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ANTI-GALLAI FUZZY GRAPHS OF THE DUTCH WINDMILL $D_3(m)$ AND DOMINATION PARAMETERS

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Abstract

In this paper, we discussed about the Dutch windmill fuzzy graphs and the complete fuzzy graphs of anti-Gallai fuzzy graphs. Also we analyses some theorems and domination parameters of that anti-Gallai fuzzy graphs.

1. Introduction

The study of dominating sets in graphs was started by two different authors, viz. Ore, 1962 and Berge, 1962. In 1965, L. A. Zadeh [1] introduced a mathematical frame work to explain the concepts of uncertainty in real life through the publication of a seminal paper. In 1975, A. Rosenfeld [2] introduced the notation of fuzzy graph theoretic concept such as paths, cycles

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and connectedness. In 1977, Cockayne and Hedetniemi have introduced the domination number and independent domination. In 1996, Van Bang Le [3] discussed about the Gallai graphs and anti-Gallai graphs. Similarly, S. Aparna Lakshmanan and S. B. Rao [4] also deliberated the Gallai graphs and anti-Gallai graphs. Further, A. Somasundram and S. Somasundram [5] have explored the domination in fuzzy graphs. In addition, the domination, independent and irredundance numbers were discussed by A. Nagoorgani and P. Vadivel [6]. In [7], the concept of Gallai type theorems and domination parameters are discussed by Gayla S. Domke et al. The concept of Dutch windmill graphs was discussed by M. R. Rajesh Kanna, R. Pradeep Kumar, and R. Jagadeesh in [11]. In our earlier work we have discussed the concept of Gallai-Type Theorems in Gallai Fuzzy Graphs on Domination Parameters and the dutch windmill $D_3(m)$ of Gallai fuzzy graphs on domination number in [12, 13].

In this paper, we discussed the Dutch windmill fuzzy graphs and the complete fuzzy graphs of anti-Gallai fuzzy graphs and some theorems of this graphs. Also we analyses the structures and fuzzy domination parameters.

2. Preliminaries

A fuzzy graph with G as the underlying set is a finite non-empty unordered pair of $G = (\sigma, \mu)$, where $\sigma : V \to [0,1]$ is a fuzzy subset, $\mu : E \to [0, 1]$ is a fuzzy relation on the fuzzy subset σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$ where $\wedge$ and $\vee$ stands for minimum and maximum. The underlying crisp fuzzy graph of $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$, where $V = \{x \in V : \sigma(x) > 0\}$ and $E = \{(x, y) \in V \times V : \mu(x, y) > 0\}$ the fuzzy order p and fuzzy size q of the fuzzy graph $G = (\sigma, \mu)$ are defined by $p = \sum v \in V \sigma(x)$ and $q = \sum x, y \in E \mu(x, y)$. Each pair $\mu = x, y$ of fuzzy vertices in σ is an fuzzy edge of G and μ is said to join x and y are fuzzy adjacent vertices, fuzzy vertex x and fuzzy edge μ are fuzzy incident with each other as are σ and μ if two distinct fuzzy edges are incident with a common fuzzy vertex, then they are called fuzzy adjacent edges. A fuzzy edge $e = xy$ of a fuzzy graph is a fuzzy effective edge if $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

$N(x) = \{y \in V / \mu(x, y) = \sigma(x) \wedge \sigma(y)\}$ is called the open neighborhood of x and $N[x] = N(x) \cup \{x\}$ is the closed neighborhood of x.

Definition 2.1. Let $G = (\sigma, \mu)$ be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v.

Definition. 2.2. A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if for every fuzzy vertex $v \in D$, $D - \{v\}$ is not a fuzzy dominating set.

Definition 2.3. Let $G = (\sigma, \mu)$ be a fuzzy graph, the two vertices in a fuzzy graph G are said to be fuzzy independent if there is no edge between them. A subset S of V is said to be fuzzy independent set for G if every two vertices of S are fuzzy independent.

Definition 2.4. Let $G = (\sigma, \mu)$ be a fuzzy graph. A fuzzy independent set S of G is said to be maximal fuzzy independent set if there is no fuzzy independent set whose cardinality is greater than the cardinality of S.

The maximum cardinality among all maximal fuzzy independent set is called fuzzy independence number of G and it is denoted by $\beta(G)$.

Definition 2.5. The maximum fuzzy cardinality among all minimal fuzzy dominating sets is called upper fuzzy domination number of G and is denoted by $\Gamma(G)$.

The Minimum fuzzy cardinality among all minimal fuzzy dominating sets is called fuzzy domination number of G and is denoted by $\gamma(G)$.

Definition 2.6. The smallest cardinality of all independent fuzzy dominating set is called independent fuzzy domination number of G and is denoted by $i(G)$.

