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FUZZY LINEAR PROGRAMMING MODEL FOR CRITICAL PATH ANALYSIS USING INTERVAL VALUED FUZZY NUMBERS

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Abstract

In this paper, we find the critical path for the project scheduling problem using Linear Programming problem with the aid of interval valued trapezoidal fuzzy numbers (IVTFN) in parametric type with an example.

1. Introduction

Linear programming or linear optimization is a field of mathematics that deals with finding optimal values or solutions that can be described with linear equations and inequalities. Very often this involves finding the minimal or maximal values, given some conditions, or constraints. Linear programming is often used for problems where no exact solution is known, for example for planning traffic flows. Linear programming is one of the main methods used in Operations research. Linear optimization is a special case of convex optimization. It forms the basis for several methods of solving problems of Integer programming. In many cases, the solutions of linear programs can be mapped to Polyhedra, which allows solving and modeling certain problems geometrically.

Minoux [2] analyses the mathematical programming theory with algorithms. Ton Shaochebg [5] extended the interval numbers and fuzzy

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numbers in linear programming. Conde [1] introduced a min-max regret approach to the critical path method with task interval times. Usha Maduriet al. [6] explained fuzzy linear programming model for critical path analysis. Thangaraj Beaula and Vijaya [4] discussed a new method for finding the critical path in a fuzzy project network. Stephen Dinagar and Abirami [3] discussed the critical path method using interval valued fuzzy numbers.

This paper is organized as follows. In section 1, we introduced the basic knowledge of Linear programming Problem to find the critical path problem. Some basic definitions which are useful for our work are given in section 2. In section 3, the definition of the interval valued fuzzy number is proposed with arithmetic operations are clearly explained as given in [3]. In section 4, a new algorithm for Linear Programming problem is explained by using interval valued trapezoidal fuzzy number. Finally, the application part of this work have been included in section 5. The conclusion part is also given in the last section.

2. Preliminaries

In this section, some important definitions and results which are useful to this work are presented.

Definition 2.1. A Fuzzy set $\widetilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

(i) $\widetilde{A}$ is convex, i.e., $\widetilde{A}(\lambda X_1 + (1-\lambda)X_2) = \text{Minimum}\,\{\widetilde{A}(X_1), \widetilde{A}(X_2)\}$, for all $X_1, X_2 \in R$ and $\lambda \in [0, 1]$

(ii) $\widetilde{A}$ is normal i.e., there exists an $X_0 \in R$ such that $\widetilde{A}(X_0) = 1$

(iii) $\widetilde{A}$ is piecewise continuous.

3. Interval Valued Fuzzy Numbers

Definition 3.1. A fuzzy number $\widetilde{A}$ is R is said to be a trapezoidal fuzzy number if its membership function $\widetilde{A} : R \to [0, 1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x - a_1)}{(a_2 - a_1)}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \frac{(x - a_4)}{(a_3 - a_4)}, & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.2. An IVTFN $\tilde{A}$ is called as zero-equivalent fuzzy number if $R(\tilde{A}) = 0$ and denoted by $\hat{0}$.

Definition 3.3. An IVTFN $\tilde{A}$ is called as zero fuzzy number if $\tilde{A} = [(0, 0, 0, 0), (0, 0, 0, 0)]$ and denoted by $\tilde{0}$.

Definition 3.4. An Interval valued fuzzy number $\tilde{A}$ on R is given by $\tilde{A} = \{X, (\mu_A^L(x), \mu_A^U(x)), x \in R\}$ and $\mu_A^L \le \mu_A^U(x)$ for all $x \in R$. And it is denoted by $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$, where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$ and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$ are the trapezoidal fuzzy numbers.

It is also noted that $a_1^U \le a_1^L, a_2^U \le a_2^L, a_3^L \le a_3^U, a_4^L \le a_4^U$.

3.5. Pictorial Representation:

Let $\tilde{A} = [(2, 4, 5, 7), (1, 3, 6, 8)]$ $\mu_{\tilde{A}}(x)$

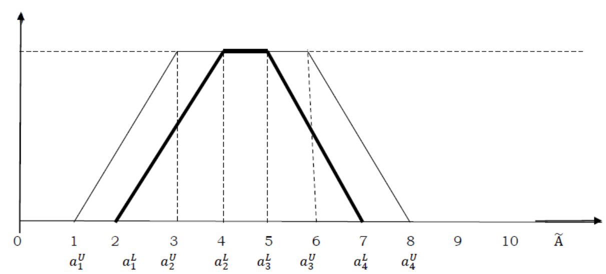

Figure 2.1. IVTFN $\tilde{A}$.

Definition 3.6. The Distance between any two IVFNS $\tilde{A}$ and $\tilde{B}$ can be defined as:

$$D(\widetilde{A}, \widetilde{B}) = \frac{1}{4} \max \{| (a_1^L + a_1^U) - (b_1^L + b_1^U) | + | (a_4^L + a_4^U) - (b_4^L + b_4^U) |,$$

$$| (a_2^L + a_2^U) - (b_2^L + b_2^U) | + | (a_3^L + a_3^U) - (b_3^L + b_3^U) |\}.$$

The Distance between any two IVFNS $\widetilde{A}$ and $\widetilde{1} = [(1, 1, 1, 1), (1, 1, 1, 1)]$ can be defined as:

$$D(\widetilde{A}, \widetilde{1}) = \frac{1}{4} \max \{| (a_1^L + a_1^U) - (1 + 1) | + | (a_4^L + a_4^U) - (1 + 1) |,$$

$$| (a_2^L + a_2^U) - (1 + 1) | + | (a_3^L + a_3^U) - (1 + 1) |\}$$

and the distance between any two IVFNS $\widetilde{A}$ and $\widetilde{0} = [(0, 0, 0, 0), (0, 0, 0, 0)]$ can be defined as:

$$D(\widetilde{A}, \widetilde{0}) = \frac{1}{4} \max \{| (a_1^L + a_1^U) - (0 + 0) | + | (a_4^L + a_4^U) - (0 + 0) |,$$

$$| (a_2^L + a_2^U) - (0 + 0) | + | (a_3^L + a_3^U) - (0 + 0) |\}.$$

4. Linear Programming Problem in Parametric Form of IVTFNS

Methodology 4.1. If all the parameters of fuzzy linear programming in critical path problem are represented by interval valued fuzzy numbers $[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ then the steps of the proposed method are as follows:

