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WHALE OPTIMIZATION ALGORITHM FOR COMBINED HEAT AND POWER ECONOMIC DISPATCH

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Abstract

This paper solves Combined Heat and Power Economic Dispatch (CHPED) using Whale optimization algorithm which is inspired from the prey predator concept of whales. The problem includes power loss and thermal production in transmission systems. The objective of CHPED problem is to minimize the overall fuel cost used for the production of power and heat with respect to equality constraints (Power and Heat). The Whale optimization algorithm (WOA) is inspired by hunting strategy of whales. Whale optimization algorithm is used for solving CHPED system. The system used for solving CHPED using whale optimization consists of 4 power only units, one heat only unit and 2 cogeneration units. For comparison, NSGA-II, Grey Wolf Optimizer and Bee Colony Optimization are used. The results demonstrate that the whale optimization algorithm on CHPED achieves better performance when compared with the existing algorithms.

1. Introduction

The process of Economic dispatch problem main process of allocating generation points to the generation units so that the system load is balanced with constraints economically. The maximum energy disused in process of

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conversion, is changed form of heat, the energy efficiency of the combined cycle of fossil fuel into electricity is about 50-65%. The unit of thermal achieved, the heat energy is not change into electric power which can be used for industrial purpose. The cogeneration is defined as the method of combined heat and power. It is used to recover and it also indicates about the utilization of heat. In a combined head and power cogeneration system, the production of head and power generation is extracted from one fuel source [1]. The combined heat and power generation is to increase the efficiency of conversion. CHP distributes the power as well as heat over the units, in order to reduce the fuel cost. The integration of heat units and power units is complex because of the multiple demands. The feasibility of the CHP is heat along one axis and power along other axis. The CHP which includes cogeneration units and economic dispatch problem is called CHPED problem.

The classical and iterative method to solve CHPED is proposed by Rooijers and Robert [2]. This problem consists of 1 power unit, 1 heat-only unit and 2 cogeneration units. Guo et al. [3] proposed Lagrangian relaxation technique to solve DHPED problem. Evolutionary programming was handling to solve CHPED and a multi-objective swarm optimization algorithm (PSO) is incorporated. This problem contains of 4 power generation units, 2 cogeneration units and 1 heat-only unit. In this problem, the authors have included power losses among different generations [4, 5]. The combination of genetic algorithm, differential evolution and tabu search is used to handle CHPED in [6]. In this paper, the authors use genetic algorithm as base level search and tabu search is used for local search. Genetic algorithm with tabu search fine tune the optimal solution. The Ant colony optimization algorithm and Bee colony are proposed to solve CHPED in [7, 8]. The authors proposed ant colony search algorithm by extracting the positive feedback which avoids premature convergence and helps to find the feasible solution. The author used Bio-inspired algorithm to solve scheduling problem [23]. In this paper, the problem CHPED is explained using Whale Optimization algorithm (WOA). The algorithm is inspired by the foraging behaviour of whale bubble-net strategy. The reminder of the work flowis organized as follows. Section 2 summarizes the related work on CHPED and evolutionary algorithms to solve CHPED. Section 3 describes the Whale Optimization Algorithm and its flowchart. Section 4 explains the problem formulation of the CHPED. Section 5 shows the empirical result of WOA on solving CHPED. Finally section 6 concludes the work.

2. Literature Survey

A. M. Elaiw et al. [9] proposes combined differential evolution (DE) and sequential quadratic programming (SQP) to figure out CHPEED (Combined Head and Power Economic Emission Dispatch) problem. Because of valve value affects this process carried by using nonsmooth and non-convex cost function. To carry a fine tuning the Differential evolution incorporates a global optimizer, and SQP. This method helps to resolve the optimal solution at the final stage of the problem. The testing process carried effectively and the results are compared to the other set of algorithms, to show the better results of proposed hybrid DE-SQP.

