Advances and Applications in Mathematical Sciences
Volume 21, Issue 10, August 2022, Pages 6049-6069

RELIABILITY FORECAST FOR PEEL REMOVER PLANT OF RICE BY EMPLOYING ALGEBRA OF LOGICS

REENA, M. M. GUPTA and A. KUMAR

Research Scholar
Department of Statistics
Meerut College, Meerut U. P., India

Department of Statistics
Meerut College, Meerut U. P., India

Department of Mathematics
Brahmanand P. G. College
Kanpur-208004, U. P., India
E-mail: KALSANIA08@gmail.com

Abstract

In this paper, the authors have considered a peel remover plant of Rice by employing algebra of logic. Several processes occur in manufacturing the area, e.g., synthesis, decomposition, crystallization, and recovery. This paper discusses the peel remover process with some redundant (parallel and standby) arrangements. The system configuration is shown in figure 1. The authors have used the algebra of logic and Boolean function technique for the mathematical model and its solution. Reliability and mean time to failure, i.e., M.T.T.F. for the considered system, have been evaluated by using Mathematica. Some particular cases have also been given to improve the practical utility of the model. Graphical illustration followed by numerical example has appended in last to highlight the significant results.

Introduction

India has the second position in the production of rice and first position in exporter of rice. In the financial year 1980, rice production was 53.6 million tons while now it has been increased up to 120 million tons in the financial year 2020-21. Rice is also a prominent grain of this country. This is the subject of pride that the largest area of India is used for rice cultivation

2020 Mathematics Subject Classification: 90B25, 06E30.
Keywords: Reliability, Availability, M. T. T. F. and Boolean function technique.
Received December 16, 2021; Accepted March 13, 2022

because this is the principal food crop. It is that rice is the dominant crop of India. Rice is the principal food crop, and being a tropical plant, it grows pleasantly in hot and humid weather. Rice is mainly flourished in rain-fed areas that experienced heavy annual rainfall. So that it is fundamentally known as the Kharif crop in India. The temperature required for the flourished rice crop is around 25 degrees Celsius and above 100 cm average rainfall. Irrigation is the other technique for Rice grown in those areas that have comparatively less rainfall. Rice is the leading food of eastern and southern parts of India.

Most rice varieties are roughly 20% rice husk, 11% bran layers, and 69% starchy endosperm, also referred to as the total milled rice.

An ideal milling process will outcome in the following fractions: About 20% husk, about 8-12% bran founding on the milling process, and about 68-72% milled Rice or white Rice counting on the variety. Total milled Rice carries whole grains or head rice and is broken. The by-products in rice milling are rice husk, rice germ and bran layers, and fine broken.

There are three types of milling systems used; one-step, two-step, and multistage milling. The main intention of the rice milling systems is to separate the hull and the bran of the rice kernel, which produces the Rice eatable and free from impurities. Depending on the customer's requirements, the Rice should have the minimum number of broken kernels possible (typically 12-15%). Here we are using multistage milling ice milling at the commercial level is generally multistage. This method necessitates a more complex system to minimize grain breakage. This is done by the thermal build-up and reducing stress. There are eight steps in the multistage process listed. Rice kernels are 20% rice husk, 11% rice bran, and 69% starchy endosperm (milled Rice). The endosperm section is generally packaged and sold for either direct consumption or further processing. The other 31% of the rice kernel becomes waste by-products. The by-products of the milling process are fine broken Rice, rice germ, husks, and bran. These products were discarded as waste. However, rice bran, rice husk, and broken Rice have industrial and bioactive applications for humans and animals.

System Descriptions.

The peel remover plant consists of seven sub-systems. The whole process of peel removal has been divided into seven sub-processes: A, B, C, D, E, F, and G. The sub-system A is an elevator with five units connected in series for inputting the paddy transporting other materials. Sub-system B is Cleaner which has two parallel units connected by a switching device. Sub-systems C and D are husking and Separator, respectively connected in series. Sub-system E is a whitener having two parallel units connected by a switching device. Sub-system F and G are polishing and grading units, respectively connected in series. All these sub-systems are connected in series, and output comes through the sub-system G. This process of decomposition of urea is of 1-out-of-7. F nature.

Elevator. An elevator is an inclination transport vehicle that brings people and goods to different building floors. With the help of the elevator, things can now be moved very quickly and efficiently. These elevators or elevators are generally powered by electric motors that either drive traction cables and counterweight systems or pump hydraulic fluid to the elevator a cylindrical piston. These are five identical units working in series and cause complete failure of the system if they fail.

Cleaning. The function of the cleaner is to remove all impurities and unfilled grains from the paddy. These are two identical units working in parallel connected by a switching device on the failure of one unit second will be online.

Husking. Paddy is put in the husking machine, which works to remove the top shell. Failure of this sub-system cause complete failure of the system.

Separation. After husking, Rice is separated, and paddy is separated by paddy separator machine. Failure of this sub-system cause complete failure of the system.

Whitening. The whitening machine is then used to separate the brown rice layer and part of the microbes from the Rice. These are two identical units working in parallel connected by a switching device on the failure of one unit second will be online.

Polishing. Then, the Rice is polished with a polishing machine. These are two identical units working in series on the failure of one unit cause complete failure of the system.

Grading. After this, small pieces of Rice are separated by a grading machine. Then it is packed and sent to the market. Failure of this sub-system cause complete failure of the system.

Assumptions. The following assumptions have been associated with this model: -

At time $t = 0$, all the units of the system are good.

(1) The reliability of every unit of the system is known in advance.

(2) The system used is non-repairable.

(3) The transition from one unit to another is a hundred percent reliable and takes no time.

