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SPLIT RELATIVELY PRIME EDGE DOMINATION NUMBER OF GRAPHS

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Abstract

In this paper, we introduced the concept of Split Relatively Prime Edge Domination Number of Graphs. Let $G = (V, E)$ be a connected graph, then a set $M \subseteq E$ is said to be split relatively prime edge dominating set if M is an edge dominating set with at least 2 edges and for every pair of edges e_i and e_j in M such that $(\deg e_i, \deg e_j) = 1$ and the induced subgraph $\langle E - M \rangle$ is disconnected and the minimum split relatively prime edge domination number is denoted by $\beta_{srpd}(G)$.

1. Introduction

Graph theory is the study of graphs that concerns with the relationship among edges and vertices. For graph theoretical terms, we refer to Harary [2] and for terms related to edge domination we refer to Arumugam [1]. The concept of edge domination was introduced by Mitchell [5] and Hedetniemi [5]. A subset M of E is said to be an edge dominating set in G if every edge in $E - M$ is adjacent to at least one edge in M. A set $S \subseteq V$ is said to be

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relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that $(\deg u, \deg v) = 1$. The minimum cardinality of a relatively prime dominating set is called relatively prime domination number and it is denoted by $\gamma_{rpd}(G)$. Jayasekaran [3] formulate the concept of relatively prime domination in graphs. The concept of split edge domination was introduced by Kulli, Janakiram [4]. A set $M \subseteq E(G)$ is said to be split edge dominating set if F is an edge dominating set and induced subgraph $\langle E - F \rangle$ is disconnected. The minimum cardinality of split edge dominating set in G is the split edge domination number and is denoted by $\beta_s(G)$. In this paper we define split relatively prime edge dominating set $\beta_{srpd}(G)$ and we obtain $\beta_{srpd}(G)$ for various classes of graphs.

2. Definition and Main Results

Definition 2.1. Let $G = (V, E)$ be a connected graph, then a set $M \subseteq E$ is said to be split relatively prime edge dominating set iff

(i) M is an edge dominating set with at least 2 elements and for every pair of edges e_i and e_j in M such that $(\deg e_i, \deg e_j) = 1$.

(ii) The induced subgraph $\langle E - M \rangle$ is disconnected and it is denoted by $\beta_{srpd}(G)$.

Theorem 2.1. *Let P_n be a path graph of order $n > 4$ then*

$$\beta_{srpd}(P_n) = \begin{cases} 2 & for\, n = 5, 6 \\ 3 & for\, n = 7, 8 \\ 0 & otherwise. \end{cases}$$

Proof of theorem 2.1.

Case 1. $n = 5, 6$.

In this case $M = \{e_2, e_n\}$ is a dominating set. Also $(d(e_2), d(e_n)) = (2, 1) = 1$. Therefore, $M = \{e_2, e_n\}$ is relatively prime edge dominating set. The set $\langle E - M \rangle = \{e_1, e_3, e_{n-1}\}$ is an induced subgraph which is disconnected. Therefore $\{e_1, e_3\}$ is a split relatively prime edge dominating set and hence $\beta_{srpd}(P_n) = 2$.

Case 2: $n = 7, 8$.

In this case $M = \{e_1, e_4, e_n\}$ is a dominating set. $(d(e_1), d(e_4)) = (1, 2) = 1$, $(d(e_4), d(e_n)) = (2, 1) = 1$, $(d(e_1), d(e_n)) = (1, 1) = 1$. This case $M = \{e_1, e_4, e_n\}$ is a dominating set. Also, therefore $M = \{e_1, e_4, e_n\}$ is a relatively prime edge dominating set. The set $\langle E - M\rangle = \{e_2, e_3, e_5, \ldots, e_{n-1}\}$ is a split relatively prime edge dominating set and hence $\beta_{srpd}(P_n) = 3$.

Case 3. $n \geq 9$.

Clearly any edge dominating set at least two internal edges $e_i, e_j, 2 \leq i \neq j < n-1$ and $(d(e_i), d(e_j)) = (2, 2) \neq 1$. It does not satisfy the condition for relatively prime edge domination.

Hence $\beta_{srpd}(P_n) = 0$.

Observation 2.1. Let P_n be a path graph then $\beta_{srpd}(P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$ where $5 \leq n \leq 8$ and n is the number of vertices.

Theorem 2.2. *Let P_n be a path graph, if M is a split relatively prime edge dominating set then M is an edge independent set.*

Proof of theorem 2.2. Let $\{e_1, e_2, \ldots, e_n\}$ be the edge set of the path P_n. Let us assume that M be a split relatively prime edge dominating set. We have to prove that set M is independent set. Let us consider M is not an independent set. That is, some edges in set M are adjacent to each other.

For any path, initial and terminal edges have degree one and all the internal edges have degree two.

Case 1. If the adjacent edges are internal edges, then the edges in the set M should have same degree two.

Case 2. If the adjacent edges are not internal edges, then the induced subgraph $\langle E - M\rangle$ is connected.

Above two cases give the contradiction to our assumption that M is split relatively prime edge dominating set. Therefore, M is an independent set.