Nitin Narang et al. [10] proposed technique, civilized swarm optimization (CSO) is choose as global search technique. To carry the local search Powell's pattern search (PPS) technique is used. The Civilized swarm optimization is having characteristic of particle swarm optimization (PSO) and society civilization algorithm (SCA). In CSO, the civilization is formed by mutually interacting societies and the updation process carried by knowing the position of society particles. This update operation was conducted under the guidance of own leader by knowing their best positions. The enhancement process is carried by PPS to know the best performing particle of CSO. The conjugate search direction method is used in PPS instead of going for Hessian matrix for optimizing the function. The process requires powerful constraints to tackle CHP dispatch problem and it is based on mutual dependency of demand and capacity of heat-power of generating units.

Thang Trung Nguyen et al. [11] propose a cuckoo search algorithm (CSA) for handling the (CHPED). The aim of using CSA is to reduce the entire cost of the fuel to generate electricity and contribute heat to demand of the load capacity. The proposed CSA process is used implemented in engineering optimization problems. It produces few control parameters, to improve quality of solution and quick computational time. The validation process of the proposed algorithm done to showcase it robustness and performance. This process contains five different number of systems, comprises of three systems with quadratic fuel cost function to remove the loss occur during transmission and non-convex fuel cost function used in remaining two systems. The results are compared with other methods to show that the proposed method produce solution with high quality and quick computational time than many other methods.

Akhilesh Gotmare et al. [12] discuss about the exhaustive review on structured to chaotic search approaches and digital filter design. The theme of this article is to use infinite impulse response adaptive filters in various systems. To estimate chaotic system a Hammerstein models as used to review on various swarm and evolutionary computing. These schemes used for identifying system and to carry digital filter. The paper is also features about as a quick reference for a few popular evolutionary computing algorithms.

D. B. Prakash and C. Lakshminarayana [13] propose a Whale Optimization Algorithm improve the reliability and stability of the system. It also used to reduce the line losses and regulate the bus voltage. The algorithm finds an optimal size and capacitors placement for a complex designed distributed system. The factors consider for solving the problem is multi objectives. The factors are reducing operational cost and to suppress power loss with inequality constraints on voltage limits.

Seyedali Mirjalili and Andrew Lewis [14] explain about nature-inspired related meta-heuristic optimization algorithm, known as Whale Optimization Algorithm (WOA). It imitates the social behavior of humpback whales. The strategy used by whale for carrying hunting process is known as bubble-net. The testing process is done with 29 mathematical optimization problems and 6 structural design problems. The final solutions are optimized and it is compared with other swarm based algorithms. It is proved to be very competing compared to the state-of-art meta-heuristic algorithms as well as conventional methods.

Julien Favier et al. [15] discuss the span wise geometrical motion of waves of the leading edge with infinite wing. The Reynolds number monitoring process is carried in the text of passive separation control. The important mechanism is inspired by the humpback whale flippers; the model is done using wavy leading edge. In the sinusoidal function whose amplitude and wavelength constitute the parameters of control. The separation of boundary is mentioned for carrying modifications of the performances of hydro dynamic.

Haider J. Touma [16] explains the meta-heuristic optimization called as Whale Optimization Algorithm method to produce an optimized solution for Economic Dispatch problem. The achieved outcome in this study is compared with other algorithms such as Particle Swarm Optimization, Ant Colony optimization and Genetic Algorithm. By using WOA the scheduling process for economic operation of power plants is done to achieve less generation cost for the power utilities.

Indrajit N. Trivedi et al. [17] propose a novel randomization and adaptive technique is integrated with WOA to improve quality. The logarithmic spiral function used for covering wide area in exploration phase. To obtain global optimal solution an effective adaptive whale optimization Algorithm (AWOA) carried. The technique also produces a faster convergence with less parameter dependency.

H. A. Shayanfar et al. [18] explain about a Particle Swarm Optimization with Improved Inertia Wight (PSOIIW) to handle Combined Heat and Power Economic Dispatch (CHPED) problem. The population based global search and optimization method used to provide an optimized solution for the CHPED problem. The proposed technique has a strong capacity to identify optimized solution with short span of time. The performance of the proposed technique contains power units (four), cogeneration units (two) and one heat unit. These factors are compared to other sets of algorithms such as Particle Swarm Optimization (PSO) and Evolutionary Programming (EP) and Real-coded Genetic Algorithm (RGA) for obtaining total operating cost, better solution at less computational time.