(4) The state of every unit of the system is either good or bad.

(5) This system is 1-out-of-7: F system.

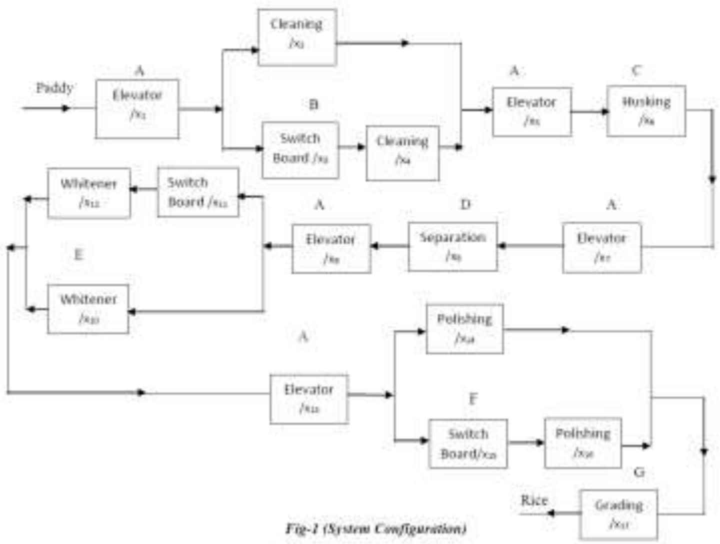

Fig-1 (System Configuration)

Nomenclature.

$x_1/x_5/x_7/x_9/x_{13}$: State of sub-system A.

x_2/x_3 : States of sub-system B.

x_6 : States of sub-system C.

x_8 : States of sub-system D.

x_{10}, x_{12} : State of sub-system E

x_{14}, x_{16} : State of sub-system F

x_{17} : State of sub-system G

x_3, x_{11}, x_{15} : State of switching device.

$x_i (i = 1, 2, \ldots, 17)$: 0, in a bad state; 1, in a good state.

x_i' : Negation of x_i, $\forall i = 1, 2, \ldots, 17$.

$\wedge/\vee$: Conjunction / Disjunction.

$|\ |$: Logical Matrix.

$R_i (i = 1, 2, \ldots, 17)$: Reliability of i^{th} unit of the system.

$Q_i (i = 1, 2, \ldots, 17)$: Unreliability of i^{th} unit of the system and, $= 1 - R_i$.

$a_i (i = 1, 2, \ldots, 17)$: The failure rate of the i^{th} unit of the system.

Formulation of Mathematical Problem

By employing the Boolean function technique, the conditions of capability for the successful operation of the considered system in terms of the logical matrix are expressed as:

$$F(x_1, x_2, \ldots, x_{17}) = \begin{vmatrix} x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14} & x_{17} & & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{15} & x_{16} & x_{17} & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{11} & x_{12} & x_{13} & x_{14} & x_{17} & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{11} & x_{12} & x_{13} & x_{15} & x_{16} & x_{17} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14} & x_{17} & & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{15} & x_{16} & x_{17} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{11} & x_{12} & x_{13} & x_{14} & x_{17} & \\ x_1 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{11} & x_{12} & x_{13} & x_{15} & x_{16} & x_{17} \end{vmatrix} \quad (1)$$

Solution of the Problem

Applying algebra of logics, equation (1) may be rewritten as:

$$F(x_1, x_2, \ldots, x_{17}) = \left| x_1 x_5 x_6 x_7 x_8 x_9 x_{13} x_{17} \wedge f(x_2, x_3, x_4, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16}) \right| \quad (2)$$

where,

$$f(x_2, x_3, x_4, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16}) = \begin{vmatrix} x_2 & x_{10} & x_{14} & & & \\ x_2 & x_{10} & x_{15} & x_{16} & & \\ x_2 & x_{11} & x_{12} & x_{14} & & \\ x_2 & x_{11} & x_{12} & x_{15} & x_{16} & \\ x_3 & x_4 & x_{10} & x_{14} & & \\ x_3 & x_4 & x_{10} & x_{15} & x_{16} & \\ x_3 & x_4 & x_{11} & x_{12} & x_{14} & \\ x_3 & x_4 & x_{11} & x_{12} & x_{15} & x_{16} \end{vmatrix} \quad (3)$$

Now substituting

$A_1 = | x_2 \; x_{10} \; x_{14} |$

$A_2 = | x_2 \; x_{10} \; x_{15} \; x_{16} |$

$A_3 = | x_2 \; x_{11} \; x_{12} \; x_{14} |$

$A_4 = | x_2 \; x_{11} \; x_{12} \; x_{15} \; x_{16} |$

$A_5 = | x_3 \; x_4 \; x_{10} \; x_{14} |$

$A_6 = | x_3 \; x_4 \; x_{10} \; x_{15} \; x_{16} |$

$A_7 = | x_3 \; x_4 \; x_{11} \; x_{12} \; x_{14} |$

and $A_8 = | x_3 \; x_4 \; x_{11} \; x_{12} \; x_{15} \; x_{16} |$

in equation (3), it can be written as:

$$f(x_2, x_3, x_4, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16}) \begin{vmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \end{vmatrix} \tag{4}$$

Using the process of orthogonalization, we can write equation (4) as:

$f(x_2, x_3, x_4, x_{10}, x_{11}, x_{12}, x_{14}, x_{15}, x_{16})$

$$\begin{vmatrix} A_1 & & & & & & & \\ A_1' & A_2 & & & & & & \\ A_1' & A_2' & A_3 & & & & & \\ A_1' & A_2' & A_3' & A_4 & & & & \\ A_1' & A_2' & A_3' & A_4' & A_5 & & & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6 & & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6' & A_7 & \\ A_1' & A_2' & A_3' & A_4' & A_5' & A_6' & A_7' & A_8 \end{vmatrix} \tag{5}$$