Theorem 2.3. *Let $S_{m,n}$ be a double star, then*

$$\beta_{srpd}(S_{m,n}) = \begin{cases} 2 & (d(e_i), d(e_j)) = 1 \\ 0 & otherwise. \end{cases}$$

Proof of theorem 2.3. Let $\{e_1, e_2, \ldots, e_n\}$ be the edge set of the double star graph. Consider the edge e_j as the cut edge of double star graph. Also, this edge e_j dominates all the other edges of the graph. But by the definition of relatively prime edge dominating set, the edge dominating set should contain at least two elements. Let us take the pendant edge set as $\{e_1, e_2, \ldots, e_{j-1}, e_{j+1}, \ldots, e_n\}$ and choose any of the pendant edge 'e_i' with e_j. Therefore, the edge dominating set become $= \{e_i, e_j\}$. If $(d(e_i), d(e_j)) = 1$, then set $\{e_i, e_j\}$ is relatively prime edge dominating set. Also, the induced subgraph $\langle E - M \rangle$ is disconnected. Since the edge e_j is a cut edge whose removal disconnects the graph. Therefore, the set M is split relatively prime edge dominating set. Hence $\beta_{srpd}(S_{m,n}) = 2$

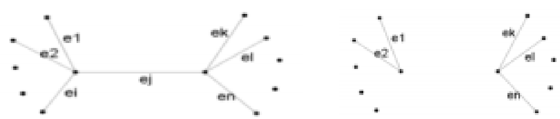

Figure 1.2. Double star graph and its induced subgraph $\langle E - M \rangle$.

Theorem 2.4. *For any cycle C_n/ complete graph K_n/ complete bipartite graph $K_{m,n}$, $\beta_{srpd}(G) = 0$.*

Proof of theorem 2.4. Let $\{e_1, e_2, \ldots, e_n\}$ be the edge set of the graph. Consider M as the edge dominating set.

For any cycle graph/complete graph/complete bipartite graph each edge has same degree.

That is each edge in set M has same degree. Therefore, M is not a relatively prime edge dominating set. Hence $\beta_{srpd}(G) = 0$.

Theorem 2.5. *Let G be a connected graph, then $\beta_{srpd}(G) \leq \beta_0(G)$ where $\beta_0(G)$ is the minimum edge covering number.*

Proof of theorem 2.5. Consider a connected graph G with n vertices. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertex set and $\{e_1, e_2, \ldots, e_n\}$ be the edge set of the graph. As every edge is incident with exactly two vertices. That is $\left\lceil \frac{n}{2} \right\rceil$ number of edges are needed to cover all the n vertices.

Let us consider the connected graph as path P_n, from observation 3.1 we have, $\beta_{srpd}(P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$ which implies $\left\lfloor \frac{n}{3} \right\rfloor + 1 \leq \left\lfloor \frac{n}{2} \right\rfloor$. That is $\beta_{srpd}(G) \leq \beta_0(G)$.

Theorem 2.6. *Let G be a connected graph, then $\lambda(G) \leq \beta_{srpd}(G)$ where $\lambda(G)$ is the edge connectivity.*

Proof of theorem 2.6. Consider a graph with n edges $\{e_1, e_2, \ldots, e_n\}$.

Case 1. If e_i is a cut edge of the graph then the edge connectivity of the graph is one, that is

$$\lambda(G) = 1. \tag{1}$$

Let M be the edge dominating set of the graph. By the definition of relatively prime edge dominating set, the set M should contain at least two elements, that is

$$\beta_{srpd}(G) \geq 2. \tag{2}$$

From (1) and (2) we get

$$\lambda(G) \leq \beta_{srpd}(G).$$

Case 2. If e_i is not a cut edge, that is $\lambda(G) > 1$.

Assume that M is a β_{srpd} set of graph G.

Also, we have for any connected graph G may have at most $n - 1$ cut edges.

Let $\lambda(G) = n - 1$.

But this is a contradiction to the minimality of split edge domination.

Therefore, $\lambda(G) \leq \beta_{srpd}(G)$.

Theorem 2.7. *For any graph* G, $\beta_{srpd}(G) \leq n - \Delta'(G)$ *where* $\Delta'(G)$ *is the maximum edge degree and* n *is the number of edges. Also, the equality holds for path graph, unicyclic graph, lollipop graph and tadpole graph.*

Proof of theorem 2.7. Let $E = \{e_1, e_2, \ldots, e_n\}$ be the edge set of the graph G. Consider set M is the β_{srpd} set of G. That is M dominates $N(M)$ and edges in $E - N(M)$ dominate themselves. Let $e_i \in M$ be an edge with maximum degree $\Delta'(G)$. Then $E - \{M\}$ forms an induced subgraph which is disconnected. Therefore $E - N(M)$ is a split relatively prime edge dominating set of cardinality less than or equal to $n - \Delta'(G)$. Hence $\beta_{srpd}(G) \leq n - \Delta'(G)$.

Theorem 2.8. *Let* P_m *be a pan graph, then* $\beta_{srpd}(P_m) = 2$ *for* $4 \leq m \leq 6$.

Proof of theorem 2.8. Let P_m be a pan graph with cycle C_m. By definition, the m-pan graph is the graph obtained by joining a cycle graph C_m to a singleton graph K_1 with a bridge. By the result for cycle graph, $\beta_{srpd}(C_m)$ is zero. Let e_i, e_j be the adjacent edges on the bridge between C_m and K_1 and let e_n take as the bridge. Clearly all the edges have degree 2 except e_i, e_j where e_i, e_j have degree 3. Let $\{e_1, e_2, \ldots, e_i, e_j, \ldots, e_n\}$ be the edge set of the graph. Let $M = \{e_m, e_i\}$ be an edge dominating set. Then $(d(e_m), d(e_j)) = (2, 3) = 1$, which is relatively prime. Therefore, $M = \{e_m, e_i\}$ is a relatively prime edge dominating set. Also, if the induced subgraph $\langle E - M \rangle$ is disconnected then it is a split edge dominating set.

Hence $\beta_{srpd}(p_m) = 2$.

3. Conclusion

In this paper, we surveyed selected results on split relatively prime edge domination number of graphs. These results establish key relationships between the relatively prime numbers and the dominating sets in graphs. We find results for some standard graphs and also, some results on edge independent set, edge covering and edge connectivity of graphs in this paper.

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