \ldots, n$, where the computation is performed modulo $2n$.

(2) For $m > 1$, we require $m + r$ be even. Then, $C(2n, m, r)$ is obtained by first taking the disjoint union of m copies of C_{2n}, namely $C_{2n}(1), C_{2n}(2), \ldots, C_{2n}(m)$, where for each $i = 1, 2, \ldots, m$, $C_{2n}(i) = (i, 0)(i, 1) \ldots (i, 2n)$. Next, for each odd $i = 1, 2, \ldots, m-1$ and each even $k = 0, 1, 2, \ldots, 2n-2$, and edge is drawn to join (a_i, a_k) to (a_{i+1}, a_k). whereas for each even $i = 1, 2, \ldots, m-1$ and each odd $k = 0, 1,$

$2, \ldots, 2n-1,$ an edge is drawn to join (a_i, a_k) to (a_{i+1}, a_k). Finally, for each odd $k = 0, 1, 2, \ldots, 2n-1,$ an edge is drawn to join (a_i, a_k) to (a_m, a_{k+r}). An edge in $C(2n, m, r)$ which is neither a brick edge nor a hooking edge is called a flat edge.

Definition 2.3. The shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G, namely G itself and G' and by joining each vertex u in G to the neighbors of the corresponding vertex u' in G'.

Example 2.1.

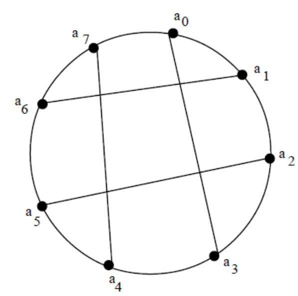

Figure 2.1. The brick product graph $C(8, 1, 3)$.

Edge detour set $= \{a_0, a_3, a_6\}$

Edge detour dominating set $= \{a_0, a_3, a_6\}$

Equitable edge detour domination $= \deg|\, a_0 - a_3 \,| = 3 - 3 = 0.$

$\deg|\, a_3 - a_6 \,| = 3 - 3 = 0.$

$\deg|\, a_0 - a_6 \,| = 3 - 3 = 0.$

$$\gamma_{ed}^{e}\{C(8, 1, 3)\} = 3.$$

Example 2.2.

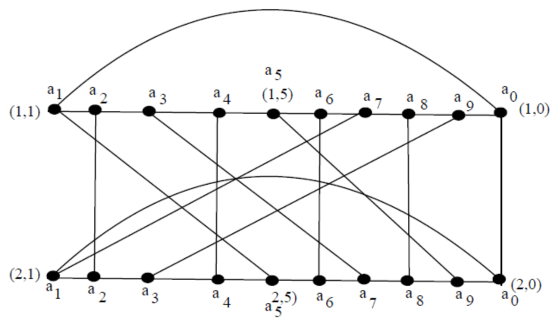

Figure 2.2. The brick product graph $C(10, 2, 4)$.

Edge detour set $= \{(1, a_1), (2, a_0)\}$

Edge detour dominating set $= \{(1, a_1), \{(1, a_4), (1, a_7), (1, a_9), \{(2, a_3), (2, a_6), \{(2, a_8), (2, a_0)\}\}\}\}$

Equitable edge detour domination $= \deg |(1, a_1) - (1, a_4)| = 3 - 3 = 0.$

Similarly, the difference of all the vertices is 0.

$\gamma_{ed}^{e}\{C(10, 2, 4)\} = n - 2 = 10 - 2 = 8.$

Theorem 2.1. *$G = C(2n, 1, 3)$. Then $\gamma_{ed}^{e}(G) = n - 1$, for $n \geq 3$, where $2n \equiv k(\text{mod } 3)$, and $k = 1, 2$.*

Proof. Let $V(G) = \{a_0, a_1, a_2, \ldots, a_{2n-1}\}$ and $E(G) = \{(a_i, a_{i+1})\}$, $i = 0, 1, 2, \ldots, 2n - 1$, modulo $2n$. $E(G) = \{(a_{2k}, a_{2k+r})\}$, $i = 0, 1, 2, \ldots, n$, modulo $2n$. Take any three vertices from a circle to form an edge detour set. The set $S = \{a_1, a_2, a_3\}$ forms an minimum edge detour set, for $2n \equiv k(\text{mod } 3)$, and $k = 1, 2$.