Definition 2.7. A fuzzy graph G is said to be connected if every pair of its fuzzy vertices are connected. Otherwise it is disconnected.

Definition: 2.8. Let G_i denote the induced fuzzy sub graph of G with fuzzy vertex set V_i, clearly the sub graphs $G_1, G_2, \ldots, G_n$ are connected and are called the fuzzy components of G.

Definition 2.9. A fuzzy graph $G = (\sigma, \mu)$ is complete fuzzy graph if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Definition 2.10. Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If $d_G(v) = k$ for all $v \in V$, that is if each fuzzy vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph.

Definition 2.11. Let $G = (\sigma, \mu)$ be a fuzzy graph such that its crisp graph is a cycle, then G is called a fuzzy cycle if there does not exists a unique edge (x, y) such that $\mu(x, y) = \wedge\{\mu(u, v); (u, v) > 0\}$.

Definition 2.12. A fuzzy graph whose fuzzy edge set is empty, it is called a null fuzzy graph or a totally disconnected fuzzy graphs.

3. The Anti-Gallai Fuzzy Graph of the Dutch Windmill Graphs

In this section, we define the Dutch windmill fuzzy graph, the complete fuzzy graph of anti-Gallai fuzzy graphs, and some important theorems are discussed.

Definition 3.1. The Dutch windmill fuzzy graph is denoted by $D_n(m)$ and it is the fuzzy graph obtained by taking m copies of the fuzzy cycle C_n with a fuzzy vertex in common. The Dutch windmill fuzzy graph is also called as friendship fuzzy graph if $n = 3$. The Dutch windmill fuzzy graph $D_n(m)$ contains $(n-1)m + 1$ fuzzy vertices and mn fuzzy edges.

Definition 3.2. The anti-Gallai fuzzy graph $\Delta(G)$ of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are fuzzy adjacent in $\Delta(G)$, if they are incident in G and lie on a triangle in G. The line fuzzy graph $L(G)$ of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are adjacent in $L(G)$ if they are fuzzy incident in G. This concept of anti-Gallai fuzzy graph will be applied to the Dutch windmill $D_n(m)$, and the complete fuzzy graph. Also to analyses the construction of structures and fuzzy domination parameters of anti-Gallai fuzzy graphs.

Theorem 3.3. *Let* $D_3(m), m \geq 2$ *be a Dutch windmill fuzzy graphs. Then* $\Delta D_3(m)$ *is disconnected with regular fuzzy graph of m components.*

Proof. Let $D_3(m), m \geq 2$ be a Dutch windmill fuzzy graph. To Prove $\Delta(D_3(m))$ is disconnected with regular fuzzy graphs of m components. Here, the fuzzy vertex set $V = \{v, y_1, y_2, \ldots, y_{2m}\}$ and the fuzzy edge set $E = \{x_1, x_2, \ldots, x_{3m}\}$ are in $D_3(m)$. Also, every pair of its fuzzy vertices has exactly one common fuzzy neighbor in $D_3(m)$. By the definition of the anti-Gallai fuzzy graph $\Delta(D_3(m))$ the fuzzy vertex set $|E(G)| = |V(G)|$. Here, the number of edges in $\Delta(D_3(m))$ is $3m$, $m \geq 2$, where m is a fuzzy triangles of $D_3(m)$. Suppose $D_3(m)$ contains m triangles, then $\Delta(D_3(m))$ is disconnected with regular fuzzy graphs of m components.

Hence the theorem.

Example 3.4. Let us consider $D_2(2)$ be a Dutch windmill fuzzy graphs or friendship fuzzy graph which is called a butterfly fuzzy graphs.

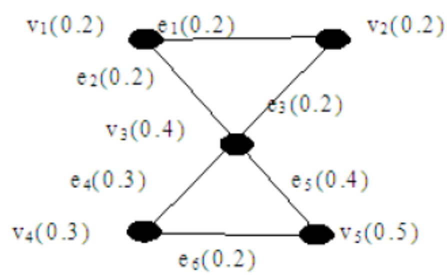

Figure 3.4.1. Dutch windmill fuzzy graph $D_3(m), m = 2, (D_3(2)) = 0.4,$ $i(D_3(3)) = 0.4, \beta(D_3(2)) = 0.7, \Gamma(D_3(2)) = 0.7.$

By using the definition $\Delta(D_3(2))$, we have the following

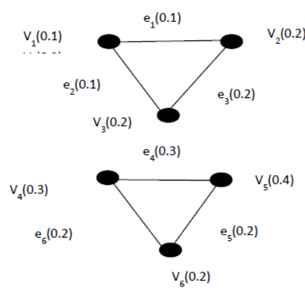

Figure 3.4.2(a). G_1 and **Figure 3.4.2(b).** G_2

$\Delta(D_3(2)) = G_1, G_2$ is disconnected of the anti-Gallai fuzzy graph. $\gamma(G_1 + G_2) = 0.3, i(G_1 + G_2) = 0.3, \beta(G_1 + G_2) = 0.7, \Gamma(G_1 + G_2) = 0.7.$