3. Whale Optimization Algorithm

3.1. Whale Optimization Algorithm

Whale Optimization algorithm is inspired from hunting performance of humpback whales. Their hunting technique is called as bubble net feeding method [22]. The working flow of Whale optimization algorithm is depicted in Figure 1. Whales hunting has been defined in 3 ways. 1. Encircling the prey, 2. Bubble-net attacking method, 3. Search for a prey.

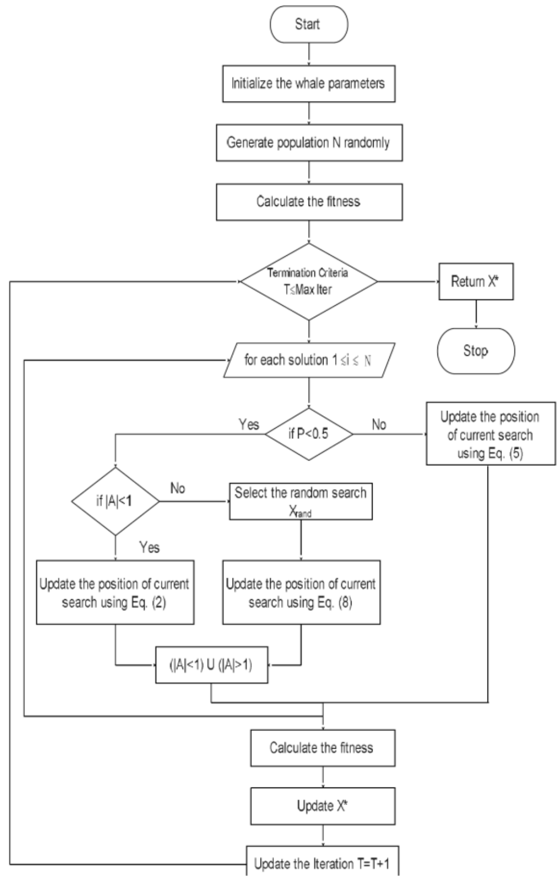

Figure 1. Flowchart of Whale Optimization Algorithm.

Normally the humpback whales knew the exact position of its prey and encircles the bubbles around the prey, but in real world optimization problem the optimal solutions are not known in prior. So Whale optimization algorithm considers the current best solution as its prey and search for a better solution around the current best $(\vec{X}^*)$.

3.1.1. Encircling Prey

The change in position with respected to the current best solution the other whales used to update its position using following equation.

$$\vec{D} = |\ \vec{C} \cdot \vec{X}^*(t) - \vec{X}(t)\ |, \tag{1}$$

where $\vec{D}$ is the distance vector represents the distance between current whale position $(\vec{X})$ and the best whale position $\vec{X}^*$. t represents the iteration number. $\vec{C}$ is the coefficient vector by which the direction towards current best solution can be achieved. Once the distance vector $\vec{D}$ is determined the solution for next iteration can be updated as follows

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D}, \tag{2}$$

where $\vec{A}$ represents a coefficient vector. The coefficient vectors $\vec{A}$ and $\vec{C}$ are calculated in each iteration as follows

$$\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a} \tag{3}$$

$$\vec{C} = 2 \cdot \vec{r}, \tag{4}$$

where $\vec{a}$ has the range [2,0] which can be decreased linearly with respect to the increase in iteration number. And $\vec{r}$ is a random vector range [0, 1]

3.1.2. Bubble-Net Attacking Method

Whale hunts their prey in 2 ways. One is shrinking encircling mechanism which has been explained above. The other one is spiral updating position. In spiral updating position the whales do not exhibit a discontinued circle between its current and predecessor position. There exists a continuous path which resembles a spiral. If this manner is applicable in hunting prey, then the above equation can be rephrased as follows.

$$\vec{X}(t+1) = \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t), \tag{5}$$

where $\vec{D} = | \vec{X}^*(t) - \vec{X}(t) |$ represents the distance between best whale and current whale. Here b is defined as a constant value and l is a random number range [–1, 1].

