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POWER DOMINATOR CHROMATIC NUMBERS OF JAHANGIR AND ASSOCIATED GRAPH

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Abstract

Power Dominator Chromatic (PDC) number was recently introduced concept by combining the notions of power domination and coloring of graphs. A power dominator chromatic number for a simple connected graph $G = (V, E)$ is a proper coloring of G in which every vertex in V power "dominates every vertex of some color class in G. $\chi_{pd}(G)$ is used to represent the power dominator chromatic number of a graph. $\chi_{pd}(G)$ is the least number of colors needed for a power dominator coloring of G. In this paper, we attain the power dominator chromatic number for Jahangir graph" and middle, line and total graph of Jahangir graph.

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1. Introduction

Domination [9] in graphs is a well-investigated area of research in graph theory. A set $S \subseteq V$ in a graph G is a dominating set of a vertex set V, if each vertex in $V - S$ is adjacent to some vertex in S. The cardinality of a minimum dominating set in G is the dominating number of G and is denoted by $\gamma(G)$. Coloring graphs is another major research area in graph theory. Coloring of a graph G is a proper coloring [1] of a graph G, is an assignment of colors to the vertices of the graph in which no two adjacent vertices have the same color. The chromatic number $\chi(G)$ of the graph G is the least number of colors needed in a proper coloring of G. For a graph G, dominator coloring [7] is a proper coloring of a graph in which every vertex of G dominates all the vertices of at least one color class. The concept of power domination [10] in a graph was introduced by Haynes et al. for observing the state of an electric power system with the following monitoring rules.

1. Each vertex that is incident to an observed edge is observed.

2. Each edge joining two observed vertices is observed.

3. If a vertex is incident to a total of $n > 1$ edges and if $n - 1$ of these edges are observed, then all of these edges are observed.

The concept of power dominator chromatic number [[4], [5], [6], [7]] was recently introduced by combining the notions of power domination and coloring concepts in graphs. For a simple connected graph $G = (V, E)$, power dominator coloring is a proper coloring of G in which each vertex of V power dominates every vertex of some color class in G. The power dominator chromatic number $\chi_{pd}(G)$ is the least number of colors needed for a power dominator coloring" of G. Here we obtain $\chi_{pd}(G)$ for Jahangir graph and its derived graphs like middle, line and total graphs.

2. Basic Definitions

(i) Jahangir graph [2] denoted by $J_{m,n}$ for $n \geq 3$, is a graph with $mn + 1$ vertices containing of a cycle C_{mn} with one extra vertex which is adjacent to n vertices of C_{mn} and having a distance m to each other on $C_{mn} \cdot J_{2,n}$ is a n-Gear graph.

(ii) The middle graph [8] for a connected graph G represented by $M(G)$ is the graph whose vertex set is union of $V(G)$ and $E(G)$. Two vertices in $M(G)$ are adjacent if

- The vertices are adjacent edges of G or
- One is a vertex of G and other is an edge incident with it.

(iii) The line graph [8] of a connected graph G is denoted by $L(G)$ is a graph in which

- All the vertex of $L(G)$ represents the edges of G
- Two vertices in $L(G)$ are adjacent if and only if their corresponding edges share a common end point.

(iv) The total graph [8] of a graph G is a graph represented by $T(G)$ in which the vertex set of T corresponds to the vertices and edges of G and two vertices in $T(G)$ are adjacent if and only if their corresponding elements are either adjacent or incident in G. Generalization of total graphs are known as line graph.