Step 1. Suppose all the parameters $\widetilde{A}_{ij}$ and $\widetilde{X}_{ij}$ are represented by $[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ type interval valued fuzzy numbers $[(a_{ij}^L, b_{ij}^L, c_{ij}^L, d_{ij}^L), (a_{ij}^U, b_{ij}^U, c_{ij}^U, d_{ij}^U)]$ and $[(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$ respectively. Then

Maximize

$$\sum_{(i, j) \in E} [(a_{ij}^L, b_{ij}^L, c_{ij}^L, d_{ij}^L), (a_{ij}^U, b_{ij}^U, c_{ij}^U, d_{ij}^U)] \otimes [(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)].$$

Subject to

$$\sum_{(i,j)\in E}[(x_{1j}^L, y_{1j}^L, z_{1j}^L, \gamma_{1j}^L), (x_{1j}^U, y_{1j}^U, z_{1j}^U, \gamma_{1j}^U)] = [(1,1,1,1), (1,1,1,1)]$$

$$\sum_{(i,j)\in E}[(x_{ij}^L, y_{ji}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$$

$$= [(x_{jk}^L, y_{jk}^L, z_{jk}^L, \gamma_{jk}^L), (x_{jk}^U, y_{jk}^U, z_{jk}^U, \gamma_{jk}^U)], i \neq 1, k \neq n.$$

$$\sum_{(i,n)\in E}[(x_{in}^L, y_{in}^L, z_{in}^L, \gamma_{in}^L), (x_{in}^U, y_{in}^U, z_{in}^U, \gamma_{in}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)]$$

$$[(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$$

is a non-negative IVFNS $\forall(i, j) \in E$.

Step 2. The ranking function to fuzzy linear programming of fuzzy critical path problems may be written as:

Maximize

$$\mathcal{R}\left(\sum_{(i,j)\in E}[(a_{ij}^L, b_{ij}^L, c_{ij}^L, d_{ij}^L), (a_{ij}^U, b_{ij}^U, c_{ij}^U, d_{ij}^U)] \otimes [(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]\right).$$

Subject to

$$\sum_{(i,j)\in E}[(x_{1j}^L, y_{1j}^L, z_{1j}^L, \gamma_{1j}^L), (x_{1j}^U, y_{1j}^U, z_{1j}^U, \gamma_{1j}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)]$$

$$\sum_{(i,j)\in E}[(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$$

$$= [(x_{jk}^L, y_{jk}^L, z_{jk}^L, \gamma_{jk}^L), (x_{jk}^U, y_{jk}^U, z_{jk}^U, \gamma_{jk}^U)], i \neq 1, k \neq n.$$

$$\sum_{(i,n)\in E}[(x_{in}^L, y_{in}^L, z_{in}^L, \gamma_{in}^L), (x_{in}^U, y_{in}^U, z_{in}^U, \gamma_{in}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)]$$

$$[(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$$

is a non-negative IVTFNS fuzzy number $\forall(i, j) \in E$.

Now, the crisp linear programming problem becomes:

Maximize

$$\mathcal{R}\left(\sum_{(i,j)\in E}[(a_{ij}^{L},b_{ij}^{L},c_{ij}^{L},d_{ij}^{L}),(a_{ij}^{U},b_{ij}^{U},c_{ij}^{U},d_{ij}^{U})]\otimes[(x_{ij}^{L},y_{ij}^{L},z_{ij}^{L},\gamma_{ij}^{L}),(x_{ij}^{U},y_{ij}^{U},z_{ij}^{U},\gamma_{ij}^{U})]\right).$$

Subject to

$$\sum_{(1,\ j)\in E} x_{1j}^{L} = 1,\ \sum_{(1,\ j)\in E} y_{1j}^{L} = 1,\ \sum_{(1,\ j)\in E} z_{1j}^{L} = 1,\ \sum_{(1,\ j)\in E} \gamma_{1j}^{L} = 1$$

$$\sum_{(1,\ j)\in E} x_{1j}^{U} = 1,\ \sum_{(1,\ j)\in E} y_{1j}^{U} = 1,\ \sum_{(1,\ j)\in E} z_{1j}^{U} = 1,\ \sum_{(1,\ j)\in E} \gamma_{1j}^{U} = 1$$

$$\sum_{i:(i,\ j)\in A} x_{ij}^{L} = 1,\ \sum_{j:(j,\ k)\in A} x_{jk}^{L},\ \sum_{i:(i,\ j)\in A} x_{ij}^{U} = \sum_{j:(j,\ k)\in A} x_{jk}^{U}\ i \neq 1,\ k \neq n,$$

$$\sum_{i:(i,\ j)\in A} y_{ij}^{L} = \sum_{j:(j,\ k)\in A} y_{jk}^{L},\ \sum_{i:(i,\ j)\in A} y_{ij}^{U} = \sum_{j:(j,\ k)\in A} y_{jk}^{U}\ i \neq 1,\ k \neq n,$$

$$\sum_{i:(i,\ j)\in A} z_{ij}^{L} = \sum_{j:(j,\ k)\in A} z_{jk}^{L},\ \sum_{i:(i,\ j)\in A} z_{ij}^{U} = \sum_{j:(j,\ k)\in A} z_{jk}^{U}\ i \neq 1,\ k \neq n,$$

$$\sum_{i:(i,\ j)\in A} \gamma_{ij}^{L} = \sum_{j:(j,\ k)\in A} \gamma_{jk}^{L},\ \sum_{i:(i,\ j)\in A} \gamma_{ij}^{U} = \sum_{j:(j,\ k)\in A} \gamma_{jk}^{U}\ i \neq 1,\ k \neq n,$$

$$\sum_{i:(i,\ n)\in E} x_{in}^{L} = 1,\ \sum_{i:(i,\ n)\in E} y_{in}^{L} = 1,\ \sum_{i:(i,\ n)\in E} z_{in}^{L} = 1,\ \sum_{i:(i,\ n)\in E} \gamma_{in}^{L} = 1$$

$$\sum_{i:(i,\ n)\in E} x_{in}^{U} = 1,\ \sum_{i:(i,\ n)\in E} y_{in}^{U} = 1,\ \sum_{i:(i,\ n)\in E} z_{in}^{U} = 1,\ \sum_{i:(i,\ n)\in E} \gamma_{in}^{U} = 1$$

$$y_{ij}^{L} - x_{ij}^{L} \geq 0,\ y_{ij}^{U} - x_{ij}^{U} \geq 0,\ z_{ij}^{L} - y_{ij}^{L} \geq 0,\ z_{ij}^{U} - y_{ij}^{U} \geq 0,$$

$$\gamma_{ij}^{L} - z_{ij}^{L} \geq 0,\ \gamma_{ij}^{U} - z_{ij}^{U} \geq 0.$$

$$x_{ij}^{L},\ y_{ij}^{L},\ z_{ij}^{L},\ \gamma_{ij}^{L} \geq 0,\ \forall(i,\ j) \in E \text{ and } x_{ij}^{U},\ y_{ij}^{U},\ z_{ij}^{U},\ \gamma_{ij}^{U} \geq 0,\ \forall(i,\ j) \in E.$$

Step 3. Find the solution $x_{ij}^{L}, x_{ij}^{U}, y_{ij}^{L}, y_{ij}^{U}, z_{ij}^{L}, z_{ij}^{U}, \gamma_{ij}^{L}, \gamma_{ij}^{U}$ by solving the crisp Linear Programming problem, which is obtained in step 2.