Using algebra of logics, we may obtain the following:

$$A'_1 = \begin{vmatrix} x'_2 & & \\ x_2 & x'_{10} & \\ x_2 & x_{10} & x'_{14} \end{vmatrix} \tag{6}$$

$$A'_2 = \begin{vmatrix} x'_2 & & & \\ x_2 & x'_{10} & & \\ x_2 & x_{10} & x'_{15} & \\ x_2 & x_{10} & x_{15} & x'_{16} \end{vmatrix} \tag{7}$$

$$A'_3 = \begin{vmatrix} x'_2 & & & \\ x_2 & x'_{11} & & \\ x_2 & x_{11} & x'_{12} & \\ x_2 & x_{11} & x_{12} & x'_{14} \end{vmatrix} \tag{8}$$

$$A'_4 = \begin{vmatrix} x'_2 & & & & \\ x_2 & x'_{11} & & & \\ x_2 & x_{11} & x'_{12} & & \\ x_2 & x_{11} & x_{12} & x'_{15} & \\ x_2 & x_{11} & x_{12} & x_{15} & x'_{16} \end{vmatrix} \tag{9}$$

$$A'_5 = \begin{vmatrix} x'_3 & & & \\ x_3 & x'_4 & & \\ x_3 & x_4 & x'_{10} & \\ x_3 & x_4 & x_{10} & x'_{14} \end{vmatrix} \tag{10}$$

$$A'_6 = \begin{vmatrix} x'_3 & & & & \\ x_3 & x'_4 & & & \\ x_3 & x_4 & x'_{10} & & \\ x_3 & x_4 & x_{10} & x'_{15} & \\ x_3 & x_4 & x_{10} & x_{15} & x'_{16} \end{vmatrix} \tag{11}$$

$$A'_7 = \begin{vmatrix} x'_3 & & & & \\ x_3 & x'_4 & & & \\ x_3 & x_4 & x'_{11} & & \\ x_3 & x_4 & x_{11} & x'_{12} & \\ x_3 & x_4 & x_{11} & x_{12} & x'_{14} \end{vmatrix} \tag{12}$$

$$A_8' = \begin{vmatrix} x_3' & & & & & \\ x_3 & x_4' & & & & \\ x_3 & x_4 & x_{11}' & & & \\ x_3 & x_4 & x_{11} & x_{12}' & & \\ x_3 & x_4 & x_{11} & x_{12} & x_{15}' & \\ x_3 & x_4 & x_{11} & x_{12} & x_{15} & x_{16}' \end{vmatrix} \tag{13}$$

Now, we have

$$A_1' \; A_2 = \begin{vmatrix} x_2' & & \\ x_2 & x_{10}' & \\ x_2 & x_{10} & x_{14}' \end{vmatrix} \wedge |\, x_2 \;\; x_{10} \;\; x_{15} \;\; x_{16} \,| = |\, x_2 \;\; x_{10} \;\; x_{14}' \;\; x_{15} \;\; x_{16} \,| \tag{14}$$

Similarly,

$$A_1' \, A_2' \, A_3' = |\, x_2 \;\; x_{10}' \;\; x_{11} \;\; x_{12} \;\; x_{14} \,| \tag{15}$$

$$A_1' \, A_2' \, A_3' \, A_4 = |\, x_2 \;\; x_{10}' \;\; x_{11} \;\; x_{12} \;\; x_{14}' \;\; x_{15} \;\; x_{16} \,| \tag{16}$$

$$A_1' \, A_2' \, A_3' \, A_4' \, A_5 = |\, x_2' \;\; x_3 \;\; x_4 \;\; x_{10} \;\; x_{14} \,| \tag{17}$$

$$A_1' \, A_2' \, A_3' \, A_4' \, A_5' \, A_6 = |\, x_2' \;\; x_3 \;\; x_4 \;\; x_{10} \;\; x_{14}' \;\; x_{15} \;\; x_{16} \,| \tag{18}$$

$$A_1' \, A_2' \, A_3' \, A_4' \, A_5' \, A_6' \, A_7 = |\, x_2' \;\; x_3 \;\; x_4 \;\; x_{10}' \;\; x_{11} \;\; x_{12} \;\; x_{15} \;\; x_{16} \,| \tag{19}$$

$$A_1' \, A_2' \, A_3' \, A_4' \, A_5' \, A_6' \, A_7' \, A_8 = |\, x_2' \;\; x_3 \;\; x_4 \;\; x_{10}' \;\; x_{11} \;\; x_{12} \;\; x_{14} \;\; x_{15} \;\; x_{16} \,| \tag{20}$$

By making use of equation (14) through (20) in equation (5) and then in equation (2), we get:

$F(x_1, x_2, \ldots, x_{17}) =$

$$\begin{vmatrix} x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14} & x_{17} & & & & & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14}' & x_{15} & x_{16} & x_{17} & & & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}' & x_{11} & x_{12} & x_{13} & x_{14} & x_{17} & & & & \\ x_1 & x_2 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}' & x_{11} & x_{12} & x_{13} & x_{14}' & x_{15} & x_{16} & x_{17} & & \\ x_1 & x_2' & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14} & x_{17} & & & & \\ x_1 & x_2' & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{13} & x_{14}' & x_{15} & x_{16} & x_{17} & & \\ x_1 & x_2' & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}' & x_{11} & x_{12} & x_{13} & x_{15} & x_{16} & x_{17} & \\ x_1 & x_2' & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}' & x_{11} & x_{12} & x_{13} & x_{14}' & x_{15} & x_{16} & x_{17} \end{vmatrix} \tag{21}$$

