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NEW KINDS OF GLOBAL EDGE DOMINATION IN FUZZY GRAPH USING STRONG ARC - NEW APPROACH

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Abstract

In this paper we establish the relation between strong arc domination number and global domination number (new approach) of some standard graphs using strong arcs. Global edge domination number using strong arc is defined and Global edge domination number of some standard graphs are discussed. Also various new kinds of global edge domination number using strong arc is introduced and some results are discussed.

1. Introduction

Fuzzy graph is the generalization of the ordinary graph. Therefore it is natural that though fuzzy graph inherits many properties similar to those of ordinary graph, it deviates at many places. The earliest idea of dominating sets date back to the origin of game of chess in India over 400 years ago in which placing the minimum number of chess piece (such as Queen, Knight etc.,) over chess board so as to dominate all the squares of chess board was

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investigated. The formal mathematical definition of domination was given by Ore. O in 1962. In 1975 A. Rosenfeld introduced the notion of fuzzy graph and several analogs of theoretic concepts such path, cycle and connected.

A. Somasundaram and S. Somasundaram discussed the domination in fuzzy graph using effective arc. C. Y. Ponnappan and V. Senthil Kumar discussed the domination in fuzzy graph using strong arc. C. Y. Ponnappan, S. Basheer Ahamed and P. Surulinathan discussed about edge domination number using effective edges. Also C. Y. Ponnappan and S. Basheer Ahamed dicussed about various new kinds of edge domination number using effective edges. Here Global edge domination number using strong arc is discussed. Also various kinds of Global edge domination number using strong arcs are discussed. Before introducing global edge domination in fuzzy graphs using strong arcs, we are placed few preliminary definitions and results for new one.

2. Preliminaries

Domination in Fuzzy Graph Using Strong Arc

Definition 2.1. An arc (u, v) of the fuzzy graph $G(\sigma, \mu)$ is called a strong arc if $\mu(u, v) = \mu^{\infty}(u, v)$ else arc (u, v) is called non strong. Strong neighborhood of $u \in V$ is $Ns(u) = \{v \in V : \text{arc } (u, v) \text{ is strong}\}$.

$Ns[u] = Ns(u)U\{u\}$ is the closed neighborhood of u. The minimum cardinality of strong neighborhood $\delta s(G) = \min\{|Ns(u)| : u \in V(G)\}$. Maximum cardinality of strong neighborhood $\Delta s(G) = \max\{|Ns(u)| : u \in V(G)\}$.

Definition 2.2. Let $G(\sigma, \mu)$ be a fuzzy graph. Let u, v be two nodes of $G(\sigma, \mu)$. We say that u dominates v if edge (u, v) is a strong arc. A subset D of V is called a dominating set of $G(\sigma, \mu)$ if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of a graph G is called the strong arc dominating number and is denoted by $\gamma_s(G)$ and the corresponding set is called minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(G)]$.

3. Global Domination in Fuzzy Graph Using Strong Arc

Definition 3.1. Let $G(\sigma, \mu)$ be a fuzzy graph. Let u, v be two nodes of $G(\sigma, \mu)$. We say that u dominates v if edge (u, v) is a strong arc. A subset D of V is called a dominating set of $G(\sigma, \mu)$ if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of a graph G is called the strong arc dominating number and is denoted by $\gamma_s(G)$ and the corresponding set is called minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(G)]$. The minimum fuzzy cardinality taken over all dominating sets of a graph $\overline{G}$, where $\overline{G}$ is the complement of the fuzzy graph G, is called the strong arc dominating number of $\overline{G}$ and is denoted by $\gamma_s(\overline{G})$ and the corresponding set is called the minimum strong arc dominating set of $\overline{G}$. The number of elements in the minimum strong arc dominating set is denoted by $n[\gamma_s(G)]$. Fuzzy Global Dominating set using strong arc is the set which is the corresponding dominating set of $\min\{\gamma_s(G), \gamma_s(\overline{G})\}$ and is denoted by $\gamma_{gs}(G)$. The number of elements of fuzzy global dominating set using strong is denoted by $n[\gamma_s(G)]$.

Note. Here we consider the fuzzy graphs with non effective edges.

Definition 3.2. If $n[\gamma_s(G)] = n[\gamma_s(\overline{G})]$ and G and $\overline{G}$ have different dominating sets then the domination is called Semi Perfect Fuzzy Global Domination using strong arc and the cardinality of the corresponding dominating set is denoted by $\gamma_{spgs}(G)$.