Theorem 3.5. *Let K_n be a complete fuzzy graph. Then $\Delta(K_n)$ is regular fuzzy graph.*

Proof. Let K_n be a complete fuzzy graph with n vertices. To Prove $\Delta(K_n)$ is regular fuzzy graph. K_n has every pair of its n fuzzy vertices adjacent. By the definition of the anti-Gallai fuzzy graphs $\Delta(K_n)$, the fuzzy vertex set $|E(G)| = |V(G)|$. Here the fuzzy edge set is $(p-2)q$ of $\Delta(K_n)$.

Where p is a fuzzy vertex and q is a fuzzy edge in K_n. Since, each fuzzy edge has form a triangle in K_n, then $\Delta(K_n)$ is regular fuzzy graph.

Hence the theorem.

Example 3.6. Let us consider $K_n, n \geq 3$ be a complete fuzzy graph with n vertices.

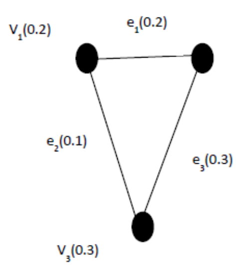

Figure 3.6.1(a). K_3

$\gamma(K_3) = 0.2, i(K_3) = 0.2,$
$\beta(K_3) = 0.4, \Gamma(K_3) = 0.4.$

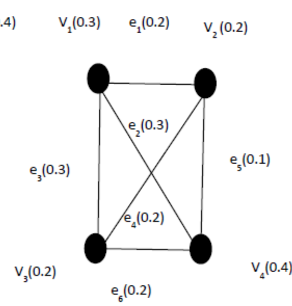

Figure 3.6.1(b). K_4

$K_4 \gamma(K_4) = 0.2, i(K_4) = 0.2,$
$\beta(K_4) = 0.4, \Gamma(K_4) = 0.4.$

By using the definition of $\Delta(K_n)$, we have the following,

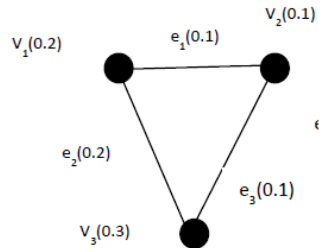

Figure 3.6.2(a). $\Delta(K_3), n=3$ is regular of the anti-Gallai fuzzy graph. $\gamma(K_3)=0.1, i(K_3)=0.1, \beta(K_3)=0.3, \Gamma(K_3)=0.3.$

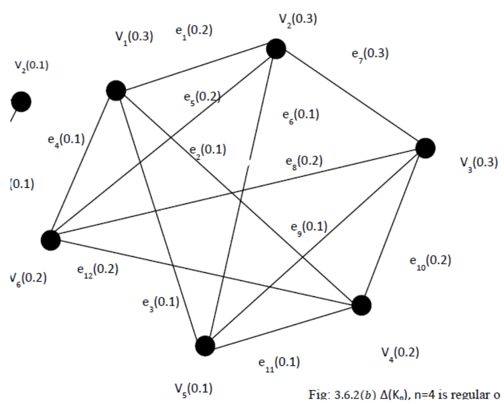

Figure 3.6.2(b). $\Delta(K_n), n=4$ is regular of the anti-Gallai fuzzy graph. $\gamma(K\ )=0.1, i(K_4)=0.1, \beta(K_4)=0.6, \Gamma(K_4)=0.6.$

Definition 3.7. A fuzzy graph is obtained from P_n whose end vertices are identical with a fuzzy vertex of two distinct C_3 and is denoted by $G=C_3P_nC_3$ as follows.

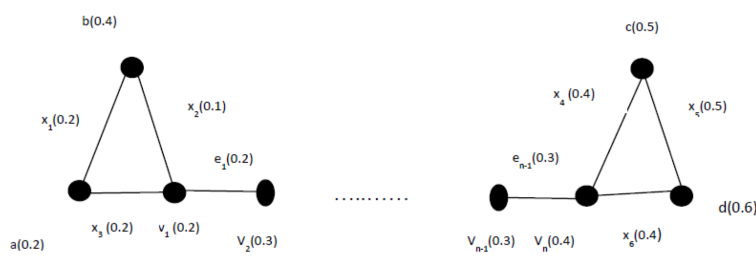

Figure 3.7.1. A fuzzy graph $G=C_3P_nC_3$.