Finally, the probability of profitable operation, i.e., reliability of the system as a whole, is given by

$R_S = P_r\{F(x_1, x_2, \ldots, x_{17}) = 1\}$

$= R_1R_5R_6R_7R_8R_9R_{13}R_{17}\,[R_2R_{10}R_{14} + R_2R_{10}Q_{14}R_{15}R_{16} + R_2Q_{10}R_{11}R_{12}R_{14} + R_2Q_{10}R_{11}R_{12}Q_{14}R_{15}R_{16} + Q_2R_3R_4R_{10}R_{14} + Q_2R_3R_4R_{10}Q_{14}R_{15}R_{16} + Q_2R_3R_4 Q_{10}R_{11}R_{12}R_{15}R_{16} + Q_2R_3R_4Q_{10}R_{11}R_{12}Q_{14}Q_{15}R_{16}]$

$= R_1R_5R_6R_7R_8R_9R_{13}R_{17}\,[R_2R_{10}R_{14} + R_2R_{10}R_{15}R_{16} + R_2R_{11}R_{12}R_{14} + R_3R_4 R_{10}R_{14} + R_2R_{11}R_{12}R_{15}R_{16} + R_3R_4R_{10}R_{15}R_{16} + R_3R_4R_{11}R_{12}R_{15}R_{16} + R_3R_4 R_{11}R_{12}R_{15}R_{16} + R_2R_{10}R_{11}R_{12}R_{14}R_{15}R_{16} + R_2R_3R_4R_{10}R_{14}R_{15}R_{16} + R_2R_3R_4 R_{10}R_{11}R_{12}R_{15}R_{16} + R_2R_3R_4R_{10}R_{11}R_{12}R_{15}R_{16} + R_2R_3R_4R_{11}R_{12}R_{14}R_{15}R_{16} + R_3R_4R_{10}R_{11}R_{12}R_{14}R_{15}R_{16} - R_2R_{10}R_{14}R_{15}R_{16} - R_2R_{10}R_{11}R_{12}R_{14} - R_2R_3 R_4R_{10}R_{14} - R_2R_{10}R_{11}R_{12}R_{15}R_{16} - R_2R_{11}R_{12}R_{14}R_{15}R_{16} - R_2R_3R_4R_{10}R_{15}R_{16} - R_3R_4R_{10}R_{14}R_{15}R_{16} - R_2R_3R_4R_{11}R_{12}R_{15}R_{16} - R_3R_4R_{10}R_{11}R_{12}R_{15}R_{16} - R_2 R_3R_4R_{11}R_{12}R_{15}R_{16} - R_3R_4R_{10}R_{11}R_{12}R_{15}R_{16} - R_3R_4R_{11}R_{12}R_{14}R_{15}R_{16} - R_2 R_3R_4R_{10}R_{11}R_{12}R_{14}R_{15}R_{16}]$

(22)

Some Particular Cases

Case i. If reliability of each component of the system is R

In this case, equation (22) yields:

$$R_S = R^{11} + 3R^{12} - R^{13} - 2R^{14} - 3R^{15} + 4R^{16} - R^{17} \quad (23)$$

Case ii. When all failures follow Weibull time distribution

Let $a_i (i = 1, 2, \ldots, 17)$ be the failure rate of i^{th} component of the complex system and α be a positive parameter than; in this case, reliability of the whole system, at instant t, can be obtained from equation (22) as:

$$(1)\ R_{SW}(t) = \sum_{i=1}^{14} \exp\cdot\{-\lambda_i t^{\alpha}\} - \sum_{j=1}^{13} \exp\cdot\{-\mu_j t^{\alpha}\} \quad (24)$$

where λ_i's and μ_j's are given as under:

$\lambda_1 = c + a_2 + a_{10} + a_{14}$

$\lambda_2 = c + a_2 + a_{10} + a_{15} + a_{16}$

$$\lambda_3 = c + a_2 + a_{11} + a_{12} + a_{14}$$

$$\lambda_4 = c + a_3 + a_4 + a_{10} + a_{14}$$

$$\lambda_5 = c + a_2 + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\lambda_6 = c + a_3 + a_4 + a_{10} + a_{15} + a_{16}$$

$$\lambda_7 = c + a_3 + a_4 + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\lambda_8 = c + a_3 + a_4 + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\lambda_9 = c + a_2 + a_{10} + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

$$\lambda_{10} = c + a_2 + a_3 + a_4 + a_{10} + a_{14} + a_{15} + a_{16}$$

$$\lambda_{11} = c + a_2 + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\lambda_{12} = c + a_2 + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\lambda_{13} = c + a_2 + a_3 + a_4 + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

$$\lambda_{14} = c + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

$$\mu_1 = c + a_2 + a_{10} + a_{14} + a_{15} + a_{16}$$

$$\mu_2 = c + a_2 + a_{10} + a_{11} + a_{12} + a_{14}$$

$$\mu_3 = c + a_2 + a_3 + a_4 + a_{10} + a_{14}$$

$$\mu_4 = c + a_2 + a_{10} + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\mu_5 = c + a_2 + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

$$\mu_6 = c + a_2 + a_3 + a_4 + a_{10} + a_{15} + a_{16}$$

$$\mu_7 = c + a_3 + a_4 + a_{10} + a_{14} + a_{15} + a_{16}$$

$$\mu_8 = c + a_2 + a_3 + a_4 + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\mu_9 = c + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\mu_{10} = c + a_2 + a_3 + a_4 + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\mu_{11} = c + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{15} + a_{16}$$

$$\mu_{12} = c + a_3 + a_4 + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

$$\mu_{13} = c + a_2 + a_3 + a_4 + a_{10} + a_{11} + a_{12} + a_{14} + a_{15} + a_{16}$$

and $c = a_1 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{13} + a_{17}$

(2) Mean time to failure (M.T.T.F)

