Advances and Applications in Mathematical Sciences
Volume 20, Issue 8, June 2021, Pages 1509-1515
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AN EXTENDED APPLICATION OF AVERAGE SUM METHOD

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Abstract

In job-machine allocation problems, all jobs are supposed to be executed on the given machine. For unbalanced assignment problem where the number of jobs is more than the number of machines, the existing approaches assign some jobs to a dummy machine which means these jobs are left without execution but in real life everyone is interested to get all jobs must be performed at minimum cost. To overcome this situation we propose the extended application of average sum method to solve the unbalanced assignment problem with the condition that every machine can perform more than one job and same job cannot be assigned to more than one machine. The method is well illustrated with numerical example.

1. Introduction

The importance of Assignment Problem is due to the fact that the actual allocation of jobs to the available machine provides the best solution to the given problem. There exist many approaches which have been developed for solving the assignment problems and getting the best result. Some of the important methods for solving assignment problem available in the literature are Hungarian method developed by Kuhn [1] and Average Sum Method by Dubey et al. [5]. These methods are based on the assumptions that one machine can perform only one job. If the number of job is more than the number of machine then the existing methods allocate some jobs to the (dummy) machine which is actually not exist. In actual practice, all jobs must be performed on proper machines and no single job can be left without execution.

2020 Mathematics Subject Classification: 90C05, 90C10, 90C59.

Keywords: unbalanced assignment problem, hungarian assignment method, average sum method, optimization.

Received February 13, 2020; Accepted January 12, 2021

Several researchers presented different methods for solving unbalanced assignment problem in which all jobs are executed. Kumar [2], Yadaiah and Haragopal [3] and Betts and Vasko [4] discus the method in which they allow the decision maker to assign more than one job. Quazzafi et al. [6] given a new form of mathematical representation of unbalanced assignment problem in which a single machine can do more than one job and there is no restriction on the number of jobs that can be assigned to a given machine.

In this paper we propose an extended application of Average Sum Method for solving unbalanced assignment problems in which all jobs are performed on the given machines at the minimum cost. This paper also discussed the assumptions and formulation given by Quazzafi et al. [6] and Dubey et al. [5]. The method is competent and suitable for assigning all jobs to the machines optimally. The algorithmic form of this method gives more accuracy and efficiency to the result and can be applied on the several set of input data to test the performance and effectiveness of the algorithm.

2. Formulation

Assuming that an unbalanced assignment problem having m machines and n jobs, where $n > m$. Also assume that c_{ij} be the associated cost of i^{th} machine to perform j^{th} job where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$; the main task is to obtain optimal assignment policy (job machine allocation) such that all the jobs must be executed at minimum cost. Here the numbers of jobs are more than number of machines and one machine can do more than one job and no same job is assigned to more than one machine. This can be stated in the form of $m \times n$ cost matrix $[c_{ij}]$ or real numbers (Table 1).

Table 1.

	J_1	J_2	$\cdots$	J_n
M_1	C_{11}	C_{12}	$\cdots$	C_{1n}
M_2	C_{21}	C_{22}	$\cdots$	C_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M_m	C_{m1}	C_{m2}	$\cdots$	C_{mn}

$$\text{Let } x_{ij} = \begin{cases} 1, \text{ if } i^{th} \text{ machine is assigned } j^{th} \text{ job} \\ 0, \text{ otherwise} \end{cases}.$$

The mathematical model is stated as

Minimize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}. \tag{1}$$

Subject to constraints

$$\sum_{j=1}^{n} x_{ij} \geq 1; \text{ for } i = 1, 2, \ldots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = 1; \text{ for } j = 1, 2, \ldots, n \tag{3}$$

$$x_{ij} = 0 \text{ or } 1. \tag{4}$$

3. Algorithm

Step 1. Input m, n.

Step 2. Find the minimum element (cost) in each column and subtract it from the elements of respective column that creates at least one zero in each column.

Step 3. Select the minimum element in each row and subtract it from the elements of respective row.

Step 4. Draw the minimum number of lines that covers all the zeros, if the number of lines is equal to the number of machines (row) then go to step 6 else step 5.

Step 5. Select the smallest element from the uncovered elements and subtract it from each uncovered element.