Definition 3.3. If the minimum dominating set D of G which is the dominating set of both G and $\overline{G}$ using strong arc is called Fuzzy Perfect

Global Dominating Set of G using strong arc and the cardinality of D is denoted by $\gamma_{pgs}(G)$.

4. Global Edge Domination in Fuzzy Graph using Strong Arc

Definition 4.1. Let G be a fuzzy graph. A subset $D_s'(G)$ of E (E is edge set of G) is said to be an edge dominating set of G using strong arc if for every edge in $E - D_s'(G)$ is adjacent to at least one strong arc in $D_s'(G)$. The minimum fuzzy cardinality of an edge dominating set of G is called the edge domination umber of G using strong arc and is denoted by $\gamma_s'(G)$. Similarly $\gamma_s'(G)$ is the edge domination number of $\overline{G}$ using strong arc. Then the global edge domination of G using strong arc $\gamma_{gs}'(G)$ is the minimum of $\gamma_s'(G)$ and $\gamma_s'(G)$. (i.e.) $\gamma_{gs}'(G) = \min\{\gamma_s'(G), \gamma_s'(G)\}$.

Example 4.2. Consider the graph G with vertices u_1, u_2, u_3, u_4 such that $\sigma(u_1) = 0.3, \sigma(u_2) = 0.4, \sigma(u_3), 0.5, \sigma(u_4) = 0.4$ and the edges $e_i, i = 1, 2, 3, 4$ such that $\mu(e_1) = \mu(u_1, u_2) = 0.1, \mu(e_2) = \mu(u_2, u_3) = 0.2, \mu(e_3) = \mu(u_3, u_4) = 0.2, \mu(e_4) = \mu(u_1, u_4) = 0.1$. Here $D_s'(G) = \{e_1, e_4\}$ and $\gamma_s'(G) = 0.2$.

Then $\overline{G}$ is the graph having the same vertices with the edges $e_i, i = 1, 2, 3, 4, 5, 6$ such that $\mu(e_1) = \mu(u_1, u_2) = 0.2, \mu(e_2) = \mu(u_2, u_3) = 0.2, \mu(e_3) = \mu(u_3, u_4) = 0.2, \mu(e_4) = \mu(u_1, u_4) = 0.2, \mu(e_5) = \mu(u_1, u_3) = 0.3, \mu(e_6) = \mu(u_2, u_4) = 0.4$.

In $\overline{G}$ all arcs are strong arcs, $D_s'(G) = \{e_1, e_4\}$ and $\gamma_s'(G) = 0.4$, $\gamma_{gs}'(G) = \min\{0.2, 0.4\} = 0.2$.

Theorem 4.3. *If G is a strong complete fuzzy graph K_n, then $n[\gamma_{gs}'(G)] = \left\lfloor \frac{n}{2} \right\rfloor$, where n is the number of vertices.*

Theorem 4.4. *If G is a fuzzy path FP_n, then $n[\gamma_{gs}'(G)] = \left\lfloor \frac{n-1}{2} \right\rfloor$, where n is the number of vertices.*

Theorem 4.5. *Let G be a fuzzy cycle C_n with strong arcs, then* $n[\gamma_{gs}'(G)] = \left\lceil \frac{n}{3} \right\rceil$, *where n is the number of vertices.*

Proof. Let $G = C_n$ be a fuzzy cycle, then $n[\gamma_{gs}'(G)] = \left\lceil \frac{n}{3} \right\rceil$. $\overline{G}$ is a complete fuzzy graph not necessarily all arcs as strong arcs. $\therefore n[\gamma_{gs}'(G)] = \left\lfloor \frac{n}{2} \right\rfloor$, where n is the number of vertices. Therefore $n[\gamma_s'(G)] \leq n[\gamma_{gs}'(\overline{G})]$. $\therefore \gamma_s'(G) \leq \gamma_s'(\overline{G})$, $\gamma_{gs}'(G) = \min\{\gamma_s'(G), \gamma_s'(\overline{G})\}$ $= \gamma_s'(G)$. (i.e.) $\gamma_{gs}'(G) = \gamma_s'(G)$. Therefore $n[\gamma_{gs}'(G)] = n[\gamma_s'(G)] = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 4.6. *For any fuzzy graph with strong arcs without isolated edges,* $\gamma_s'(G) \leq \frac{q}{2}$, *where* $q = \sum \mu(e_i)$.