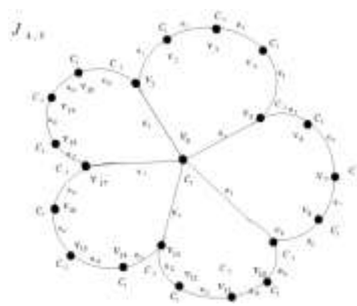
Theorem 3.1. *Let $J_{m,n}$, $m \geq 2$, $n > 3$ be the Jahangir graph, then*

$$\chi_{pd}(J_{m,n}) = \begin{cases} \frac{n}{2} + 2, & n \text{ is even} \\ \left\lceil \frac{n}{2} \right\rceil + 2, & n \text{ is odd} \end{cases}$$

Proof. In $J_{m,n}$ the vertex set is $\{v_0, v_1, v_2, v_3, v_4, \ldots, v_{mn-1}, v_{mn}\}$. The vertex v_0 is of degree n, the vertices $\{v_1, v_{m+1}, v_{2m+1}, v_{3m+1}, \ldots, v_{m(n-1)+1}\}$ are of degree 3 and the remaining vertices are of degree 2. The vertices $\{v_1, v_{m+1}, v_{2m+1}, v_{3m+1}, \ldots, v_{m(n-1)+1}\}$ which is having degree 3 will power dominate all the remaining vertices in the graph. But the vertices which are of degree 2 will power dominate only the end vertices of the path of length $m-1$ in each petals of the graph. So either one of the end vertices of the path must have different color.

To power dominating the vertices of degree 2, assign $\left\lceil \frac{n}{2} \right\rceil$ colors alternatively in the vertex set $\{v_1, v_{m+1}, v_{2m+1}, v_{3m+1}, \ldots, v_{m(n=1)+1}\}$ when n is odd or assign $\frac{n}{2}$ colors alternatively in the vertex set $\{v_1, v_{m+1}, v_{2m+1}, v_{3m+1}, \ldots, v_{m(n-1)+1}\}$ when n is even. Assign color C_1 to the vertex v_0 and remaining vertices in the graph can be colored by the color class C_1 and C_2. By this assignment of colors all the vertices of $J_{m,n}$ will power dominate at least one color.

Example 3.1.1. The power dominator coloring of $J_{4,5}$ is presented below



$\chi_{pd}(J_{4,5}) = 5.$

Corollary 3.1.2. *If* $m = 2, J_{2,n}, n > 3$ *be the n-Gear graph denoted by* G_n, *then*

$$\chi_{pd}(G_n) = \begin{cases} \frac{n}{2} + 2, & n \text{ is even} \\ \left\lceil \frac{n}{2} \right\rceil + 2, & n \text{ is odd.} \end{cases}$$

Proof. In Jahangir graph, the vertices which are adjacent to middle vertex are connected by a path. But the PDC number of a path is 2. So theorem 3.1 remains same.

Theorem 3.2. *Let* $J_{m,n}, m \geq 2, n > 3$ *be the Jahangir graph, the PDC number of middle graph of* $J_{m,n}$,

$$\chi_{pd}(M(J_{m,n})) = \begin{cases} \frac{nm}{2} + n + 1, \; m \; is \; even \\ \frac{n(m-1)}{2} + n + 3, \; m \; is \; odd \end{cases}$$

Proof. Let $V(J_{m,n}) = mn + 1$ and $E(J_{m,n}) = n(m+1)$. From the middle graph definition $V(M(J_{m,n})) = V(J_{m,n}) \cup E(J_{m,n})$, two vertices are adjacent if they are adjacent edges of $J_{m,n}$ or one is a vertex of $J_{m,n}$ and other is an edge incident with it. So the middle graph of $J_{m,n}$ contains a clique of order $n+1$, whereas the n vertices which are connected to the middle vertex in $J_{m,n}$ and the middle vertex forms a clique.

Notice that $M(J_{m,n})$ contains a clique $\langle e_i' \rangle \cup \{v_0\}$, $i = 1, 2, 3, \ldots, n$ of order $n+1$ and assign color C_i $(1 \leq i \leq n+1)$ to the clique. The color C_1 can be assigned to the vertex v_0 which is a middle vertex in $J_{m,n}$. The vertex v_0 is adjacent only with vertices in the clique. The color C_1 also assigned to the vertices v_j $(1 \leq j \leq mn)$. Except the vertices $v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ in the vertex set v_j $(1 \leq j \leq mn)$ will power dominate only the two vertices $\{e_j, e_{j+1}\}$. Any one of the vertices of $\{e_j, e_{j+1}\}$ must have a different color. Further assignment of colors based on following cases.