By using the definition of $\Delta(G = C_3P_nC_3)$, we have the following

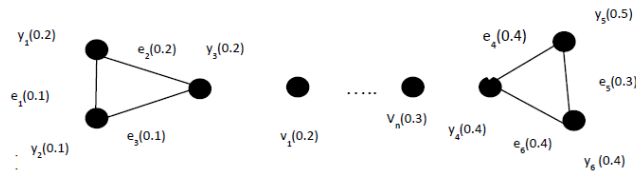

Figure 3.7.2. The anti-Gallai fuzzy graph $\Delta(G = C_3P_nC_3)$ is disconnected of $n+1$ components.

Example 3.8. The fuzzy graph for the sequence $G = C_3P_2C_3$ as follows.

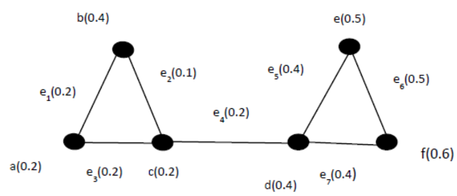

Figure 3.8.1. A fuzzy graph $G = C_3P_2C_3, \gamma(G = C_3P_2C_3) = 0.6$, $i(G = C_3P_2C_3) = 0.6, \beta(G = C_3P_2C_3) = 1.0, \Gamma(G = C_3P_2C_3) = 1.0$.

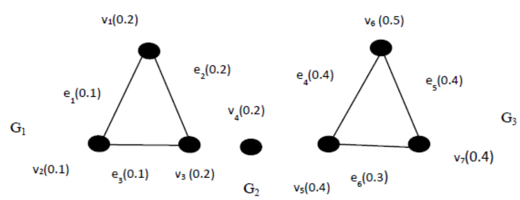

Figure 3.8.2. The anti-Gallai fuzzy graph $\Delta(G = C_3P_2C_3) = G_1, G_2, G_3$ is disconnected of three components. $\gamma(G_1 + +G_2 + G_3) = 0.7$, $i(G_1 + +G_2 + G_3) = 0.7, \beta(G_1 + +G_2 + G_3) = 0.9, \Gamma(G_1 + +G_2 + G_3) = 0.9$.

Theorem 3.9. *Let $G = C_3P_nC_3$ be a fuzzy graph. Then $\Delta(G)$ is disconnected of $n+1$ components, it is regular fuzzy graph of two components and other $n-1$ components are null graph.*

Proof. Let $G = C_3P_nC_3$ be a fuzzy graph, where C_3 is a cycle and P_n is a $n-1$ path of G.

To prove $\Delta(G)$ is disconnected of $n+1$ components, it is regular fuzzy graph of two components and other $n-1$ components are null graph. The fuzzy vertex set $V = \{y_1, y_2, \ldots, y_m\}$ and the fuzzy edge set $E = \{x_1, x_2, \ldots, x_{m+1}\}$ are in G.

A fuzzy graph for the sequence $G = C_3P_nC_3$ contains two distinct fuzzy triangles.

By the definition of the anti-Gallai fuzzy graphs $\Delta(K_n)$, the fuzzy vertex set $|E(G)| = |V(G)|$. Here, the fuzzy edge set is $3t$, where t is the total fuzzy triangle of G.

Hence, $\Delta(G)$ is disconnected of $n+1$ components, it is regular fuzzy graph of two components and other $n-1$ components are null graph.

Hence the Theorem.

4. Domination Chain

In this section, we discussed the domination parameters of the anti – Gallai fuzzy graphs.

Theorem. 4.1. *For any anti-Gallai fuzzy graph, we have* $\gamma(\Delta(G)) \leq i(\Delta(G)) \leq \beta(\Delta(G)) \leq \Gamma(\Delta(G))$.

Theorem 4.2. *Let* $D_3(m)$ *be a Dutch windmill fuzzy graph contains m triangles with a domination number* $\gamma(G) = 1$. *Then* $\Delta(D_3(m)) = G_1, G_2, G_3$ *is disconnected with the domination numbers* $\gamma(D_3(m)) = m$.

Proof. Let $D_3(m)$ be a Dutch windmill fuzzy graph. To Prove $\Delta(D_3(m)) = G_1, G_2, \ldots, G_m$ is disconnected with the domination numbers $\gamma(D_3(m)) = m$. Suppose $D_{3(m)}$ contains m fuzzy triangles of G. Since $D_3(m)$ is a Dutch windmill fuzzy graph with a domination number $\gamma(D_3(m)) = 1$, every pair of its fuzzy vertices has exactly one common neighbor. By the definition of the anti-Gallai fuzzy graph $\Delta(D_3(m)), \Delta(D_3(m)) = G_1, G_2, \ldots, G_m$ is disconnected of m components. That is, $\Delta(D_3(m))$ have the domination numbers $\gamma(D_{3(m)}) = m$.

Hence the theorem.

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