Step 4. Find the fuzzy solution $\widetilde{X}_{ij}$ by putting the values of x_{ij}^L, y_{ij}^L, z_{ij}^L, γ_{ij}^L, x_{ij}^U, y_{ij}^U, z_{ij}^U, γ_{ij}^U in $\widetilde{X}_{ij} = [(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)]$ and also calculate the maximum total completion fuzzy time by putting the values of $\widetilde{X}_{ij}$ in $\sum_{(i,j)\in E} \widetilde{A}_{ij} \otimes \widetilde{X}_{ij}$.

Step 5. Find the fuzzy critical path by combining all the activities (i, j) such that $\widetilde{X}_{ij} = [(1, 1, 1, 1), (1, 1, 1, 1)]$.

5. Numerical Illustration

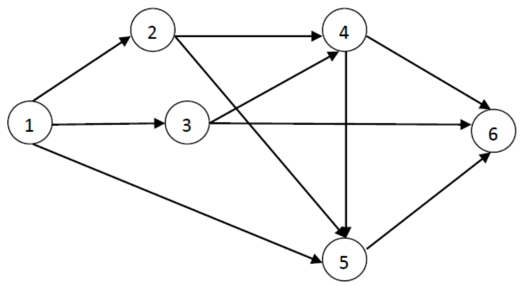

Figure 5.1. Network Project.

Activity A_{ij}	Fuzzy activity times (Hours) $\widetilde{FET}_{ij}$
1-2	[(2,2,3,4),(1,1,4,5)]
1-3	[(2,3,3,6),(1,2,4,7)]
1-5	[(2,3,4,5),(1,2,5,6)]
2-4	[(2,2,4,5), (1,1,5,6)]
2-5	[(2,2,5,8), (1,1,6,9)]
3-4	[(1,1,2,2), (0,0,3,3)]
3-6	[(7,8,11,15), (6,7,12,16)]
4-5	[(2,2,3,5), (1,2,4,6)]
4-6	[(3,3,4,6), (2,2,5,7)]
5-6	[(1,1,1,2), (0,0,2,3)]

Step 1. The given problem may be formulated as follows:

Maximize

$\{[(2, 2, 3, 4), (1, 1, 4, 5)] \otimes [(x_{12}^L, y_{12}^L, z_{12}^L, \gamma_{12}^L), (x_{12}^U, y_{12}^U, z_{12}^U, \gamma_{12}^U)] \oplus$

$\{[(2, 3, 3, 6), (1, 2, 4, 7)] \otimes [(x_{13}^L, y_{13}^L, z_{13}^L, \gamma_{13}^L), (x_{13}^U, y_{13}^U, z_{13}^U, \gamma_{13}^U)] \oplus$

$\{[(2, 3, 4, 5), (1, 2, 5, 6)] \otimes [(x_{15}^L, y_{15}^L, z_{15}^L, \gamma_{15}^L), (x_{15}^U, y_{15}^U, z_{15}^U, \gamma_{15}^U)] \oplus$

$\{[(2, 2, 4, 5), (1, 1, 5, 6)] \otimes [(x_{24}^L, y_{24}^L, z_{24}^L, \gamma_{24}^L), (x_{24}^U, y_{24}^U, z_{24}^U, \gamma_{24}^U)] \oplus$

$\{[(2, 2, 5, 8), (1, 1, 6, 9)] \otimes [(x_{25}^L, y_{25}^L, z_{25}^L, \gamma_{25}^L), (x_{25}^U, y_{25}^U, z_{25}^U, \gamma_{25}^U)] \oplus$

$\{[(1, 1, 2, 2), (0, 0, 3, 3)] \otimes [(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L), (x_{34}^U, y_{34}^U, z_{34}^U, \gamma_{34}^U)] \oplus$

$\{[(7, 8, 11, 15), (6, 7, 12, 16)] \otimes [(x_{36}^L, y_{36}^L, z_{36}^L, \gamma_{36}^L), (x_{36}^U, y_{36}^U, z_{36}^U, \gamma_{36}^U)] \oplus$

$\{[(2, 2, 3, 5), (1, 2, 4, 6)] \otimes [(x_{45}^L, y_{45}^L, z_{45}^L, \gamma_{45}^L), (x_{45}^U, y_{45}^U, z_{45}^U, \gamma_{45}^U)] \oplus$

$\{[(3, 3, 4, 6), (2, 2, 5, 7)] \otimes [(x_{46}^L, y_{46}^L, z_{46}^L, \gamma_{46}^L), (x_{46}^U, y_{46}^U, z_{46}^U, \gamma_{46}^U)] \oplus$

$\{[(1, 1, 1, 2), (0, 0, 2, 3)] \otimes [(x_{56}^L, y_{56}^L, z_{56}^L, \gamma_{56}^L), (x_{56}^U, y_{56}^U, z_{56}^U, \gamma_{56}^U)]\}.$

Subject to the constrains

$[(x_{12}^L, y_{12}^L, z_{12}^L, \gamma_{12}^L), (x_{12}^U, y_{12}^U, z_{12}^U, \gamma_{12}^U)] \oplus [(x_{13}^L, y_{13}^L, z_{13}^L, \gamma_{13}^L),$

$(x_{13}^U, y_{13}^U, z_{13}^U, \gamma_{13}^U)] \oplus$

$[(x_{15}^L, y_{15}^L, z_{15}^L, \gamma_{15}^L), (x_{15}^U, y_{15}^U, z_{15}^U, \gamma_{15}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)],$