Case (i). When n is even

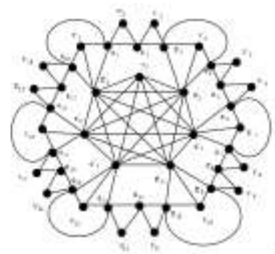
There is odd number of vertices in v_j $(1 \leq j \leq mn)$ except the vertices $v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ will power dominate only the two vertices $\{e_{j-1}, e_j\}$. We have to assign $\frac{m}{2}$ different colors for each side of $M(J_{m,n})$. So to power dominate v_j $(1 \leq j \leq mn)$, $\frac{nm}{2}$ different colors should assign.

Case (ii). When n is odd

There is an even number of vertices in v_j $(1 \leq j \leq mn)$ except the vertices $v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ will power dominates only the two vertices $\{e_{j-1}, e_j\}$. We have to assign $\frac{m-1}{2}$ different color class for each side of

$M(J_{m,n})$. So to power dominate v_j $(1 \leq j \leq mn)$, $\frac{n(m-1)}{2} + 2$ different colors should assign.

Example 3.2.1. The middle graph for Jahangir graph $J_{3,6}$ is presented below



$M(J_{3,6})$

The color classes of $M(J_{3,6})$ are as follows

$C_1 = \{v_0, v_1, v_2, v_3, \ldots, v_{18}\}$, $C_2 = \{e_1'\}$, $C_3 = \{e_2'\}$, $C_4 = \{e_3'\}$, $C_5 = \{e_4'\}$, $C_6 = \{e_5'\}$, $C_7 = \{e_6'\}$, $C_8 = \{e_2\}$, $C_9 = \{e_5\}$, $C_{10} = \{e_8\}$, $C_{11} = \{e_{11}\}$, $C_{12} = \{e_{14}\}$, $C_{13} = \{e_{17}\}$, $C_{14} = \{e_1, e_4, e_7, e_{10}, e_{13}, e_{16}\}$, $C_{15} = \{e_3, e_6, e_9, e_{12}, e_{15}, e_{18}\}$.

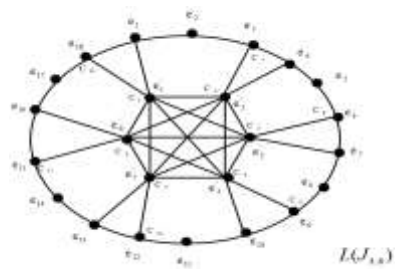
Corollary 3.2.2. *The PDC number of Gear graph* $G_n, n \geq 3$ *is* $\chi_{pd}(G_n) = 2n + 1$.

Proof. Gear graph is special case of Jahangir graph. While m takes the value two in $J_{m,n}$ which gives result for Gear graph G_n.

Theorem 3.3. *For any* $m > 2, n > 3$, *the PDC number of line graph of Jahangir graph* $J_{m,n}$, $\chi_{pd}(L(J_{m,n})) = 2n$.

Proof. Let $V(J_{m,n}) = mn + 1$ and $E(J_{m,n}) = n(m+1)$. From the line graph definition $V(L(J_{m,n})) = E(J_{m,n})$ and two vertices in $L(J_{m,n})$ are adjacent if and only if their corresponding edges share a common end point. So line graph of $J_{m,n}$ contains a clique in the middle, which is of order n, whereas the n vertices corresponding to the edges having end point as v_0 in $J_{m,n}$. Note that $L(J_{m,n})$ contains a clique $\langle e_i' \rangle$ where $(1 \leq i \leq n)$. Assign colors $C_i (1 \leq i \leq n)$ to the clique. We have to assign colors to the remaining nm vertices which is in the circumstances of $L(J_{m,n})$. Among those nm vertices there are $n(m-2)$ vertices which will power dominates only the two vertices which are end vertices of the path formed by $m-2$ vertices n times. To power dominate nm vertices in the circumstances of $L(J_{m,n})$, we have to assign another n colors for those vertices.

