Advances and Applications in Mathematical Sciences
Volume 20, Issue 11, September 2021, Pages 2975-2985

EVALUATION OF PERFORMANCE MEASURES OF FUZZY QUEUES WITH PREEMPTIVE PRIORITY USING DIFFERENT FUZZY NUMBERS

B. KALPANA

Department of Mathematics
Emerald Heights College for Women
Ooty, Tamilnadu, India

Abstract

This paper investigates the performance measures in crisp values for preemptive priority fuzzy queues. This paper gives a procedure to convert fuzzy environment to crisp environment by Yager ranking Index in order to analyse the performance measures of fuzzy queues. Ranking the fuzzy numbers is an important characteristic in fuzzy environment. We take both the arrival time and service time as fuzzy numbers and derive the performances measure for triangular, trapezoidal and pentagonal fuzzy numbers. An example is given to derive the performance measures of 3-priority queues.

1. Introduction

Queueing models are applied in several fields such as transportation engineering, service industry, production, communication systems, health care and information processing systems. The waiting discipline where customers are served with respect to their order of arrival is frequently found in queuing models, but then again many real queueing systems follow priority discipline model. A priority mechanism is a useful method that allows different customer types of customers to receive different performance level. Priority queueing have many applications such as communication network, call centres and hospitals etc. Priority schemes are also known for their ease of implementation. Many queuing works is devoted to analyse priority queues. There are two possible segments in priority situation, the

2010 Mathematics Subject Classification: 68M20.
Keywords: Performance measures, Preemptive priority, Trapezoidal fuzzy number, Triangular fuzzy number, Pentagonal fuzzy number.
Received July 7, 2021; Accepted August 7, 2021

preemption and non-preemption. In the preemptive priority a customer with high priority is permitted to go into service immediately even if the lower priority is already in service. The preemptive priority queues are useful for performance evaluation of production, manufacturing, inventory controls and computer systems. The service of low customer is interrupted when on arrival of customer belonging to a higher class arrives, and will be restarted from the point of pause, when all the queues of higher priority have been emptied. In this situation the lower priority class customers are completely unseen and do not affect in any way the queues of the higher classes. Here the queuing model considered is one where few customers are given a priority service over routine. Given a model of this kind we require to find the performance measures.

The parameters in the preemptive priority may be fuzzy. The fuzzy queuing models are more realistic and practical than classical ones. Queuing models along with fuzzy increases their application. Uncertainty is determined by fuzzy set theory. Dissimilar to the classic model that considers arrivals to follow a Poisson process and exponentially distributed service times, the arrival rate in numerous real conditions is more possibility than probabilistic and are not denoted by exact terms. Bellman and Zadeh [1] presented the idea of fuzziness so that inexact information could be solved by decision making problems. The researchers like Li and Lee [10], Buckley [2], Negi and Lee [12], Kaufmann [9], Kao et al. [8], Chen [3, 4] has examined fuzzy queues by Zadeh's extension principle. Parametric linear programming approach to derive the membership functions of the system in fuzzy queues has been derived by Kao et al. [8]. Recent developments on fuzzy numbers by random variables can be used to analyse the queueing system. Zadeh, L. A [16] has presented the idea of fuzzy probabilities and the properties of fuzzy probability Markov chains were discussed. Buckley [2] studies multi-server queues with finite and infinite capacity queueing models with arrivals and departures follow possibilistic pattern. Chen [3] suggests a strategy of parametric programming in order to derive membership functions of the fuzzy queues.

Further, the conversion of fuzzy queues to crisp queues has also been widely discussed in works and many methods and approaches have been used. One of such methods is the robust ranking method [15]. Non-preemptive priority fuzzy queues have been explained by many authors where fuzzy problem are reduced to crisp problem. Most of the previous work have been done on non-preemptive queues. In this paper we study the preemptive priority queue and henceforth derive the performance measures of fuzzy queues using Yager ranking technique.

Definitions

Fuzzy set

A fuzzy set is characterized by a membership function mapping elements of a domain space, or universe of discourse X to the unit interval $[0, 1]$. (i. e.) $A = \{(x, \mu_A(x)); x \in X\}$. Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from $[0, 1]$.