Proof. Let G be a fuzzy graph with strong arcs with 'n' number of vertices. ∴ G has at most nc_2 edges. Then $D_s'(G)$ be the edge dominating set of G using strong arcs and $D_s'(G)$ has at most $\left\lfloor \frac{n}{2} \right\rfloor$ edges. ∴ The fuzzy edge domination number of G using strong arc is at most $\frac{q}{2}$. $\therefore \gamma_s'(G) \leq \frac{q}{2}$.

Corollary 4.7. *Let G be a fuzzy graph such that G and $\overline{G}$ have no isolated edges. Then* $\gamma_s'(G) + \gamma_s'(\overline{G}) \leq q$, $q = \sum \mu(e_i)$, $e_i \in G$.

Theorem 4.8. *Let $G = K_{1, m}$ be a fuzzy graph with strong arcs. Then* $\gamma_{gs}'(G) = \gamma_s'(G)$ *and* $n[\gamma_{gs}'(G)] = 1$.

Proof. Let $G = K_{1, m}$ be a fuzzy graph with all arcs as strong arcs. Then $\gamma_s'(G) = \min\{\mu(e_i), i = 1, 2, 3, \ldots, m$. $\therefore n[\gamma_s'(G)] = 1$. For $G = K_{1, m}$, $\overline{G}$ will be a fuzzy complete graph with $(m+1)$ vertices, not necessarily all arcs are strong arcs. $\therefore n[\gamma_s'(\overline{G})] = \left\lfloor \frac{m+1}{2} \right\rfloor$. $\therefore \gamma_s'(\overline{G}) > \gamma_s'(G)$.

Therefore $\gamma_{gs}'(G) = \min\{\gamma_s'(G), \gamma_s'(\overline{G})\} = \gamma_s'(G)$ and $n[\gamma_{gs}'(G)] = 1$.

Results 4.9. Let G be a fuzzy graph with all edges as strong arcs.

Result 4.9.1. $G = P_2 \times P_2$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.2. $G = P_2 \times P_3$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 4$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.3. $G = P_2 \times P_4$ $n[\gamma_s'(G)] = 3$; $n[\gamma_s'(\overline{G})] = 4$, $n[\gamma_{gs}'(G)] = 3$

Result 4.9.4. $G = D_{2^3}$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.5. $G = D_{3^3}$ $n[\gamma_s'(G)] = 3$; $n[\gamma_s'(\overline{G})] = 3$, $n[\gamma_{gs}'(G)] = 3$

Result 4.9.6. $G = D_{4^3}$ $n[\gamma_s'(G)] = 4$; $n[\gamma_s'(\overline{G})] = 4$, $n[\gamma_{gs}'(G)] = 4$

Result 4.9.7. $G = D_{n^3}$ $n[\gamma_s'(G)] = n$; $n[\gamma_s'(\overline{G})] = n$, $n[\gamma_{gs}'(G)] = n$

Result 4.9.8. $G = W_3$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.9. $G = W_4$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.10. $G = W_5$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 3$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.11. $G = W_6$ $n[\gamma_s'(G)] = 3$; $n[\gamma_s'(\overline{G})] = 3$, $n[\gamma_{gs}'(G)] = 2$

Result 4.9.12. $G = W_7$ $n[\gamma_s'(G)] = 3$; $n[\gamma_s'(\overline{G})] = 4$, $n[\gamma_{gs}'(G)] = 3$

Result 4.9.13. $G = Sh_4$ $n[\gamma_s'(G)] = 1$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)]$ can't be predicted.

Result 4.9.14. $G = Sh_5$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 2$, $n[\gamma_{gs}'(G)]$ can't be predicted.

Result 4.9.15. $G = Sh_6$ $n[\gamma_s'(G)] = 2$; $n[\gamma_s'(\overline{G})] = 3$, $n[\gamma_{gs}'(G)]$ can't be predicted.