$[(x_{24}^L, y_{24}^L, z_{24}^L, \gamma_{24}^L), (x_{24}^U, y_{24}^U, z_{24}^U, \gamma_{24}^U)] \oplus [(x_{25}^L, y_{25}^L, z_{25}^L, \gamma_{25}^L),$

$(x_{25}^U, y_{25}^U, z_{25}^U, \gamma_{25}^U)]$

$= [(x_{12}^L, y_{12}^L, z_{12}^L, \gamma_{12}^L), (x_{12}^U, y_{12}^U, z_{12}^U, \gamma_{12}^U)],$

$[(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L), (x_{34}^U, y_{34}^U, z_{34}^U, \gamma_{34}^U)] \oplus [(x_{36}^L, y_{36}^L, z_{36}^L, \gamma_{36}^L),$

$(x_{36}^U, y_{36}^U, z_{36}^U, \gamma_{36}^U)]$

$= [(x_{13}^L, y_{13}^L, z_{13}^L, \gamma_{13}^L), (x_{13}^U, y_{13}^U, z_{13}^U, \gamma_{13}^U)],$

$[(x_{45}^L, y_{45}^L, z_{45}^L, \gamma_{45}^L), (x_{45}^U, y_{45}^U, z_{45}^U, \gamma_{45}^U)] \oplus [(x_{46}^L, y_{46}^L, z_{46}^L, \gamma_{46}^L),$

$(x_{46}^U,\ y_{46}^U,\ z_{46}^U,\ \gamma_{46}^U)]$

$= [(x_{24}^L, y_{24}^L, z_{24}^L, \gamma_{24}^L), (x_{24}^U, y_{24}^U, z_{24}^U, \gamma_{24}^U)] \oplus [(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L),$

$(x_{34}^U,\ y_{34}^U,\ z_{34}^U,\ \gamma_{34}^U)],$

$[(x_{56}^L,\ y_{56}^L,\ z_{56}^L,\ \gamma_{56}^L),\ (x_{56}^U,\ y_{56}^U,\ z_{56}^U,\ \gamma_{56}^U)] = [(x_{15}^L,\ y_{15}^L,\ z_{15}^L,\ \gamma_{15}^L),$

$(x_{15}^U,\ y_{15}^U,\ z_{15}^U,\ \gamma_{15}^U)] \oplus$

$[(x_{25}^L,\ y_{25}^L,\ z_{25}^L,\ \gamma_{25}^L),\ (x_{25}^U,\ y_{25}^U,\ z_{25}^U,\ \gamma_{25}^U)] \oplus [(x_{45}^L,\ y_{45}^L,\ z_{45}^L,\ \gamma_{45}^L),$

$(x_{45}^U,\ y_{45}^U,\ z_{45}^U,\ \gamma_{45}^U)],$

and

$[(x_{36}^L,\ y_{36}^L,\ z_{36}^L,\ \gamma_{36}^L),\ (x_{36}^U,\ y_{36}^U,\ z_{36}^U,\ \gamma_{36}^U)] \oplus [(x_{46}^L,\ y_{46}^L,\ z_{46}^L,\ \gamma_{46}^L),$

$(x_{46}^U,\ y_{46}^U,\ z_{46}^U,\ \gamma_{46}^U)] \oplus$

$[(x_{56}^L,\ y_{56}^L,\ z_{56}^L,\ \gamma_{56}^L),\ (x_{56}^U,\ y_{56}^U,\ z_{56}^U,\ \gamma_{56}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)]$

Where {[(2, 3, 3, 6), (1, 2, 4, 7)], [(2, 3, 4, 5), (1, 2, 5, 6)], [(2, 2, 4, 5), (1, 1, 5, 6)] [(2, 2, 5, 8), (1, 1, 6, 9)], [(1, 1, 2, 2), (0, 0, 3, 3)], [(7, 8, 11, 15), (6, 7, 12, 16)] [(2, 2, 3, 5), (1, 2, 4, 6)], [(3, 3, 4, 6), (2, 2, 5, 7)], [(1, 1, 1, 2), (0, 0, 2, 3)]} are non-negative Interval valued fuzzy numbers.

Step 2. Using ranking function to the fuzzy linear programming problem, step 1, may be written as,

Maximize

$\mathcal{R}\{[(2, 2, 3, 4), (1, 1, 4, 5)] \otimes [(x_{12}^L,\ y_{12}^L,\ z_{12}^L,\ \gamma_{12}^L),\ (x_{12}^U,\ y_{12}^U,\ z_{12}^U,\ \gamma_{12}^U)] \oplus$

$\{[(2, 3, 3, 4), (1, 2, 4, 7)] \otimes [(x_{13}^L,\ y_{13}^L,\ z_{13}^L,\ \gamma_{13}^L),\ (x_{13}^U,\ y_{13}^U,\ z_{13}^U,\ \gamma_{13}^U)] \oplus$

$\{[(2, 3, 4, 5), (1, 2, 5, 6)] \otimes [(x_{15}^L,\ y_{15}^L,\ z_{15}^L,\ \gamma_{15}^L),\ (x_{15}^U,\ y_{15}^U,\ z_{15}^U,\ \gamma_{15}^U)] \oplus$

$\{[(2, 2, 4, 5), (1, 1, 5, 6)] \otimes [(x_{24}^L,\ y_{24}^L,\ z_{24}^L,\ \gamma_{24}^L),\ (x_{24}^U,\ y_{24}^U,\ z_{24}^U,\ \gamma_{24}^U)] \oplus$

$\{[(2, 2, 5, 8), (1, 1, 6, 9)] \otimes [(x_{25}^L, y_{25}^L, z_{25}^L, \gamma_{25}^L), (x_{25}^U, y_{25}^U, z_{25}^U, \gamma_{25}^U)] \oplus$

$\{[(1, 1, 2, 2), (0, 0, 3, 3)] \otimes [(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L), (x_{34}^U, y_{34}^U, z_{34}^U, \gamma_{34}^U)] \oplus$

$\{[(7, 8, 11, 15), (6, 7, 12, 16)] \otimes [(x_{36}^L, y_{36}^L, z_{36}^L, \gamma_{36}^L), (x_{36}^U, y_{36}^U, z_{36}^U, \gamma_{36}^U)] \oplus$

$\{[(2, 2, 3, 5), (1, 2, 4, 6)] \otimes [(x_{45}^L, y_{45}^L, z_{45}^L, \gamma_{45}^L), (x_{45}^U, y_{45}^U, z_{45}^U, \gamma_{45}^U)] \oplus$