The expression for MTTF is given by

$$MTTF = \int_0^\infty R_{SW}(t)dt$$

$$= \sum_{i=1}^{14} \frac{1}{\alpha} \frac{\Gamma(1/\alpha)}{(\lambda_i)^{1/\alpha}} - \sum_{j=1}^{13} \frac{1}{\alpha} \frac{\Gamma(1/\alpha)}{(\mu_j)^{1/\alpha}} \tag{25}$$

(3) System's life time distribution P.D.F:

The system life time p.d.f. is given by

$$f(t) = -\frac{d}{dt} R_{SW}(t)$$

$$= \alpha t^{\alpha-1} \left[\sum_{i=1}^{14} \lambda_i \exp \cdot \{-\lambda_i t^\alpha\} - \sum_{j=1}^{13} \mu_j \exp \cdot \{-\mu_j t^\alpha\} \right] \tag{26}$$

(4) System's Hazard rate:

The system's hazard rate can be obtained by using the formula

$$r(t) = \frac{f(t)}{R_{SW}(t)} = \frac{\alpha t^{\alpha-1} \sum_{i=1}^{14} \lambda_i \exp \cdot \{-\lambda_i t^\alpha\} - \sum_{j=1}^{13} \mu_j \exp \cdot \{-\mu_j t^\alpha\}}{\sum_{i=1}^{14} \exp \cdot \{-\lambda_i t^\alpha\} - \sum_{j=1}^{13} \exp \cdot \{-\mu_j t^\alpha\}} \tag{27}$$

(5) Moments:

$$E(T) = \int_0^\infty t f(t) dt$$

$$= \int_0^\infty t\alpha t^{\alpha-1}\left[\sum_{i=1}^{14}\lambda_i \exp\cdot\{-\lambda_i t^\alpha\} - \sum_{j=1}^{13}\mu_j \exp\cdot\{-\mu_j t^\alpha\}\right]dt$$

$$= \sum_{i=1}^{14}\frac{1}{\alpha}\frac{\Gamma(1/\alpha)}{(\lambda_i)^{1/\alpha}} - \sum_{j=1}^{13}\frac{1}{\alpha}\frac{\Gamma(1/\alpha)}{(\mu_j)^{1/\alpha}} \tag{28}$$

Also, $$E(T^2) = \sum_{i=1}^{14}\frac{2}{\alpha}\frac{\Gamma(2/\alpha)}{(\lambda_i)^{2/\alpha}} - \sum_{j=1}^{13}\frac{2}{\alpha}\frac{\Gamma(2/\alpha)}{(\mu_j)^{2/\alpha}} \tag{29}$$

In general

$$E(T^r) = \sum_{i=1}^{14}\frac{r}{\alpha}\frac{\Gamma(r/\alpha)}{(\lambda_i)^{r/\alpha}} - \sum_{j=1}^{13}\frac{r}{\alpha}\frac{\Gamma(r/\alpha)}{(\mu_j)^{r/\alpha}} \tag{30}$$

(6) Variance of time to failure:

The variance of time to failure is given by

$V(t) = E(T^2) - [E(T^2)]^2$

$$= \frac{2}{\alpha}\Gamma\frac{2}{\alpha}\left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)^{2/\alpha}} - \sum_{j=1}^{13}\frac{1}{(\mu_j)^{2/\alpha}}\right] - \left[\frac{1}{\alpha}\Gamma\frac{1}{\alpha}\right]^2\left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)^{1/\alpha}} - \sum_{j=1}^{13}\frac{1}{(\mu_j)^{1/\alpha}}\right]^2 \tag{31}$$

Case iii. When all failures follow exponential time distribution.

As we know, the exponential distribution is a specific case of Weibull distribution $\alpha = 1$. Hence, the reliability of the whole system at an instant's, in this case, is given by:

(1) $$R_{SE}(t) = \sum_{i=1}^{14}\exp\cdot\{-\lambda_i t\} - \sum_{j=1}^{13}\exp\cdot\{-\mu_j t\} \tag{32}$$

Also, the expression for M.T.T.F., in this case, is

(2) M.T.T.F. $= \int_0^\infty R_{SE}(t)dt$

$$= \sum_{i=1}^{14}\left\{\frac{1}{\lambda_i}\right\} - \sum_{j=1}^{13}\left\{\frac{1}{\mu_j}\right\} \tag{33}$$

(3) System's life time distribution:

$$f(t) = \left[\sum_{i=1}^{14}\lambda_i \exp\cdot\{-\lambda_i t\} - \sum_{j=1}^{13}\mu_j \exp\cdot\{-\mu_j t\}\right] \tag{34}$$

(4) System's Hazard rate:

$$r(t) = \frac{f(t)}{R_{SE}(t)} = \frac{\sum_{i=1}^{14}\lambda_i \exp\cdot\{-\lambda_i t\} - \sum_{j=1}^{13}\mu_j \exp\cdot\{-\mu_j t\}}{\sum_{i=1}^{14}\exp\cdot\{-\lambda_i t\} - \sum_{j=1}^{13}\exp\cdot\{-\mu_j t\}} \tag{35}$$