Step 6. Start from the first zero cost in modified matrix obtained, allocate the average sum given by Dubey et al. [5] of each cost of row and column perpendicular to it by the following formula and allocate the value in the small bracket ( ).

$$\text{Average Cost} = \frac{\text{Sum of cost perpendicular to zero}}{\text{No. of cost added}}.$$

Step 7. Continue the assignment of average sum of the all zero cost entries and assign the largest allocated value sum in the matrix. If there is tie in the largest value then assign that value which has lowest cost in the original problem.

Step 8. After assigning the cost, delete the respective column and again find the average sum cost of each zero value.

Step 9. Repeat step 6, 7 and 8 until all the jobs get assigned.

Step 10. End.

4. Numerical Illustration

Here we take the same numerical example as considered by Quazzafi et al. [6] in which five machines are available for assigning eight jobs with associated costs

Step 1. Input 5, 8 (Table 2)

Table 2.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
M_1	300	250	180	320	270	190	220	260
M_2	290	310	190	180	210	200	300	190
M_3	280	290	300	190	190	220	230	260
M_4	290	300	190	240	250	190	180	210
M_5	210	200	180	170	160	140	160	180

Step 2. Find the minimum cost in each column and subtract the same from the respective column that creates at least one zero in each column.

Step 3. If any row is left without creating a zero then select minimum cost in that row and subtract it from all the elements of that row for obtaining at least one (Table 3).

Table 3.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
M_1	90	50	0	150	110	50	60	80
M_2	70	100	0	0	40	50	130	0
M_3	50	70	100	0	10	60	50	60
M_4	70	90	0	60	80	40	10	20
M_5	0	0	0	0	0	0	0	0

Step 5. Draw the minimum number of lines to cover all the zeros, here four lines are drawn to cover all zeros. The number of lines is not equal to the number of machines (Table 4). Now go to step 6.

Table 4.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
M_1	90	50	0	150	110	50	60	80
M_2	70	100	0	0	40	50	130	0
M_3	50	70	100	0	10	60	50	60
M_4	70	90	0	60	80	40	10	20
M_5	0	0	0	0	0	0	0	0

Step 6. Select the smallest cost (i.e. 10) from the uncovered costs and subtract it from each uncovered cost. Now draw the lines to cover all zeros. Here five lines are drawn which are equal to the number of rows (Table 5).

Table 5.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
M_1	80	40	0	150	100	40	50	80
M_2	60	90	0	0	30	40	120	0
M_3	40	60	100	0	0	50	40	60
M_4	60	80	0	60	70	30	0	20
M_5	0	0	0	0	0	0	0	0

Step 7. For assignment, start from the first zero cost in above modified matrix so obtained, allocate the average sum formula by Dubey et al. [5] of each cost of row and column perpendicular to it by the following formula and allocate the value in the small bracket ( ) (Table 6).

$$\text{Average Cost} = \frac{\text{Sum of cost perpendicular to zero}}{\text{No. of cost added}}.$$

Table 6.

	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8
M_1	80	40	0(58.1)	150	100	40	50	80
M_2	60	90	0(40)	0(50)	30	40	120	0(36.3)
M_3	40	60	100	0(50.9)	0(50)	50	40	60
M_4	60	80	0(38.1)	60	70	30	0(48.1)	20
M_5	0(21.8)	0(24.5)	0(9.0)	0(19.0)	0(18.1)	0(14.5)	0(18)	0(14.5)

Step 8. Continue the assignment of average sum of all the zero cost entries and assign the largest allocated value sum in the matrix. Here largest value is C_{13} thus we assign job J_3 to machine M_1. After assigning delete the corresponding column (Table 7).

Table 7.

	J_1	J_2	J_4	J_5	J_6	J_7	J_8
M_1	80	40	150	100	40	50	80
M_2	60	90	0	30	40	120	0
M_3	40	70	0	0	50	40	60
M_4	60	80	60	70	30	0	20
M_5	0	0	0	0	0	0	0

Step 9. Continue steps 7 and 8 until all jobs are assigned.

Step 10. End.

The assignment of jobs which minimizes the overall cost is shown in Table 8.

Table 8.

Machine	Jobs	Cost
M_1	J_3	180

M_2	J_4, J_8	180, 190
M_3	J_5	180
M_4	J_7	190
M_5	J_1, J_2, J_6	210, 200, 140
Total Cost		1470

5. Conclusion

This method fulfils the requirement of efficiency and optimally assigning the jobs to the available machine. All jobs are performed on given machine and the machines are allowed to perform more than one job. The obtained result of the above numerical problem is same as given by Quazzafi et al. [6].

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