$\{[(3, 3, 4, 6), (2, 2, 5, 7)] \otimes [(x_{46}^L, y_{46}^L, z_{46}^L, \gamma_{46}^L), (x_{46}^U, y_{46}^U, z_{46}^U, \gamma_{46}^U)] \oplus$

$\{[(1, 1, 1, 2), (0, 0, 2, 3)] \otimes [(x_{56}^L, y_{56}^L, z_{56}^L, \gamma_{56}^L), (x_{56}^U, y_{56}^U, z_{56}^U, \gamma_{56}^U)]\}.$

Subject to the constrains

$$[(x_{12}^L, y_{12}^L, z_{12}^L, \gamma_{12}^L), (x_{12}^U, y_{12}^U, z_{12}^U, \gamma_{12}^U)] \oplus [(x_{13}^L, y_{13}^L, z_{13}^L, \gamma_{13}^L),$$

$$(x_{13}^U, y_{13}^U, z_{13}^U, \gamma_{13}^U)] \oplus$$

$$[(x_{15}^L, y_{15}^L, z_{15}^L, \gamma_{15}^L), (x_{15}^U, y_{15}^U, z_{15}^U, \gamma_{15}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)],$$

$$[(x_{24}^L, y_{24}^L, z_{24}^L, \gamma_{24}^L), (x_{24}^U, y_{24}^U, z_{24}^U, \gamma_{24}^U)] \oplus [(x_{25}^L, y_{25}^L, z_{25}^L, \gamma_{25}^L),$$

$$(x_{25}^U, y_{25}^U, z_{25}^U, \gamma_{25}^U)]$$

$$= [(x_{12}^L, y_{12}^L, z_{12}^L, \gamma_{12}^L), (x_{12}^U, y_{12}^U, z_{12}^U, \gamma_{12}^U)],$$

$$[(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L), (x_{34}^U, y_{34}^U, z_{34}^U, \gamma_{34}^U)] \oplus [(x_{36}^L, y_{36}^L, z_{36}^L, \gamma_{36}^L),$$

$$(x_{36}^U, y_{36}^U, z_{36}^U, \gamma_{36}^U)]$$

$$= [(x_{13}^L, y_{13}^L, z_{13}^L, \gamma_{13}^L), (x_{13}^U, y_{13}^U, z_{13}^U, \gamma_{13}^U)]$$

$$[(x_{45}^L, y_{45}^L, z_{45}^L, \gamma_{45}^L), (x_{45}^U, y_{45}^U, z_{45}^U, \gamma_{45}^U)] \oplus [(x_{46}^L, y_{46}^L, z_{46}^L, \gamma_{46}^L),$$

$$(x_{46}^U, y_{46}^U, z_{46}^U, \gamma_{46}^U)]$$

$$= [(x_{24}^L, y_{24}^L, z_{24}^L, \gamma_{24}^L), (x_{24}^U, y_{24}^U, z_{24}^U, \gamma_{24}^U)] \oplus [(x_{34}^L, y_{34}^L, z_{34}^L, \gamma_{34}^L), (x_{34}^U, y_{34}^U, z_{34}^U, \gamma_{34}^U)]$$

$$[(x_{56}^L, y_{56}^L, z_{56}^L, \gamma_{56}^L), (x_{56}^U, y_{56}^U, z_{56}^U, \gamma_{56}^U)] = [(x_{15}^L, y_{15}^L, z_{15}^L, \gamma_{15}^L), (x_{15}^U, y_{15}^U, z_{15}^U, \gamma_{15}^U)] \oplus$$

$$[(x_{25}^L, y_{25}^L, z_{25}^L, \gamma_{25}^L), (x_{25}^U, y_{25}^U, z_{25}^U, \gamma_{25}^U)] \oplus [(x_{45}^L, y_{45}^L, z_{45}^L, \gamma_{45}^L), (x_{45}^U, y_{45}^U, z_{45}^U, \gamma_{45}^U)],$$

and

$$[(x_{36}^L, y_{36}^L, z_{36}^L, \gamma_{36}^L), (x_{36}^U, y_{36}^U, z_{36}^U, \gamma_{36}^U)] \oplus [(x_{46}^L, y_{46}^L, z_{46}^L, \gamma_{46}^L), (x_{46}^U, y_{46}^U, z_{46}^U, \gamma_{46}^U)]$$

$$[(x_{56}^L, y_{56}^L, z_{56}^L, \gamma_{56}^L), (x_{56}^U, y_{56}^U, z_{56}^U, \gamma_{56}^U)] = [(1, 1, 1, 1), (1, 1, 1, 1)].$$

Where {[(2, 3, 3, 6), (1, 2, 4, 7)], [(2, 3, 4, 5), (1, 2, 5, 6)], [(2, 2, 4, 5), (1, 1, 5, 6)] [(2, 2, 5, 8), (1, 1, 6, 9)], [(1, 1, 2, 2), (0, 0, 3, 3)], [(7, 8, 11, 15), (6, 7, 12, 16)] [(2, 2, 3, 5), (1, 2, 4, 6)], [(3, 3, 4, 6), (2, 2, 5, 7)], [(1, 1, 1, 2), (0, 0, 2, 3)]} are non-negative Interval valued fuzzy numbers.

The crisp Linear Programming problem becomes:

Maximize

$$[0.25x_{12}^L + 0.25y_{12}^L + 0.375z_{12}^L + 0.5\gamma_{12}^L + 0.125x_{12}^U + 0.125y_{12}^U + 0.5z_{12}^U + 0.625\gamma_{12}^U$$
$$+ 0.25x_{13}^L + 0.375y_{13}^L + 0.375z_{13}^L + 0.75\gamma_{13}^L + 0.125x_{13}^U + 0.25y_{13}^U + 0.5z_{13}^U + 0.875\gamma_{13}^U$$
$$+ 0.25x_{15}^L + 0.375y_{15}^L + 0.5z_{15}^L + 0.625\gamma_{15}^L + 0.125x_{15}^U + 0.25y_{15}^U + 0.625z_{15}^U + 0.75\gamma_{15}^U$$
$$+ 0.25x_{24}^L + 0.25y_{24}^L + 0.5z_{24}^L + 0.625\gamma_{24}^L + 0.125x_{24}^U + 0.125y_{24}^U + 0.625z_{24}^U + 0.75\gamma_{24}^U$$
$$0.25x_{25}^L + 0.25y_{25}^L + 0.625z_{25}^L + \gamma_{25}^L + 0.125x_{25}^U + 0.125y_{25}^U + 0.75z_{25}^U + 1.125\gamma_{25}^U$$
$$+ 0.125x_{34}^L + 0.125y_{34}^L + 0.25z_{34}^L + 0.25\gamma_{34}^L + 0x_{34}^U + 0y_{34}^U + 0.375z_{34}^U + 0.375\gamma_{34}^U$$
$$+ 0.875x_{36}^L + y_{36}^L + 1.375z_{36}^L + 1.875\gamma_{36}^L + 0.75x_{36}^U + 0.875y_{36}^U + 1.5z_{36}^U + 2\gamma_{36}^U$$
$$+ 0.25x_{45}^L + 0.375y_{45}^L + 0.375z_{45}^L + 0.625\gamma_{45}^L + 0.125x_{45}^U + 0.25y_{45}^U + 0.5z_{45}^U$$
$$+ 0.75\gamma_{45}^U + 0.375x_{46}^L + 0.375y_{46}^L + 0.5z_{46}^L + 0.75\gamma_{46}^L + 0.25x_{46}^U$$
$$+ 0.25y_{46}^U + 0.625z_{46}^U 0.875\gamma_{46}^U + 0.125x_{56}^L + 0.125y_{56}^L + 0.125y_{56}^L + 0.25\gamma_{56}^L + 0x_{56}^U$$
$$+ 0y_{56}^U + 0.25z_{56}^U + 0.375\gamma_{56}^U.$$

Maximize

$$[0.25x_{12}^L + 0.25y_{12}^L + 0.375z_{12}^L + 0.5\gamma_{12}^L + 0.125x_{12}^U + 0.125y_{12}^U + 0.5z_{12}^U + 0.625\gamma_{12}^U$$
$$0.25x_{13}^L + 0.375y_{13}^L + 0.375z_{13}^L + 0.75\gamma_{13}^L + 0.125x_{13}^U + 0.25y_{13}^U + 0.5z_{13}^U$$

$+ 0.875\gamma_{13}^{U} + 0.25x_{15}^{L} + 0.375y_{15}^{L}$

$+ 0.5z_{15}^{L} + 0.625\gamma_{15}^{L} + 0.125x_{15}^{U} + 0.25y_{15}^{U} + 0.625z_{15}^{U} + 0.75\gamma_{15}^{U} + 0.25x_{24}^{L}$

$+ 0.25y_{24}^{L} + 0.5z_{24}^{L} + 0.625\gamma_{24}^{L} + 0.125x_{24}^{U} +$

$0.125y_{24}^{U} + 0.625z_{24}^{U} + 0.75\gamma_{24}^{U} + 0.25x_{25}^{L} + 0.25y_{25}^{L} + 0.625z_{25}^{L} + \gamma_{25}^{L} + 0.125x_{25}^{U}$

$+ 0.125y_{25}^{U} + 0.75z_{25}^{U} + 1.125\gamma_{25}^{U} + 0.125x_{34}^{L} +$

$0.125y_{34}^{L} + 0.25z_{34}^{L} + 0.25\gamma_{34}^{L} + 0x_{34}^{U} + 0y_{34}^{U} + 0.375z_{34}^{U} + 0.375\gamma_{34}^{U}$

$+ 0.875x_{36}^{L} + y_{36}^{L} + 1.375z_{36}^{L} + 1.875\gamma_{36}^{L} + 0.75x_{36}^{U} + 0.875y_{36}^{U} + 1.5z_{36}^{U}$

$+ 2\gamma_{36}^{U} + 0.25x_{45}^{L} + 0.375y_{45}^{L} + 0.375z_{45}^{L} +$

$0.625\gamma_{45}^{L} + 0.125x_{45}^{U} + 0.25y_{45}^{U} + 0.5z_{45}^{U} + 0.75\gamma_{45}^{U} + 0.375x_{46}^{L} + 0.375y_{46}^{L} + 0.5z_{46}^{L}$

$+ 0.75\gamma_{46}^{L} + 0.25x_{46}^{U} + 0.25y_{46}^{U} + 0.625z_{46}^{U}$

$0.875\gamma_{46}^{U} + 0.125x_{56}^{L} + 0.125y_{56}^{L} + 0.125z_{56}^{L} + 0.$

Subject to the constraints

$x_{12}^{L} + x_{13}^{L} + x_{15}^{L} = 1, x_{12}^{U} + x_{13}^{U} + x_{15}^{U} = 1, y_{12}^{L} + y_{13}^{L} + y_{15}^{L} = 1, y_{12}^{U} + y_{13}^{U} + y_{15}^{U} = 1,$

$z_{12}^{L} + z_{13}^{L} + z_{15}^{L} = 1, z_{12}^{U} + z_{13}^{U} + z_{15}^{U} = 1, \gamma_{12}^{L} + \gamma_{13}^{L} + \gamma_{15}^{L} = 1, \gamma_{12}^{U} + \gamma_{13}^{U} + \gamma_{15}^{U} = 1,$

$x_{24}^{L} + x_{25}^{L} = x_{12}^{L}, .25\gamma_{56}^{L} + 0x_{56}^{U} + 0y_{56}^{U} + 0.25z_{56}^{U} + 0.375\gamma_{56}^{U}.$

Subject to the constraints

$x_{12}^{L} + x_{13}^{L} + x_{15}^{L} = 1, x_{12}^{U} + x_{13}^{U} + x_{15}^{U} = 1, y_{12}^{L} + y_{13}^{L} + y_{15}^{L} = 1, y_{12}^{U} + y_{13}^{U} + y_{15}^{U} = 1,$

$z_{12}^{L} + z_{13}^{L} + z_{15}^{L} = 1, z_{12}^{U} + z_{13}^{U} + z_{15}^{U} = 1, \gamma_{12}^{L} + \gamma_{13}^{L} + \gamma_{15}^{L} = 1, \gamma_{12}^{U} + \gamma_{13}^{U} + \gamma_{15}^{U} = 1,$

$x_{24}^{L} + x_{25}^{L} = x_{12}^{L}, x_{24}^{U} + x_{25}^{U} = x_{12}^{U}, y_{24}^{L} + y_{25}^{L} = u_{12}^{L}, y_{24}^{U} + y_{25}^{U} = y_{12}^{U},$

$z_{24}^{L} + z_{25}^{L} = z_{12}^{L}, z_{24}^{U} + z_{25}^{U} = z_{12}^{U}, \gamma_{24}^{L} + \gamma_{25}^{L} = \gamma_{12}^{L}, \gamma_{24}^{U} + \gamma_{25}^{U} = \gamma_{12}^{U},$