5. Fuzzy Global Non Split Edge Domination Number by Strong Arc

Definition 5.1. An edge dominating set $D_{nss}'(G)$ of a fuzzy graph G is a fuzzy non split dominating set of G using strong arc if the induced subgraph $\langle E - D_{nss}'(G)\rangle$ is connected. The non split edge domination number $\gamma_{nss}'(G)$ is the minimum fuzzy cardinality of a non split edge dominating set of G. Similarly $\gamma_{nss}'(G)$ is the minimum fuzzy cardinality of a non split edge dominating set of $\overline{G}$ using strong arcs. Then fuzzy global nonsplit edge domination number $\gamma_{gnss}'(G)$ of G is the minimum of $\gamma_{nss}'(G)$ and $\gamma_{nss}'(\overline{G})$ (i.e.) $\gamma_{gnss}'(G) = \min\{\gamma_{nss}'(G), \gamma_{nss}'(\overline{G})\}$.

Example 5.2. Consider the graph G with vertices u_1, u_2, u_3, u_4 such that $\sigma(u_1) = 0.5, \sigma(u_2) = 0.3, \sigma(u_3), 0.4, \sigma(u_4) = 0.3$ and the edges $e_i, i = 1, 2, 3, 4$ such that $\mu(e_1) = \mu(u_1, u_4) = 0.2, \mu(e_2) = \mu(u_1, u_2) = 0.2, \mu(e_3) = \mu(u_2, u_3) = 0.1, \mu(e_4) = \mu(u_3, u_4) = 0.1$.

Here $D_{nss}'(G) = \{e_3, e_4\}, \gamma_{nss}'(G) = 0.2, \langle E - D_{nss}'(G)\rangle = \{e_1, e_2\}$ is connected.

Then $\overline{G}$ is the graph having the same vertices with the edges $e_i, i = 1, 2, 3, 4, 5, 6$ such that $\mu(e_1) = \mu(u_1, u_4) = 0.1, \mu(e_2) = \mu(u_1, u_2) = 0.1, \mu(e_3) = \mu(u_2, u_3) = 0.2, \mu(e_4) = \mu(u_3, u_4) = 0.2, \mu(e_5) = \mu(u_2, u_4) = 0.3$ and $\mu(e_6) = \mu(u_1, u_3) = 0.4; e_1, e_2$ are non strong arcs.

Here $D_{nss}'(G) = \{e_3, e_4\}, \gamma_{nss}'(\overline{G}) = 0.4, \langle E - D_{nss}'(\overline{G})\rangle = \{e_1, e_2, e_5, e_6\}$ is connected. Therefore $\gamma_{gnss}'(G) = \min\{\gamma_{nss}'(G), \gamma_{nss}'(\overline{G})\} = 0.2$. And $n[\gamma_{gnss}'(G)] = 2$.

Result 5.3. Fuzzy global non split edge domination number using strong arc does not exist for $FP_n, (n \geq 5)$.

Result 5.4. For any fuzzy graph G, $\gamma_{gnss}'(G) \leq q(q = \sum \mu(u, v))$.

Result 5.5. For complete fuzzy graph K_3 with all arcs as strong arcs, $n[\gamma_{gnss}'(G)] = 1$.

For complete fuzzy graphs K_3, K_5 with all arcs as strong arcs, $n[\gamma_{gnss}'(G)] = 2$.

Result 5.6. For fuzzy cycles $FC_n(n > 4)$, fuzzy global non split edge dominating set using strong arc does not exist.

Proof. Case (i) Let $G = FC_n, (n > 4)$ be a fuzzy cycle with all arcs as strong arcs. G has at least five edges. With one edge, two edges are incident. In the remaining two edges if we select one of the edge, then we get the edge dominating set D.

But $\langle E - D\rangle$ will not be connected. Therefore non split edge dominating set does not exists.

Case (ii) If we select two consecutive edges first, then two edges will be dominated by those edges. Remaining one edge also be taken in the dominating set. But $\langle E - D\rangle$ will be disconnected. Hence the result.

Result 5.7. For $G = D_{n^3}$, $n[\gamma_{nss}'(G)] = n$; $n[\gamma_{nss}'(\overline{G})] = n$, $n[\gamma_{gnss}'(G)] = n$.

Proof. Let $G = D_{n^3}$. Then $n[\gamma_{nss}'(G)] = n$. $\overline{G}$ is a graph with $(2n+1)$ vertices and $\overline{G}$ will be a complete graph not necessarily all the arcs as strong arcs. Then $n[\gamma_{nss}'(\overline{G})] = \left\lfloor \frac{2n+1}{2} \right\rfloor = n$. $\therefore n[\gamma_{nss}'(G)] = n[\gamma_{nss}'(\overline{G})] = n$. Therefore $n[\gamma_{nss}'(G)] = n$.