Example 3.3.1. The line graph of Jahangir graph $J_{3,6}$ is presented below



Corollary 3.3.2. *For* $m = 2$ *and* $n > 3$, *the PDC number of line graph of Jahangir graph* $J_{m,n}$, $\chi_{pd}(L(J_{m,n})) = n$.

Proof. The value of $m = 2$ gives there are only n vertices in the circumstances of $L(J_{m,n})$ which are adjacent with the clique which is mentioned in the theorem 3.3. So n different colors are enough to power dominate $L(J_{m,n})$.

Theorem 3.4. *For any* $m \geq 2$, $n > 3$, *the PDC number of total graph of Jahangir graph* $J_{m,n}$,

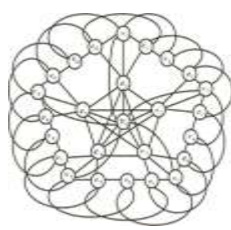
$$\chi_{pd}(T(J_{m,n})) = \begin{cases} \frac{n}{2} + 2, & n \text{ is even} \\ \left\lceil \frac{3n+2}{2} \right\rceil + 2, & n \text{ is odd.} \end{cases}$$

Proof. Let $V(J_{m,n}) = mn + 1$ and $E(J_{m,n}) = n(m+1)$. From the total graph definition $V(T(J_{m,n})) = E(J_{m,n})$ and two vertices in $T(J_{m,n})$ are adjacent if and only if corresponding elements in $T(J_{m,n})$ are either adjacent or incident in $J_{m,n}$. In $T(J_{m,n})$, v_0 be the middle vertex. The vertices $e_1, e_2, e_3, \ldots, e_n$ and v_0 are adjacent to each other and the vertices

$v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ are also adjacent with v_0. Assign colors $C_i (1 \leq i \leq n+1)$ to the vertices $e_1, e_2, e_3, \ldots, e_n$ and v_0 which will power dominate each other and power dominate the vertices $v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ also.

The outer layer of the total graph of $J_{m,n}$ contains n sides having $2nm$ number of vertices. Every vertex between each pair of $v_1, v_{m+1}, v_{2m+1}, \ldots, v_{m(n-1)+1}$ will power dominates at least one of the vertex in the pair. So we have to assign another $\frac{n}{2}$ colors for $v_1, v_{2m+1}, v_{4m+1}, \ldots, v_{m(n-2)}$ when n is even. Otherwise we have to assign $\left\lceil \frac{n}{2} \right\rceil$ colors for $v_1, v_{2m+1}, v_{4m+1}, \ldots, v_{m(n-2)}$ when n is odd.

Example 3.4.1. The Total graph of Jahangir graph $J_{2,5}$ is presented below



$T(J_{2,5})$

The color classes of $T(J_{2,5})$ is

$C_1 = \{e_1', e_3, v_6, v_8\}, C_2 = \{e_2', e_1, v_4, e_6, e_8\}, C_3 = \{e_3', v_2, v_7, e_9\}$,
$C_4 = \{e_4', e_2, v_{10}\}, C_5 = \{e_5', v_3, e_5, e_7, e_{10}\}, C_6 = \{v\}, C_7 = \{v_1\}, C_8 = \{v_5\}$,
$C_9 = \{v_9\}$.

Conclusion

In this paper, we obtained the power dominator chromatic number of Jahangir graph and power dominator chromatic numbers of middle, line and total graph of Jahangir graph. This paper can be extended to identify the graph families for which dominator and power dominator chromatic numbers are equal.

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