α-cut of a fuzzy number

The α-cut of a fuzzy number $A(x)$ is defined as $A(\alpha) = \{x : \mu(x) \geq \alpha, \alpha \in [0, 1]\}$.

Addition of two Triangular fuzzy numbers can be performed as $(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

Triangular fuzzy number

Fuzzy Triangular number can be represented by $\widetilde{A}(a, b, c)$ where $a < b < c$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x - a}{b - a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{c - x}{c - b}, & b \leq x \leq c \\ 0, & \text{otherwise.} \end{cases}$$

Trapezoidal fuzzy number

Fuzzy Trapezoidal number can be represented by $\widetilde{A}(a, b, c, d)$ where

$a < b < c < d$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & \text{otherwise.} \end{cases}$$

Pentagon fuzzy number

Fuzzy Trapezoidal number can be represented by $\widetilde{A}(a, b, c, d, e)$ where $a < b < c < d < e$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-b}{c-b}, & b \le x \le c \\ 1, & x = c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ \dfrac{e-x}{e-d}, & d \le x \le e \\ 0, & \text{otherwise.} \end{cases}$$

Definition. A fuzzy set $\widetilde{A}$ is convex subset of Z if and only if $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda x_2)) \ge \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$ for all $x_1, x_2 \in X$ and $\lambda \in [0, 1]$.

Yager ranking Index

Fuzzy problem is solved by reformulating into crisp problem by using Yager ranking Index. To solve the problem the fuzzy numbers are defuzzified into crisp ones by ranking method. Yager suggested the following index to order the fuzzy numbers. $Y(\widetilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^L + a_\alpha^U)\, d\alpha$, $Y(\widetilde{a})$ has the properties of linearity and additivity.

Mathematical formulation

A FM/FM/1 queuing system with a single server and 3-priority queues is studied. The inter arrival time $\widetilde{A}_\iota$, $i = 1, 2, 3$ of the first, second and third priority queues and service times are approximately known are given by the fuzzy sets

$$A_\iota = \{(x, \mu_{A_l}(X))/x \in X\}$$

$$\tilde{S} = \{(y, \mu_{\tilde{S}}(y))\ /y \in Y\}$$

Where X and Y are crisp universal sets of the inter arrival time and inter service time respectively and $\mu_{A_\iota}(x), i = 1, 2, 3$ and $\mu_{\tilde{S}}(y)$ are the respective membership functions. Using α-cut the inter arrival times and service time can be denoted by different levels of confidence intervals $[0, 1]$. Therefore a fuzzy preemptive priority queue can be reformulated to family of crisp queues with different α-cuts. Using the notion of α-cut the FM/FM/1 queue with 3-priority queues are reformulated to M/M/1 queue with 3-priority customers with equal service rates, i.e., $\mu_1 = \mu_2 = \mu_3 = \mu$. Further $\rho_1 = \frac{\lambda_1}{\mu_1}$, $\rho_2 = \frac{\lambda_2}{\mu_2}$, $\rho_3 = \frac{\lambda_3}{\mu_3}$

$$\rho = \rho_1 + \rho_2 + \rho_3, \rho = \frac{\lambda}{\mu}, \lambda = \lambda_1 + \lambda_2 + \lambda_3, \rho = \frac{\lambda_1 + \lambda_2 + \lambda_3}{\mu}, \sigma_k = \sum_{i=1}^{i=k} \rho_i, \sigma_0 = 0$$

Without loss of generality let us assume the performance measures for 3-priority queues.

From the traditional queuing theory, the waiting time in the queue is

$$W_{q,i} = \frac{\frac{1}{\tilde{\mu}}(1 - \sigma_{i-1}) + \frac{1}{\tilde{\mu}^2}\sum_{j=1}^{p}\tilde{\lambda}_j}{(1 - \sigma_{i-1})(1 - \sigma_i)} - \frac{1}{\tilde{\mu}}$$

The expected queue size is

$$L_q = \sum_{i=1}^{r} L_q^{(i)} = \sum_{i=1}^{r} \lambda_i (W_{q,\,i})$$

From which we can derive

$$W_q^i = \frac{\frac{1}{\tilde{\mu}}}{(1 - \sigma_{i-1})(1 - \sigma_i)} - \frac{1}{\tilde{\mu}}$$

$$W_q^{(1)} = \frac{1}{\mu}\left[\frac{\mu}{\mu - \lambda_1} - 1\right] = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu}$$

$$W_q^{(2)} = \frac{\mu}{(\mu - \lambda_1)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu}$$

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu}$$

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu}$$

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu}$$

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1 - \frac{\lambda}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu},$$

where λ_1, λ_2, λ_3 are the arrival rates of first, second and third priority units and μ is the service rate.