Result 5.8. For W_4 with all arcs as strong arcs, $n[\gamma_{nss}'(G)] = n[\gamma_{nss}'(\overline{G})] = n[\gamma_{gnss}'(G)] = 2$.

6. Fuzzy Global Connected Edge Domination Number Using Strong Arc

Definition 6.1. Let G be a fuzzy graph. An edge dominating set $D_{cs}'(G)$ of a fuzzy graph G using strong arc is a connected edge dominating set with $\langle D_{cs}'(G)\rangle$ is connected. The connected edge domination number $\gamma_{cs}'(G)$ of G is the minimum fuzzy cardinality of connected edge dominating set using strong arc of G. Similarly $D_{cs}'(\overline{G})$ is a connected edge dominating set of $\overline{G}$

using strong arc. The connected edge domination number $\gamma_{cs}'(\overline{G})$ of $\overline{G}$ is the minimum fuzzy cardinality of connected edge dominating set using strong arc of $\overline{G}$. Then fuzzy global connected edge domination number using strong arc $\gamma_{gcs}'(G)$ of G is the minimum of $\gamma_{cs}'(G)$ and $\gamma_{gcs}'(G) = \min\{\gamma_{cs}'(G), \gamma_{cs}'(\overline{G})\}$.

Result 6.2. For a strong complete fuzzy graph $K_n (n > 4)$, a connected edge dominating set using strong arcs may exist but it need not be the minimal dominating set.

Example 6.3. Consider the graph G with vertices u_1, u_2, u_3, u_4 such that $\sigma(u_1) = 0.5, \sigma(u_2) = 0.2, \sigma(u_3), 0.4, \sigma(u_4) = 0.2$ and the edges $e_i, i = 1, 2, 3, 4$ such that $\mu(e_1) = \mu(u_1, u_2) = 0.1, \mu(e_2) = \mu(u_2, u_3) = 0.1, \mu(e_3) = \mu(u_3, u_4) = 0.1, \mu(e_4) = \mu(u_1, u_3) = 0.3$. Here $D_{cs}'(G) = \{e_2\}$ and $\gamma_{cs}'(G) = 0.1$.

Then $\overline{G}$ is the graph having the same vertices with the edges $e_i, i = 1, 2, 3, 4, 5, 6$ such that that $\mu(e_1) = \mu(u_1, u_2) = 0.2, \mu(e_2) = \mu(u_2, u_3) = 0.2, \mu(e_3) = \mu(u_3, u_4) = 0.1, \mu(e_4) = \mu(u_1, u_4) = 0.2, \mu(e_5) = \mu(u_1, u_3) = 0.1, \mu(e_6) = \mu(u_2, u_4) = 0.2$.

In $(\overline{G})$ the arcs (u_1, u_3) and (u_3, u_4) are non strong arcs. $D_{cs}'(\overline{G}) = \{e_1, e_2\}$ and $\gamma_{cs}'(\overline{G}) = 0.4$. $\therefore \gamma_{gcs}'(G) = 0.1$ and $n[\gamma_{gcs}'(G)] = 1$.

Result 6.4. $\gamma_{cs}'(G) + \gamma_{cs}'(\overline{G}) \leq q + q$ where $q = \sum_{uv \in \overline{G}} \mu(uv)$

7. Fuzzy Global Inverse Edge Domination Number Using Strong Arc

Definition 7.1. Let G be a fuzzy graph. A subset D_s' of E is a minimal edge dominating set of G using strong arc if $E - D_s'$ contains an edge dominatingset D_{is}' using strong arc, then D_{is}' is called an inverse edge dominating set of G with respect to D_s'. The minimum fuzzy cardinality taken over all inverse edge dominating set of G is called the Fuzzy inverse edge domination number of G and is denoted by $\gamma_{is}'(G)$. Similarly $\gamma_{is}'(\overline{G})$ is the fuzzy inverse edge domination number of $\overline{G}$. Then the fuzzy global inverse edge domination number using strong arc $\gamma_{gis}'(G)$ is defined as the minimum of $\gamma_{is}'(G)$ and $\gamma_{is}'(\overline{G})$. $\gamma_{gis}'(G) = \min\{\gamma_{is}'(G), \gamma_{is}'(\overline{G})\}$.