(5) Moments:

$$E(T) = \sum_{i=1}^{14}\frac{1}{(\lambda_i)} - \sum_{j=1}^{13}\frac{1}{(\mu_j)} \tag{36}$$

$$E(T^2) = 2\left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)^2} - \sum_{j=1}^{13}\frac{1}{(\mu_j)^2}\right] \tag{37}$$

$$E(T^r) = r\lceil r\left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)^2} - \sum_{j=1}^{13}\frac{1}{(\mu_j)^r}\right] \tag{38}$$

(6) Variance of time to failure:

$$V(T) = 2\left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)^2} - \sum_{j=1}^{13}\frac{1}{(\mu_j)^2}\right] - \left[\sum_{i=1}^{14}\frac{1}{(\lambda_i)} - \sum_{j=1}^{13}\frac{1}{(\mu_j)}\right]^2 \tag{39}$$

where, values of λ_i's and μ_j's have mentioned earlier.

Numerical Computation

For a numerical computation

(i) Setting $a_i (i = 1, 2, \ldots, 17) = 0.003$ and $\alpha \in [0.6, 1.8]$ with the space 0.2 in equation (24), (26) and (27) where time '*t*' varies between 0 to 10;

(ii) Setting $a_i (i = 1, 2, \ldots, 17) = \alpha = 0, 0.1, \ldots, 1.0$ and $\alpha \in [0.6, 1.8]$ in equation (25), one may compute the table of Reliability for Weibull time distribution $R_{SW}(t)$, Density Function $f(t)$ System's Hazard rate $r(t)$ and Mean time to failure M.T.T.F vs a by using Mathematica in the table 1, 2, 3 and 4. Corresponding graphs have shown through respective figures.

Table 1. Reliability $R_{SW}(t)$ w.r.t Time '*t*':

Time '*t*'	Reliability for Weibull Distribution $R_{SW}(t)$						
	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
0	1	1	1	1	1	1	1
1	0.973378	0.973378	0.973378	0.973378	0.973378	0.973378	0.973378
2	0.959939	0.954127	0.947495	0.939936	0.931331	0.921546	0.910438
3	0.949202	0.937136	0.922326	0.904215	0.882164	0.855464	0.82335
4	0.939936	0.921546	0.897847	0.867531	0.829122	0.781065	0.721906
5	0.931644	0.906974	0.874036	0.830587	0.774208	0.702664	0.614578
6	0.924061	0.893201	0.850871	0.79382	0.718808	0.623543	0.508327
7	0.917028	0.880087	0.828331	0.757524	0.663925	0.546194	0.40848
8	0.910438	0.867531	0.806396	0.721906	0.610301	0.472468	0.318757
9	0.904215	0.855464	0.785048	0.687113	0.558481	0.403688	0.241379
10	0.898304	0.843827	0.764268	0.65325	0.508862	0.340733	0.17723

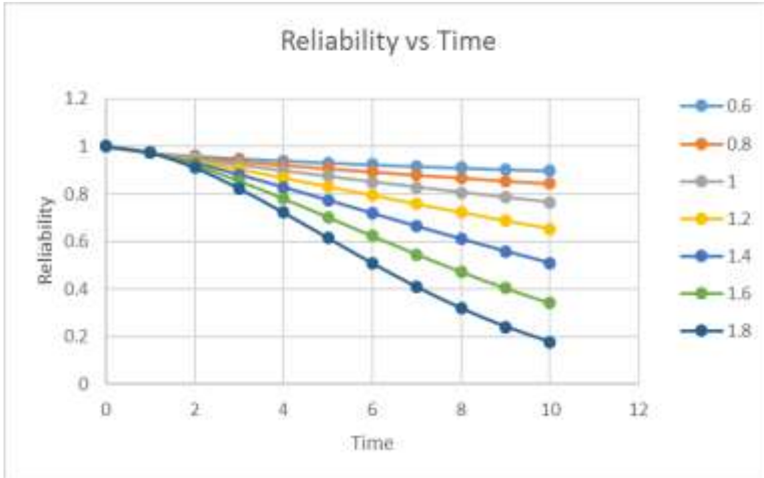

Figure 2.

Table 2. System's life time distribution w.r.t Time '*t*':

Time '*t*'	System's life time distribution P.D.F. $f(t)$						
	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
0	Indeterminate	Indeterminate	0.027	0	0	0	0
1	0.0157489	0.020999	0.026248	0.031498	0.036748	0.041997	0.0472467
2	0.0117637	0.017904	0.025522	0.034889	0.046313	0.06014	0.0767509
3	0.00988617	0.016204	0.02482	0.036352	0.051509	0.071064	0.0957935
4	0.0087223	0.015035	0.024141	0.036903	0.054249	0.077022	0.105679
5	0.0079046	0.014144	0.023485	0.036914	0.055351	0.079236	0.107815
6	0.00728679	0.013425	0.022849	0.036572	0.055284	0.078593	0.103801
7	0.0067972	0.012821	0.022234	0.035986	0.054359	0.075799	0.0952754
8	0.00639591	0.012301	0.021638	0.035226	0.0528	0.071434	0.0838029
9	0.00605861	0.011844	0.021061	0.034343	0.050775	0.065981	0.0708097
10	0.00576952	0.011436	0.020501	0.033371	0.048415	0.059845	0.0575312