For the edge detour domination, we take one vertex from every three vertex to get the edge detour dominating set.

For $n = 4$, The set $S' = \{a_1, a_3, a_6\}$ forms a minimum edge detour dominating set.

For $n = 5$, The set $S' = \{a_1, a_3, a_6, a_8\}$ forms a minimum edge detour dominating set.

For $n = 7$, The set $S' = \{a_1, a_2, a_3, a_6, a_9, a_{12}\}$ forms a minimum edge detour dominating set.

Obviously this set forms an equitable edge detour dominating set, since all the degrees are equal and their difference is 0.

i.e.) $\deg|a_1 - a_3| = 3 - 3 = 0$.

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$\deg|a_i - a_{i+1}| = 3 - 3 = 0$.

Thus, $\gamma_{ed}^{e}(G) = n - 1$.

Theorem 2.2. $G = C(2n, 1, 5)$. *Then* $\gamma_{ed}^{e}(G) = n - 1$, *for* $n \geq 4$, *where* $2n \equiv k (\bmod 5)$, *and* $k = 1, 2$.

Proof. The vertex set and the edge set are same as in theorem 2.1

The set S' forms an minimum edge detour dominating set. Any set less then S' do not form the edge detour dominating set. $S' - \{a_{2n}\}$ do not form the required set.

Also the degrees are equal. Since, the brick product graph is regular. We get the required equitable edge detour dominating set. Thus, $\gamma_{ed}^{e}(G) = n - 1$.

Theorem 2.3. $G = C(2n, 1, r)$. *Then* $\gamma_{ed}^{e}(G) = n - 1$, *for* $n > 4$, *where* $2n \equiv k (\bmod n)$, *and* $k = 1, 2$.

Proof. The vertex set and the edge set are same as in theorem 2.1

The proof follows from theorem 2.1 and 2.2.

For $n = 5$, $\gamma_{ed}^{e}(G) = n - 1 = 5 - 1 = 4$.

For $n = 7$, $\gamma_{ed}^{e}(G) = n - 1 = 7 - 1 = 6$.

In general, $\gamma_{ed}^{e}(G) = n - 1$, for $r = n$ and $n > 4$, where $2n \equiv k (\bmod n)$, and $k = 1, 2$.

Theorem 2.4. *$G = C(2n, 1, r)$, $n = r$. Then $\gamma_{ed}^{e}(G) = n$, for $n \geq 3$, where $2n \equiv 0(\bmod n)$.*

Proof. The vertex set and edge set are same as in theorem 2.1.

For $n = r = 3$, The set $S' = \{a_1, a_3, a_6\}$ forms a minimum equitable edge detour dominating set.

For $n = r = 6$, The set $S' = \{a_1, a_3, a_6, a_8\}$ forms a minimum equitable edge detour dominating set.

For $n = r = 9$, The set $S' = \{a_1, a_2, a_3, a_6, a_9, a_{12}\}$ forms a minimum edge detour dominating set. In general, $\gamma_{ed}^{e}(G) = n$, for $n \geq 3$, where $2n \equiv 0(\bmod n)$.

Theorem 2.5. *Let $G = C(2n, 2, r)$, $n = r$, where r is even. Then $\gamma_{ed}^{e}(G) = n - 2$, where $2n \equiv k(\bmod n)$ and $k = 0, 1, 2$.*

Proof. Let $V(G) = \{(1, a_0), (1, a_1), (1, a_2), \ldots, (1, a_{2n-1}), (2, a_0), (2, a_1), (2, a_2), \ldots, (2, a_{2n-1})\}$ and $E(G) = 6n$. Take r as even. The set $S = \{(1, a_0), (2, a_{2n-1})\}$ forms an edge detour set.