$x_{34}^L + x_{36}^L = x_{13}^L$, $x_{34}^U + x_{36}^U = x_{13}^U$, $y_{34}^L + y_{36}^L = y_{13}^L$, $y_{34}^U + y_{36}^U = y_{13}^U$,

$z_{34}^L + z_{36}^L = z_{13}^L$, $z_{34}^U + z_{36}^U = z_{13}^U$, $\gamma_{34}^L + \gamma_{36}^L = \gamma_{13}^L$, $\gamma_{34}^U + \gamma_{36}^U = \gamma_{13}^U$,

$x_{45}^L + x_{46}^L = x_{24}^L + x_{34}^L$, $x_{45}^U + x_{46}^U = x_{24}^U + x_{34}^U$, $y_{45}^L + y_{46}^L = y_{24}^L + y_{34}^L + y_{45}^U$

$+ y_{46}^U = y_{24}^U + y_{34}^U$, $z_{45}^L + z_{46}^L = z_{24}^L + z_{34}^L + z_{45}^U + z_{46}^U =$

$z_{24}^U + z_{34}^U$, $\gamma_{45}^L + \gamma_{46}^L = \gamma_{24}^L + \gamma_{34}^L$, $\gamma_{45}^U + \gamma_{46}^U = \gamma_{24}^U + \gamma_{34}^U$, $x_{56}^L = x_{15}^L + x_{25}^L$ $+ x_{45}^L$, $x_{56}^U = x_{15}^U + x_{25}^U + x_{45}^U$, $y_{56}^L = y_{15}^L + y_{25}^L + y_{45}^L$, $y_{56}^U = y_{15}^U + y_{25}^U$ $+ y_{45}^U$, $z_{56}^L = z_{15}^L + z_{25}^L + z_{45}^L$, $z_{56}^U = z_{15}^U + z_{25}^U + z_{45}^U$, $\gamma_{56}^L = \gamma_{15}^L + \gamma_{25}^L + \gamma_{45}^L$, $\gamma_{56}^U = \gamma_{15}^U + \gamma_{25}^U + \gamma_{45}^U$, $x_{36}^L + x_{46}^L + x_{56}^L = 1$, $x_{36}^U, + x_{46}^U + x_{56}^U = 1$, $y_{36}^L + y_{46}^L$ $+ y_{56}^L = 1$, $y_{36}^U + y_{46}^U + y_{56}^U = 1$, $z_{36}^L + z_{46}^L + z_{56}^L = 1$, $z_{36}^U + z_{46}^U + z_{56}^U = 1$, γ_{36}^L $+ \gamma_{46}^L + \gamma_{56}^L = 1$, $\gamma_{36}^U + \gamma_{46}^U + \gamma_{56}^U = 1$, $y_{12}^L - x_{12}^L \geq 0$, $y_{12}^U - x_{12}^U \geq 0$, $z_{12}^L - y_{12}^L$ ≥ 0, $z_{12}^U - y_{12}^U \geq 0$, $\gamma_{12}^L - z_{12}^L \geq 0$, $\gamma_{12}^U - z_{12}^U \geq 0$, $y_{13}^L - x_{13}^L \geq 0$, $y_{13}^U - x_{13}^U \geq 0$, $y_{15}^L - x_{15}^L \geq 0$, $y_{15}^U - x_{15}^U \geq 0$, $z_{15}^L - y_{15}^L \geq 0$, $z_{15}^U - y_{15}^U \geq 0$, $\gamma_{15}^L - z_{15}^L \geq 0$, γ_{15}^L $- z_{15}^L \geq 0$, $y_{24}^L - x_{24}^L \geq 0$, $y_{24}^U - x_{15}^U \geq 0$, $z_{24}^L - y_{24}^L \geq 0$, $z_{24}^U - y_{24}^U \geq 0$, $\gamma_{24}^L - z_{24}^L$ ≥ 0, $z_{24}^L - y_{24}^L \geq 0$, $\gamma_{24}^L - z_{24}^L \geq 0$, $y_{25}^L - x_{25}^L \geq 0$, $y_{25}^U - x_{25}^U \geq 0$, $z_{25}^L - y_{25}^L \geq 0$, $z_{25}^U - y_{25}^U \geq 0$, $\gamma_{25}^L - z_{25}^L \geq 0$, $\gamma_{25}^U - z_{25}^U \geq 0$, $y_{34}^L - x_{34}^L \geq 0$, $y_{34}^U - x_{34}^U \geq 0$, z_{34}^L $- y_{34}^L \geq 0$, $z_{34}^U - y_{34}^U \geq 0$, $\gamma_{34}^L - z_{34}^L \geq 0$, $\gamma_{34}^U - z_{34}^U \geq 0$, $y_{36}^L - x_{36}^L \geq 0$, $y_{36}^U - x_{36}^U$ ≥ 0, $z_{36}^L - y_{36}^L \geq 0$, $z_{36}^U - y_{36}^U \geq 0$, $\gamma_{36}^L - z_{36}^L \geq 0$, $\gamma_{36}^U - z_{36}^U \geq 0$, $y_{45}^L - x_{45}^L \geq 0$, $y_{45}^U - x_{45}^U \geq 0$, $z_{45}^L - y_{45}^L \geq 0$, $z_{45}^U - y_{45}^U \geq 0$, $\gamma_{45}^L - z_{45}^L \geq 0$, $\gamma_{45}^U - z_{45}^U \geq 0$, y_{46}^L $- x_{46}^L \geq 0$, $y_{46}^U - x_{46}^U \geq 0$, $z_{46}^L - y_{46}^L \geq 0$, $z_{46}^U - y_{46}^U \geq 0$, $\gamma_{46}^L - z_{46}^L \geq 0$, $\gamma_{46}^U - z_{46}^U$ ≥ 0, $y_{56}^L - x_{56}^L \geq 0$, $y_{56}^U - x_{56}^U \geq 0$, $z_{56}^L - y_{56}^L \geq 0$, $z_{56}^U - y_{56}^U \geq 0$, $\gamma_{56}^L - z_{56}^L \geq 0$, $\gamma_{56}^U - z_{56}^U \geq 0$,

and

$y_{12}^L, x_{12}^L, y_{12}^U, x_{12}^U, z_{12}^L, y_{12}^L, z_{12}^U, y_{12}^U, \gamma_{12}^L, z_{12}^L, \gamma_{12}^U, z_{12}^U, y_{13}^L, x_{13}^L, y_{13}^U,$