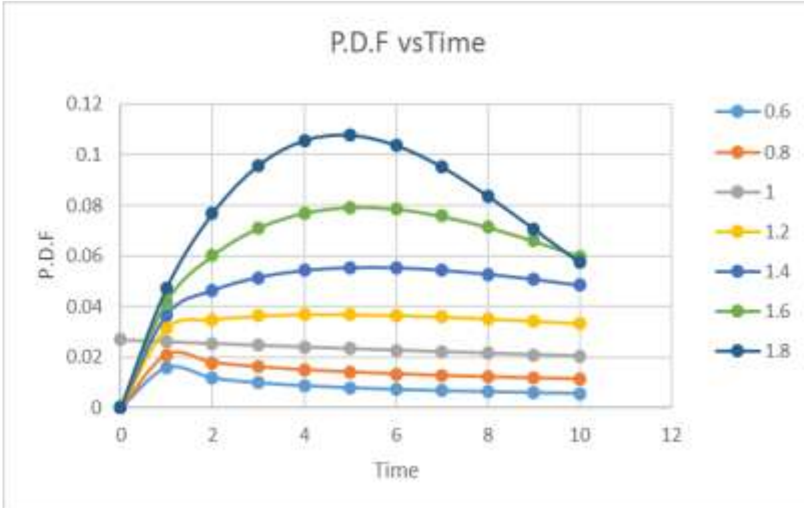

Figure 3.

Table 3. System's Hazard rate w.r.t Time 't'.

Time 't'	System's Hazard rate $r(t)$						
	α = 0.6	α = 0.8	α = 1.0	α = 1.2	α = 1.4	α = 1.6	α = 1.8
0	Indeterminate	Indeterminate	0.027	0	0	0	0
1	0.0161796	0.0215729	0.026966	0.032359	0.037753	0.043146	0.0485389
2	0.0122546	0.0187646	0.026936	0.037119	0.049728	0.065259	0.0843011
3	0.0104152	0.0172911	0.02691	0.040202	0.058389	0.083071	0.116346
4	0.00927967	0.0163149	0.026888	0.042538	0.06543	0.098611	0.146388
5	0.00848457	0.0155949	0.026869	0.044444	0.071494	0.112765	0.175429
6	0.00788561	0.0150299	0.026854	0.046071	0.076911	0.126043	0.204201
7	0.00741221	0.014568	0.026842	0.047504	0.081875	0.138776	0.233244
8	0.00702509	0.0141793	0.026833	0.048796	0.086515	0.151193	0.262905
9	0.00670041	0.0138451	0.026828	0.049981	0.090917	0.163446	0.293355
10	0.00642267	0.013553	0.026825	0.051084	0.095143	0.175637	0.324614

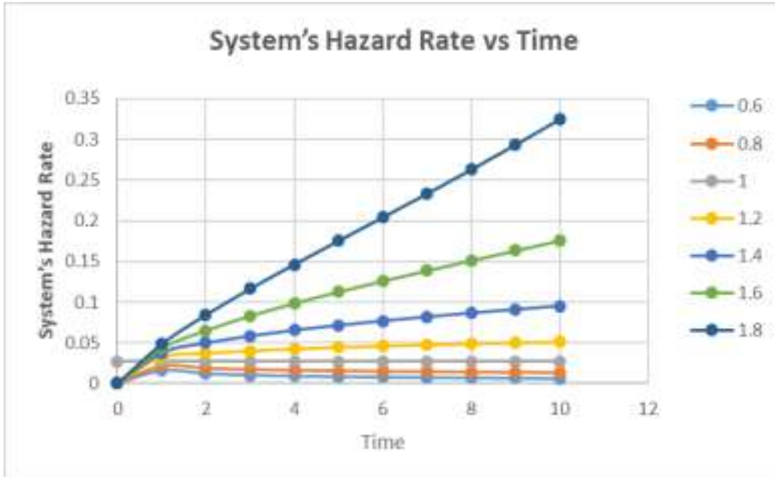

Figure 4.

Table 4. M.T.T.F w.r.t Failure Rate 'a'.

Time 'a'	Mean Time To Failure M.T.T.F.						
	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
0	Complex Infinity	Complex Infinity	Complex Infinity	Complex Infinity	Complex Infinity	Complex Infinity	Complex Infinity
0.1	1.69119	1.24915	1.08629	1.01041	0.97054	0.948192	0.935281
0.2	0.532691	0.525203	0.543144	0.567071	0.591551	0.614827	0.63636
0.3	0.271013	0.316383	0.362096	0.404478	0.442805	0.477195	0.508013
0.4	0.167787	0.220821	0.271572	0.318258	0.360554	0.398666	0.432976
0.5	0.115676	0.167071	0.217258	0.264253	0.307432	0.346769	0.382495
0.6	0.0853637	0.133023	0.181048	0.227005	0.269893	0.309423	0.345649
0.7	0.066023	0.109709	0.155184	0.19964	0.241753	0.281003	0.31728
0.8	0.0528496	0.0928436	0.135786	0.178616	0.21976	0.258503	0.294595
0.9	0.0434298	0.080133	0.120699	0.161917	0.202028	0.240157	0.275935

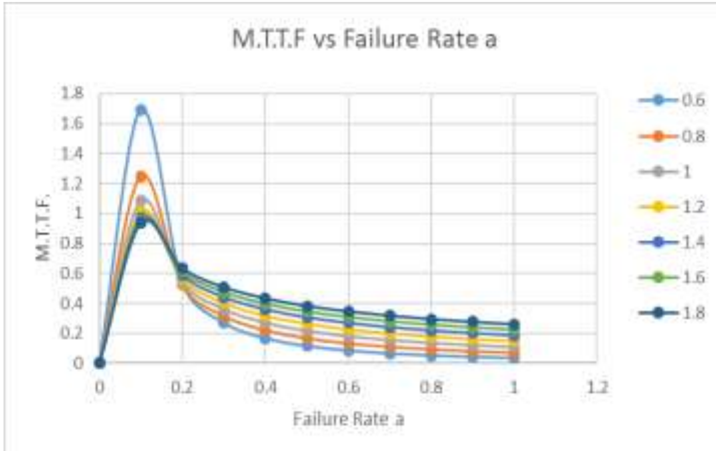

Figure 5.