The edge detour dominating set S' is obtained by taking the required vertices along with the edge detour set, so that all the vertices S' are adjacent to G. Obviously the set S' satisfy the equitable edge detour domination.

Theorem 2.6. *$G = C(2n, 1, 3)$. Then $\gamma_{ed}^{e}(D_2\{G, \{2\}\}) = 4$. Where $2n \equiv k(\bmod n)$ and $k = 0, 1, 2$.*

Proof. Consider two copies of G namely G itself and G' respectively.

In the first copy, let $V(G) = \{(a_0)_1, (a_1)_1, (a_2)_1, \ldots, (a_{2n-1})_1\}$ and $E(G) = 3n$.

In the second copy, let $V(G') = \{(a_0)_2, (a_1)_2, (a_2)_2, \ldots, (a_{2n-1})_2\}$ and $E(G') = 3n$.

Two graphs are connected by the shadow of distance $\{2\}$. Take any one vertex from G and the same shadow vertex from G'. These two vertices form an edge detour set. For domination we need some vertices so that all the vertices of G and G' are adjacent with the dominating set. Obviously, these vertices form an equitable edge detour dominating set, since difference of every two vertices is 0.

For $n = 3$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^{e} with the minimum cardinality.

For $n = 4$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^{e} with the minimum cardinality.

For $n = 5$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^{e} with the minimum cardinality.

Hence, $\gamma_{ed}^{e}(D_2\{G, \{2\}\}) = 4$. Where $2n \equiv k (\mathrm{mod}\, 3)$ and $k = 0, 1, 2$.

Theorem 2.7. $G = C(2n, 1, 3)$. *Then* $\gamma_{ed}^{e}(D_2\{G, \{3\}\}) = 4$. *Where* $2n \equiv k (\mathrm{mod}\, n)$ *and* $k = 0, 1, 2$.

Proof. Consider two copies of G namely G itself and G' respectively.

In the first copy, let $V(G) = \{(a_0)_1, (a_1)_1, (a_2)_1, \ldots, (a_{2n-1})_1\}$ and $E(G) = 3n$.

In the second copy, let $V(G') = \{(a_0)_2, (a_1)_2, (a_2)_2, \ldots, (a_{2n-1})_2\}$ and $E(G') = 3n$.

Two graphs are connected by the shadow of distance $\{3\}$. Take any one vertex from G and the same shadow vertex from G'. These two vertices form an edge detour set. For domination we need some vertices so that all the vertices of G and G' are adjacent with the dominating set. Obviously, these vertices form an equitable edge detour dominating set, since difference between degrees of every two vertices is 0.

For $n = 3$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^{e} with the minimum cardinality.

For $n = 4$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^e with the minimum cardinality.

For $n = 5$, The set $S' = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\}$ forms a minimum γ_{ed}^e with the minimum cardinality.

Hence, $\gamma_{ed}^e(D_2\{G, \{3\}\}) = 4$. Where $2n \equiv k(\text{mod } 3)$ and $k = 0, 1, 2$.

Theorem 2.8. $G = C(2n, 1, r)$, $r = n$, *Then*

(i) $\gamma_{ed}^e(D_2\{G, \{2\}\}) = 4$.

(ii) $\gamma_{ed}^e(D_2\{G, \{3\}\}) = 4$.

Proof. The vertex and edge set are same as in theorem 2.6

Here r takes only the odd values so n should be odd.

For $n = 7$, $G = C(2n, 1, r) = C(14, 1, 7)$ Then

$\gamma_{ed}^e(D_2\{G, \{2\}\}) = \{(a_0)_1, (a_0)_2, (a_3)_1, (a_3)_2\} = 4$.

Similar steps are followed for the brick product of shadow graph of distance $\{3\}$ to get the required equitable edge detour domination.

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