The author Seyedali Mirjalili, et al. [14] took half of the probability as encircling shrinking mechanism and the other half as spiral update position in exploitation phase. So the algorithm has been modelled as

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} \text{ if } p < 0.5 \\ \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) \text{ if } P \geq 0.5 \end{cases} \tag{6}$$

where p is the random number [0, 1].

3.1.3. Search for Prey

Prey search is a process of exploration. Many optimization algorithms incorporate this exploration phase in different format. Exploration is the process of accessing new search space or to deviate the guided search towards a random search. This leads to exploration of new search space which then can be used to exploit best solutions in the upcoming iterations.

In whale optimization, exploration mechanism has been defined as follows

$$\vec{D} = | \vec{C} \cdot \overrightarrow{X_{rand}} - \vec{X} | \tag{7}$$

$$\vec{X}(t+1) = \overrightarrow{X_{rand}} - \vec{A} \cdot \vec{D}. \tag{8}$$

4. Problem Formulation

The power economic and combined heat problem of a system is to regulate the unit power generation and heat production so that the system production cost of the heat and power generation is minimized while satisfying heat and power demands and other operational constraints. The cost function of the problem is to minimize the objective function

$$F(p, h) = Min \sum_{i=1}^{n_p} C_i(P_i) + \sum_{j=1}^{n_c} C_j(P_j, H_j) + \sum_{k=1}^{n_h} C_k(H_k), \tag{9}$$

where $F(p, h)$ is the total heat and power production cost of the problem; $C_i(P_i)$, $C_j(P_j, H_j)$ and $C_k(H_k)$ are the cost functions of the i^{th} conventional power unit, j^{th} cogeneration unit and k^{th} heat-only unit. P is the unit power generation, h is the unit heat production. n_p, n_c and n_h are the number of conventional power unit, cogeneration unit and heat-only units respectively.

Subject to

Equality constraints

$$\sum_{i=1}^{n_p} P_i + \sum_{j=1}^{n_c} P_j = P_d + P_L \tag{10}$$

$$\sum_{k=1}^{n_h} H_k + \sum_{j=1}^{n_c} H_j = H_d \tag{11}$$

$$P_i^{\min} \leq P_i \leq P_i^{\max},\ i = 1, 2, \ldots, n_p \tag{12}$$

$$P_j^{\min}(H_j) \leq P_j \leq P_j^{\max}(H_j),\ j = 1, 2, \ldots, n_c \tag{13}$$

$$H_j^{\min}(P_j) \leq H_j \leq H_j^{\max}(P_j),\ j = 1, 2, \ldots, n_c \tag{14}$$

$$H_k^{\min} \leq H_k \leq H_k^{\max},\ k = 1, 2, \ldots, n_h, \tag{15}$$

where $P^{\min}$ and $P^{\max}$ are the limits of the unit power capacity, and $H^{\min}$ and $H^{\max}$ are the limits of the unit heat capacity. The active power transmission loss P_L, can the calculated using the formula:

$$P_L = \sum_{i=1}^{n_c} \sum_{j=1}^{n_c} P_i PC_{i,j} P_j, \tag{16}$$

where $PC_{i,j}$ is the loss coefficient connected between i and j.

5. Experimental Results

In this section, the experimental results of the proposed system Whale Optimization Algorithm on Combined Heat and Power Dispatch Problem (Whale-CHPED) is tabulated and discussed. The proposed methodology is implemented in MATLAB version 8.3, on Intel Core i7 processor of 3.2GHz speed and 4GB RAM. The proposed system is implemented in one of the system of CHPED which consists of 4 power only units, 1 heat unit and 2 cogeneration units. The power units are considered along with its power loss during transmission. The demanded power and heat units are of 600MW and 150MWth respectively. The feasible sets for power and heat only units are given in the Appendix. The feasible region for cogeneration units of combined heat and power units are given in Figure A1 and A2. The power loss coefficient table is also given in Appendix.

Table 1. Simulation Results of Whale-CHPED, BCO, GWO, NSGA-II.