Conclusion

In this study, the author has considered an industrial problem related to the peel remover plant of rice to evaluate some essential reliability parameters using the algebra of logic and boolean function technique. Some standby and parallel units are arranged in a redundant position to improve the system's performance. Switching device has been used to online standby unit on the failure of main unit. The reliability of the whole system has been computed in three different cases. Also, the M.T.T.F. of the system, in cases failures follow exponential time distribution, has been obtained. A numerical example together with a graphical illustration has been appended in the end to highlight actual results.

A critical examination of table 1 for $R_{SW}(t)$ with t and the corresponding graph which have been shown in figure 2 reveals that the system's reliability decreases rapidly. It is also evident that the reliability tends to zero more rapidly as we increase the value of α.

Table 2 and figure 3 represent the trends for System's life time distribution P.D.F. $f(t)$. For some time, interval the density function increases and after this time interval it decreases also it is observed that from the graphs as we increase the value of α the curve becomes more peaked and its approaches to the curve of normal distribution.

Table 3 and figure 4 provides the trends of Hazard's rate or failure rate to the system and this is clear from the graph. It is also evident that the failure rate increases as we increase the value of α.

The study of M.T.T.F. and corresponding graphs shows that the graph for each value of α is same. The deep observation shows that if $\alpha \in [0, 0.1]$ the M.T.T.F. increases while if $\alpha \in [0.1, 0.2]$ and when $\alpha \in [0.2, 1.0]$ the M.T.T.F. decreases approximately in a uniform way.

References

[1] A. Kumar and Ajeeta Singh, Steady state behavior of ATM with cold standby of bank computers having internal and external repair policy, Life Cycle Reliability and Safety Engineering (2021), 2520-1352.

[2] A. Kumar, Sushil Kumar and Sunit Kumar, Availability estimation of nuclear reactor with standby generators by employing pre-emptive resume repair policy, Malaya Journal of Matematik 9(1) (2021), 829-835.

[3] Vikas Kumar and Ashish Kumar Arora, Profit Analysis and Modeling of a Power Plant with Boolean Function Algorithm, Journal of Science and Technology 06(04) (2021), 04-11.

[4] Reena Garg, Analysis of various factors of reliability using Boolean function technique in refinery unit, International Journal of Statistics and Applied Mathematics 2(5) (2017), 35-40.

[5] Surbhi Gupta and Neelam Sharma, Evaluation of some reliability parameters for solar panel by boolean algebra technique, International Journal of Education and Science Research Review 01(03) (2014), 49-58.

[6] Amit Kumar and Pooja Dhiman, Reliability Estimation of A Network Structure Using Generalized Trapezoidal Fuzzy Numbers, Journal of KONBiN 51(01) (2021), 225-241.

[7] Shalini Jindal, Reena Garg, Tarun Kumar Garg and Yashasvi Garg, Performance Modeling and Reliability Analysis of Demand and Supply Model, Journal of Xian University of Architecture and Technology XI(XII) (2019), 1009-1014.

[8] Surbhi Gupta and Neelam Sharma, Performance Evaluation of fiber optic communication Using Boolean Function Approach, Turkish Journal of Computer and Mathematics Education 12(9) (2021), 1379-1383.

[9] A. Merceline Anita, S. Kalaiarasi and G. Padmavathy, Reliability analysis of cement manufacturing process using boolean function technique, Advances in Mathematics, Scientific Journal 09(08) (2020), 5909-5916.

[10] Nicolae Jula, Costin Cepisca, Mircea Covrig, Ciprin Racuciu and Tudor Ursu, Boolean Applications in Aircraft Electric Power Systems Reliability Analysis, 2nd European Computing Conference (ECC'08) Malta, September (2008), 11-13.

[11] C. K. Goel, Ram Kishan and Dhan Pal Singh, Reliability analysis of gas-separator system by using boolean function technique, Journal of Statistics and Management Systems 10(1) (2007), 55-67.

[12] P. P. Gupta and R. K. Sharma, Reliability behaviour of a power plant by boolean function technique under arbitrary failure time distribution, Microelectron. Reliab. 26(5) (1986), 815-819.

[13] Kashiful Haq and Yusuf Perwej, A reliability forecast of a parallel power supply plant using boolean function technique under exponential and weibull distributions, International Journal of Mathematics Trends and Technology 66(12) (2020), 71-78.

[14] H. Panagiotis and Tsarauhas, Classification and calculation of primary failure modes in bread production line, Reliability Engineering and System Safety 94(2) (2009), 551-557.

[15] P. A. Ochang, P. J. Irving, and P. O. Ofem, Research on wireless network security awareness of average users, International Journal of Wireless and Microwave Technologies 6(2) (2016), 21-29.

[16] C. M. Dalah and V. V. Singh, Study of reliability measures of a two units' standby system under the concept of switch failure using copula distribution, American Journal of Computational and Applied Mathematics 4(4) (2014), 118-129.

[17] M. R. Haghifam and M. Manbachi, Reliability and availability modelling of combined heat and power (CHP) systems, International Journal of Electrical Power and Energy Systems 33(3) (2011), 385-393.

[18] P. H. Tsarouhas and G. Fourlas, Mission reliability estimation of mobile robot system, International Journal of Systems Assurance Engineering and Management 7(2) (2016), 220-228.