Numerical Example

Expected waiting time and expected number of customers in the queue for FM/FM/1 queue with 3-preemptive priority classes.

For Trapezoidal Fuzzy number

The rates of first, second and third priority with same service rates are trapezoidal fuzzy numbers represented by $A_1 = [3, 4, 6, 7]$, $A_2 = [5, 6, 8, 9]$, $A_3 = [7, 8, 10, 11]$, $\tilde{S} = [20, 21, 23, 24]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[3 + \alpha, 7 - \alpha]$, $[5 + \alpha, 9 - \alpha]$, $[7 + \alpha, 11 - \alpha]$ and $[20 + \alpha, 24 - \alpha]$ respectively.

We calculate $Y(3\ 4, 6, 7)$ by applying Yager ranking method. The membership function of the Trapezoidal fuzzy number $(3\ 4, 6, 7)$ is

$$\mu_{(x)} = \begin{cases} \frac{x-3}{1}, & 3 \le x \le 4 \\ 1, & 4 \le x \le 6 \\ \frac{7-x}{1}, & 6 \le x \le 7 \\ 0, & \text{otherwise} \end{cases}$$

The α cut of the fuzzy number $(3, 4, 6, 7)$ is $(a_\alpha^L, a_\alpha^U) = (\alpha + 3, 7 - \alpha)$ and $Y(A_1) = R(3, 4, 6, 7) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U) d\alpha = \int_0^1 0.5(10) d\alpha = 5.$

The Yager ranking indices for the fuzzy numbers A_2, A_3, S are given by $Y(A_2) = 7, Y(A_3) = 9, Y(\tilde{S}) = 22$ and hence $\lambda_1 = 5$, $\lambda_2 = 7$, $\lambda_3 = 9$, $\mu = 22$, $\lambda = 21$.

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.01336.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{1}{(\mu - \lambda_1)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.08395.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 2.1545.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.0668.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.58765.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1-\frac{\lambda}{\mu}\right)\left(1-\frac{(\lambda_1+\lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 19.3905.$$

For Triangular Fuzzy number

Let the rates of first, second and third priority with the same service rates are triangular fuzzy numbers denoted by $A_1 = [3, 6, 8], A_2 = [4, 7, 9], A_3 = [5, 8, 10], \tilde{S} = [20, 23, 25]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[3 + 3\alpha, 8 - 2\alpha], [4 + 3\alpha, 9 - 2\alpha], [5 + 3\alpha, 10 - 2\alpha],$ and $[20 + 3\alpha, 25 - 2\alpha]$ respectively.

We calculate $Y(3, 6, 8)$ by Yager ranking method.

The membership function of the triangular fuzzy number $(3, 6, 8)$ is

$$\mu_{(x)} = \begin{cases} \frac{x-3}{3}, & 3 \le x \le 6 \\ 1, & x = 6 \\ \frac{8-x}{2}, & 6 \le x \le 8 \\ 0, & \text{otherwise.} \end{cases}$$

The α cut of the fuzzy number $(3, 6, 8)$ is $(a_\alpha^L, a_\alpha^U) = (3 + 3\alpha, 8 - 2\alpha)$ for which $Y(A_1) = R(3, 6, 8) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5(3 + 3\alpha + 8 - 2\alpha)$ $d\alpha = 5.75$.

Similarly the Yager ranking indices for the fuzzy numbers A_2, A_3, S are calculated as $Y(A_2) = 6.75, Y(A_3) = 7.75, Y(\tilde{S}) = 22.75$ and hence $\lambda_1 = 5.75$, $\lambda_2 = 6.75$, $\lambda_3 = 7.75$, $\mu = 22.75$, $\lambda = 20.25$.