	BCO [21]	**GWO* [20]**	**NSGA-II [19]**	**Whale-CHPED**
P_1	43.9457	52.8074	74.5357	40.6311
P_2	98.5888	98.5398	99.3518	100.828
P_3	112.932	112.673	174.719	114.782
P_4	209.771	209.815	211.017	210.049
P_{C1}	98.8000	93.8115	100.936	57.5209
P_{C2}	44.0000	40.0000	44.1036	17.9663
H_{C1}	12.0974	29.3704*	24.3678	98.5196
H_{C2}	78.0236	75.0000*	72.5270	74.5127
H_1	59.8790	29.3704*	53.1052	42.5569
Total Power	608.038	607.649	704.664	607.368
Total Heat	150	133.741*	150	150
Total Cost	10317	10111.24*	10712.86	10290.82
Total Loss	8.0384	7.6499	104.664	7.3681

*Wrong results

Here the parameters are selected as $N = 50$, Maximum iteration as 500, $\vec{a}$ ranges (2, 0), $\vec{a}$ random variable ranges (0, 1) and l ranges (–1, 1). The proposed algorithm is compared with three existing algorithms such as Non-Dominated Sorting Genetic Algorithm-II [19], Grey Wolf Optimizer [20] and Bee Colony Optimization algorithm [21]. For comparison with proposed algorithm the simulated results are taken from the papers sited in Table 1.

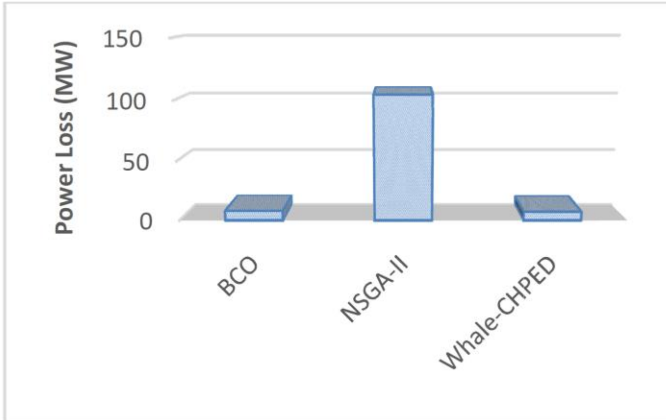

Figure 2. Comparison of Power Loss (MW).

Table 1 compares the result of simulated proposed algorithm on CHPED with other existing techniques on CHPED. ∗represents that the results given in Table 1 is wrong since it does not meet the equality constraint of Heat demand of CHPED. As per the results of Table 1, GWO results are not correct and hence that result is not considered for comparison with other algorithms.

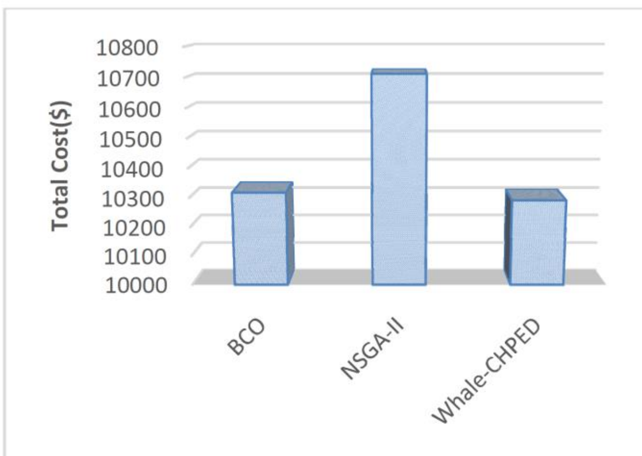

Figure 3. Comparison of Total Cost ($).

From figure 2 it can be perceived that the proposed whale-CHPED reduce the loss by 9% when compared with BCO and 100% reduction when compared with NSGA-II. From figure 3 the comparison of total cost is done for BCO, NSGA-II and the proposed algorithm. On comparing the results the proposed algorithm reduce the cost by 27$ when compared to BCO and 422$ on comparing NSGA-II.