Example 7.2. Consider the graph G with vertices u_1, u_2, u_3 such that $\sigma(u_1) = 0.3$, $\sigma(u_2) = 0.4$, $\sigma(u_3)$, 0.5 and the edges e_i, $i = 1, 2, 3$ such that $\mu(e_1) = \mu(u_1, u_2) = 0.2$, $\mu(e_2) = \mu(u_2, u_3) = 0.2$, $\mu(e_3) = \mu(u_2, u_3) = 0.3$

Here $D_s'(G) = \{e_1\}$, $E - D_s'(G) = \{e_2, e_3\}$, $D_{is}'(G) = \{e_2\}$, $\gamma_{is}'(G) = 0.2$, $n[\gamma_{is}'(G)] = 1$.

Then $\overline{G}$ is the graph having the same vertices with the edges e_i, $i = 1, 2, 3$ such that $\mu(e_1) = \mu(u_1, u_2) = 0.1$, $\mu(e_2) = \mu(u_2, u_3) = 0.1$, $\mu(e_3) = \mu(u_2, u_3) = 0.1$

Here $D_s'(\overline{G}) = \{e_1\}$, $E - D_s'(\overline{G}) = \{e_2, e_3\}$, $D_{is}'(\overline{G}) = \{e_2\}$. $\therefore \gamma_{is}'(\overline{G}) = 0.1$ $n[\gamma_{is}'(\overline{G})] = 1, \gamma_{gis}'(G) = \min\{0.2, 0.1\} = 0.1$ and $n[\gamma_{is}(G)] = 1$.

Result 7.3. If G is a strong complete fuzzy graph K_n then $n[\gamma_{gis}'(G)] = \left\lfloor \frac{n}{2} \right\rfloor$, where n is the number of vertices.

Result 7.4. For any fuzzy graph, $\gamma_s'(G) \leq \gamma_{is}'(G)$.

Result 7.5. For any fuzzy graph G, $n[\gamma_s'(G)] = n[\gamma_{is}'(G)]$.

Theorem 7.6. *Let $G = K_{1, m}$ be a fuzzy graph with all arcs as strong arcs. Then $\gamma_{gis}'(G) = \gamma_{is}'(G)$ and $n[\gamma_{gis}'(G)] = 1$.*

Proof. Let $G = K_{1, m}$ be a fuzzy graph with all arcs as strong arcs.

Then $\gamma_s'(G) = \min\{\mu(e_i)\}$, $i = 1, 2, 3, \ldots, m$ $\{\text{say}\mu(e_j)\}$. $\therefore n[\gamma_s'(G)] = 1$

Now $\gamma_{is}(G) = \min\{\mu(e_i)\}$, $i = 1, 2, 3, \ldots, j-1, j+1, \ldots, m$

Therefore $\gamma_{is}'(G) \geq \gamma_s'(G)$ and $n[\gamma_{is}'(G)] = 1$.

Now $\overline{G}$ will be a fuzzy complete graph with $(m+1)$ vertices, not necessarily all arcs as strong arcs. $\therefore n[\gamma_s'(\overline{G})] = \left\lfloor \frac{m+1}{2} \right\rfloor$. $\therefore n[\gamma_{is}'(\overline{G})] = \left\lfloor \frac{m+1}{2} \right\rfloor$.

Also $\gamma_{is}'(G) \geq \gamma_{is}'(G)$. Therefore $\gamma_{gis}'(G) = \gamma_{is}'(G)$. And $n[\gamma_{gis}'(G)] = n[\gamma_{is}'(G)] = 1$.

Theorem 7.7. *Let G be a fuzzy path FP_n with strong arcs. Then $n[\gamma_{gis}'(G)] = \left\lfloor \frac{n-1}{2} \right\rfloor$, where n is the number of vertices.*

Proof. Let G be a fuzzy path FP_n with strong arcs. Then $n[\gamma_s'(G)] = n[\gamma_{is}'(G)] = \left\lfloor \frac{n-1}{2} \right\rfloor$ and $\overline{G}$ will be a strong complete fuzzy graph with n vertices.

Therefore $n[\gamma_s'(\overline{G})] = \left\lfloor \frac{n}{2} \right\rfloor$. $\therefore n[\gamma_{is}'(\overline{G})] = \left\lfloor \frac{n}{2} \right\rfloor$. And $\left\lfloor \frac{n-1}{2} \right\rfloor \leq \left\lfloor \frac{n}{2} \right\rfloor$. $\therefore n[\gamma_{is}'(G)] = \left\lfloor \frac{n-1}{2} \right\rfloor$.

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