$x_{13}^U, z_{13}^L, y_{13}^L, z_{13}^U, y_{13}^U, \gamma_{13}^L, z_{13}^L, \gamma_{13}^U, z_{13}^U, y_{15}^L, x_{15}^L, y_{15}^U, x_{15}^U, z_{15}^L, y_{15}^L,$

$z_{15}^U, y_{15}^U, \gamma_{15}^L, z_{15}^L, \gamma_{15}^U, z_{15}^U, y_{24}^L, x_{24}^L, y_{24}^U, x_{24}^U, z_{24}^L, y_{24}^L, z_{24}^U, y_{24}^U, \gamma_{24}^L,$

$z_{24}^L, \gamma_{24}^U, z_{24}^U, y_{25}^L, x_{25}^L, y_{25}^U, x_{25}^U, z_{25}^L, y_{25}^L, z_{25}^U, y_{25}^U, \gamma_{25}^L, z_{25}^L, \gamma_{25}^U, z_{25}^U,$

$y_{34}^L, x_{34}^L, y_{34}^U, x_{34}^U, z_{34}^L, y_{34}^L, z_{34}^U, y_{34}^U, \gamma_{34}^L, z_{34}^L, \gamma_{34}^U, z_{34}^U, y_{36}^L, x_{36}^L, y_{36}^U,$

$x_{36}^U, z_{36}^L, y_{36}^L, z_{36}^U, y_{36}^U, \gamma_{36}^L, z_{36}^L, \gamma_{36}^U, z_{36}^U, y_{45}^L, x_{45}^L, y_{45}^U, x_{45}^U, z_{45}^L, y_{45}^L,$

$z_{45}^U, y_{45}^U, \gamma_{45}^L, z_{45}^L, \gamma_{45}^U, z_{45}^U, y_{46}^L, x_{46}^L, y_{46}^U, x_{46}^U, z_{46}^L, y_{46}^L, z_{46}^U, y_{46}^U, \gamma_{46}^L,$

$z_{46}^L, \gamma_{46}^U, z_{46}^U, y_{56}^L, x_{56}^L, y_{56}^U, x_{56}^U, z_{56}^L, y_{56}^L, z_{56}^U, y_{56}^U, \gamma_{56}^L, z_{56}^L, \gamma_{56}^U, z_{56}^U \geq 0$

Step 3. On solving crisp linear programming using TORA system, obtain in step2, solution is $x_{13}^L = y_{13}^L = z_{13}^L = \gamma_{13}^L = x_{36}^L = y_{36}^L = z_{36}^L = \gamma_{36}^L = 1$, $x_{13}^U = y_{13}^U = z_{13}^U = \gamma_{13}^U = x_{36}^U = y_{36}^U = z_{36}^U = \gamma_{36}^U = 1$ and the remaining all values

$$x_{12}^L, x_{12}^U, y_{12}^L, y_{12}^U, z_{12}^L, z_{12}^U, \gamma_{12}^L, \gamma_{12}^U, x_{24}^L, x_{24}^U, y_{24}^L, y_{24}^U, z_{24}^L, z_{24}^U,$$

$$\gamma_{24}^L, \gamma_{24}^U, x_{25}^L, x_{25}^U, y_{25}^L, y_{25}^U, z_{25}^L, z_{25}^U, \gamma_{25}^L, \gamma_{25}^U, x_{34}^L, x_{34}^U, y_{34}^L, y_{34}^U,$$

$$z_{34}^L, z_{34}^U, \gamma_{34}^L, \gamma_{34}^U, x_{45}^L, x_{45}^U, y_{45}^L, y_{45}^U, z_{45}^L, z_{45}^U, \gamma_{45}^L, \gamma_{45}^U, x_{46}^L, x_{46}^U,$$

$$y_{46}^L, y_{46}^U, z_{46}^L, z_{46}^U, \gamma_{46}^L, \gamma_{46}^U, x_{56}^L, x_{56}^U, y_{56}^L, y_{56}^U, z_{56}^L, z_{56}^U, \gamma_{56}^L, \gamma_{56}^U \text{ are zero.}$$

Step 4. Putting the values of $x_{ij}^L, x_{ij}^U, y_{ij}^L, y_{ij}^U, z_{ij}^L, z_{ij}^U, \gamma_{ij}^L, \gamma_{ij}^U$ in

$$\widetilde{X}_{ij} = [(x_{ij}^L, y_{ij}^L, z_{ij}^L, \gamma_{ij}^L), (x_{ij}^U, y_{ij}^U, z_{ij}^U, \gamma_{ij}^U)].$$

The solution is $\widetilde{x}_{12} = [(0, 0, 0, 0), (0, 0, 0, 0)]$, $\widetilde{x}_{13} = [(1, 1, 1, 1), (1, 1, 1, 1)]$

$\widetilde{x}_{15} = [(0, 0, 0, 0), (0, 0, 0, 0)]$, $\widetilde{x}_{24} = [(0, 0, 0, 0), (0, 0, 0, 0)]$,

$\widetilde{x}_{25} = [(0, 0, 0, 0), (0, 0, 0, 0)]$, $\widetilde{x}_{34} = [(0, 0, 0, 0), (0, 0, 0, 0)]$,

$\widetilde{x}_{36} = [(1, 1, 1, 1), (1, 1, 1, 1)]$, $\widetilde{x}_{45} = [(0, 0, 0, 0), (0, 0, 0, 0)]$,

$\widetilde{x}_{46} = [(0, 0, 0, 0), (0, 0, 0, 0)]$, $\widetilde{x}_{56} = [(0, 0, 0, 0), (0, 0, 0, 0)]$,

Step 5. Using the fuzzy solution, the fuzzy critical path is 1-3-6. Replacing the values of $x_{ij}^L, x_{ij}^U, y_{ij}^L, y_{ij}^U, z_{ij}^L, z_{ij}^U, \gamma_{ij}^L, \gamma_{ij}^U$ in step 1, the maximum total completion fuzzy time is [(9,11,14,21),(7,9,16,23)]. Hence, in this problem, the fuzzy critical path is 1-3-6 and the corresponding maximum total completion fuzzy time is [(9,11,14,21), (7,9,16, 23)] respectively.

Conclusion

In this paper, a new method has been employed to find the critical path in the project scheduling problem for the Linear Programming Problems using IVFNS. A distinct algorithmic approach has been proposed for finding the critical path in project network and also few relevant properties of the above said notions have been proposed in this work.

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