6. Conclusion

In this paper, the author has implemented whale optimization algorithm for solving CHPED in which the system consists of 4 power units, 2 cogeneration units and one heat unit. This system also considers the transmission loss of power. The intelligence during hunting of prey by the whales are properly mapped with CHPED problem and converged towards optimal solution. The results are then compared with the existing algorithms and the performance are shown using figure 2 and 3. The future work of this algorithm may be extended to the multi-objective problem Combined Heat and Power Economic Emission Dispatch.

Appendix

a. Four pure power units

$$F_{p1}(P_{F1}) = 25 + 2P_{P1} + 0.008P_{P1}^2 + |\,100 * \sin(0.042 * (P_{P1\cdot\min} - P_{P1}))\,|$$

$10 \le P_{P1} \le 75$

$$F_{p2}(P_{P2}) = 60 + 18P_{P2} + 0.003P_{P2}^2 + |\,140 * \sin(0.04 * (P_{P2\cdot\min} - P_{P2}))\,|$$

$20 \le P_{P2} \le 125$

$$F_{p3}(P_{P3}) = 100 + 2.1P_{P3} + 0.0012P_{P3}^2 + |\,160 * \sin(0.038 * (P_{P3\cdot\min} - P_{P3}))\,|$$

$30 \le P_{P3} \le 175$

$$F_{p4}(P_{P4}) = 120 + 2P_{P4} + 0.001P_{P4}^2 + |\,180 * \sin(0.037 * (P_{P4\cdot\min} - P_{P4}))\,|$$

$40 \le P_{P4} \le 250$

b. Cogeneration Units

$$F_{C1}(P_{C1}, H_{C1}) = 2650 + 14.5P_{C1} + 0.0345P_{C1}^2 + 4.2H_{C1} + 0.03H_{C1}^2$$
$$+0.031P_{C1}, H_{C1}(\$/h)$$

$$F_{C2}(P_{C2},\ H_{C2}) = 1250 + 36P_{C2} + 0.0435P_{C2}^2 + 0.6H_{C2} + 0.0276H_{C2}^2$$
$$+0.011P_{C2},\ H_{C2}(\$/h)$$

c. Heat-only unit

$$F_{k1}(H_{K1}) = 950 + 2.0109H_{K1} + 0.038H_{K1}^2(\$/h)0 \le H_{K1} \le 2695.2.$$

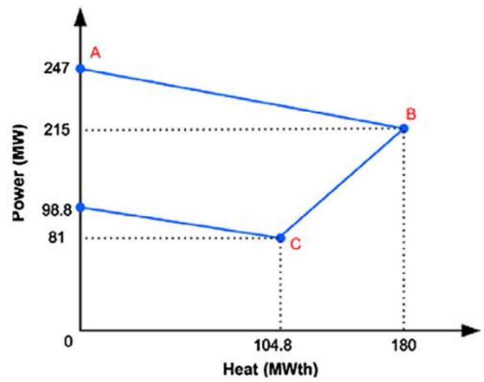

Figure A1. Heat Power Feasible Region of Cogeneration unit 1.

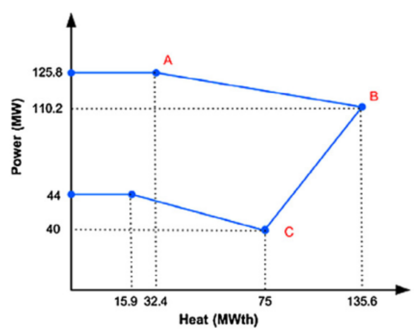

Figure A2. Heat Power Feasible Region of Cogeneration unit 2.

Power Losses Coefficient

$$PC = 10^{-6*}\begin{vmatrix} 39 & 10 & 12 & 15 & 15 & 16 \\ 10 & 40 & 14 & 11 & 15 & 20 \\ 12 & 14 & 35 & 17 & 20 & 18 \\ 15 & 11 & 17 & 39 & 25 & 19 \\ 15 & 15 & 20 & 25 & 49 & 14 \\ 16 & 20 & 18 & 19 & 14 & 45 \end{vmatrix}.$$

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