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.01486.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{\mu}{(\mu - \lambda_1)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.08660.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.8438.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.0854.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.5846.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1 - \frac{\lambda}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 6.5398.$$

For Pentagon Fuzzy number

Let the rates of first, second and third priority with the same service rates are pentagon fuzzy numbers denoted by $A_1 = [1, 2, 3, 4, 5]$, $A_2 = [3, 4, 5, 6, 7]$, $A_3 = [9, 10, 11, 12, 13]$, $\tilde{S} = [20, 23, 25, 27, 30]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[1 + \alpha, 5 - \alpha], [3 + \alpha, 7 - \alpha], [9 + \alpha, 13 - \alpha]$, and $[20 + 3\alpha,\ 30 - 3\alpha]$ respectively.

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.00545.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{\mu}{(\mu - \lambda_1)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.0268.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.20509.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.01636.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.13422.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1 - \frac{\lambda}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 2.25607.$$

Conclusion

The fuzzy preemptive priority queues are represented more exactly and the results are derived. Numerical examples for triangular, trapezoidal and pentagonal numbers are explained effectively to determine the validity of the suggested model. The ranking approach used here in the paper is more effective while transforming fuzzy queues into crisp queues and $\alpha-$ cut is used to reduce the fuzzy queues to crisp queues. The expected queue length and expected waiting time are derived. The future work can be done in examining the efficiency of this technique to other queueing models and applying different methods to find the performance measures in fuzzy preemptive queues.

References

[1] R. E. Bellman and L. A. Zadeh, Decision-making in a fuzzy environment, Management Science 17 (1970), 141-164.

[2] Buckley, J. J. and Y. Qu, On using α-cuts to evaluate fuzzy equations, Fuzzy Sets and Systems 38 (1990), 309-312

[3] S. P. Chen, Parametric nonlinear programming approach to fuzzy queues with bulk service, European Journal of Operational Research 163 (2005), 434-444.

[4] S. P. Chen, A Mathematics programming approach to the machine interference problem with fuzzy parameters, Applied Mathematics and Computation 174 (2006), 374-387.

[5] B. Kalpana and N. Anusheela, A single server retrial fuzzy queue with priority and unequal service rates, Journal of Intelligent and Fuzzy Systems 37(1) (2019), 811-820.

[6] B. Kalpana and N. Anusheela, Analysis of a single server non-preemptive fuzzy priority queue using LR method, ARPN Journal of Engineering and Applied Sciences 13(23) (2018).

[7] B. Kalpana and N. Anusheela, Analysis of fuzzy priority queue with two classes of customers and unequal service rates using mixed integer nonlinear programming approach, Journal of Engineering and Applied Sciences 14 (2018), 5030-5038.

[8] C. Kao, C. Li and S. Chen, Parametric programming to the analysis of fuzzy queues, Fuzzy Sets and Systems 107 (1999), 93-100.

[9] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, Academic Press, New York, Volume 1 (1975).

[10] R. J. Li and E. S. Lee, Analysis of fuzzy queues, Computers and Mathematics with Applications 17(7) (1989), 1143-1147.

[11] D. R. Miller, Computation of steady-state probabilities for M/M/1 priority queues, Oper. Res. 29 (1981), 945-958.

[12] D. S. Negi and E. S. Lee, Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems 46 (1992), 321-330.

[13] B. Palpandi and G. Geetharamani, Computing performance measures of fuzzy non-preemptive priority queues using robust ranking technique, Applied Mathematical Analysis 7(102) (2013), 5095-5102.

[14] M. J. Pardo and David de la Fuente, Optimizing a priority discipline queuing model using fuzzy set theory, Comput. Math Appl. 54 (2007), 267-281.

[15] R. R. Yager, A procedure for ordering fuzzy subsets of the unit interval, Inf. Sci. (Ny), 24(2) (1981), 143-161.

[16] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1978), 3-28.

[17] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

[18] H. J. Zimmermann, Fuzzy Set Theory and Its Applications, Kluwer Academic, Boston, 2001.