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A RANDOMIZED THRESHOLD DISCRETE TIME QUEUEING SYSTEM WITH SERVICE CONTROL AND REPAIRS

P. GUNASEKARAN

Department of Science and Humanities
Nehru Institute of Engineering and Technology
Coimbatore, Tamil Nadu, India
E-mail: nietgunasekaran@nehrucolleges.com
drgunasekaranpappnnan@gmail.com

Abstract

This paper discusses a randomized threshold discrete-time queueing system with starting failures in which customer arrives under three different policies. Two correspond to the LCFS (Last Come First Served) discipline, in which displacements or removal of arriving customers occur. The third strategy acts as a signal, that is, it becomes a negative customer. Also examined is the possibility of failures at each service commencement epoch. A threshold policy is adopted in this model i.e. the server provides the service only when N customers available in the system other wise the server is idle. We carry out a thorough study of the model, deriving analytical results for the stationary distribution. We obtain probability generating function PGF at an arbitrary time by using generating function technique and an analytical expressions for mean queue length and average waiting time are derived as performance measure. In numerical examples the effects of mean queue length and average waiting time are analyzed at various arrival and service rates.

1. Introduction

In Queuing models many researchers have found a lot of application in computer communications and manufacturing systems. Currently many researchers are interested in discrete queue, due to applications in a various slotted digital communicated systems and other related areas. The analysis of discrete queuing model has received considerable attention in the scientific

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literature over the past years because of its applications which are widely used in the real life.

Only at the end 20 century most of queueing models literature survey are motivated and developed. At these time most of the researchers concentrate only on continuous models and only few of them focus on discrete models because of its applications in many parts of real life such as telecommunication, computer system etc. The analysis of discrete queuing model has received considerable attention in the scientific literature over the past years because of its applications which are widely used in the real life.

Initially the discrete queues are discussed by Meisling [1], Bindsall, Ristenbatt, and Weinstein [2] and also by powell and Avi – Lizha [3].

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Recently, many researchers increase their attention in queuing system with N policy and this concept N policy is used to control the service in queuing system. This model is widely used for modelling purpose of any production and manufacturing system as well as network communication and telecommunication system. From this policy we infers that when N or more customers are present in the system the server is activated and deactivated when the server is empty. In earlier Yadin and Noor [4], ke and warg [5] were first introduce this policy. They focussed on a single removable server queuing system with finite capacity operating under N policy. Jain et al. [6] discussed unreliable $MM/1/k$ queuing model under N and F policy with multi optional phase repair and start up. Moreover Wang et al [7] has examined the concept strategic behaviour of $M/M/1$ constant retrial queue with N policy Moreno [8] discussed a discrete time single server queue model with general N policy and fixing close down times. Zhang and Tian [9] investigated a discrete time $Geo/G/1$ queue with multiple adaptive vacation.

In most of the queueing literature, it is understood that the server is always busy, but this assumption is practically not true. In queueing system a notable and unavoidable phenomenon in the service facility is its breakdown and consequent repair. The waiting time of a customer will increase with a consequence effect over the impatience of the customer until the server facility is recovered again. Indeed, queueing systems with server breakdowns are very common in communication systems. Queues with service interruptions were first studied by White and Christie [10]. In queueing theory the periods on which the service not available are called as server vacation, server break down and server interruption.

Upgrading analytical models to be applicable for discussing their performance is a very major issue, which has been handled by several researchers. Mostly all the existing models concentrate on continuous-time ones and application to networks with blocking and truncation. Works related to discrete-time systems with server interruptions with or without expulsions and vacations can be found, including those by Fiems et al. [11-12].

The $M/M/1$ queue, the $M/G/1$ queue and the $GI/M/1$ queue have all used the idea of the negative consumer. Artalejo [13], and Gelenbe [14] have all done excellent surveys on negative clients. This topic has recently been expanded to include a discrete-time queueing mechanism. Due to the packetized nature of transport protocols, the discrete-time queue is better suited to represent the behavior of time-slotted digital communication systems. Atencia and Moreno [15] looked into $Geo/Geo/1$ delays with bad consumers and the numerous removal disciplines that these customers caused.

The server is assumed to be reliable on a permanent basis in most of the literature on queueing systems with negative consumers, independent of the arrival of negative customers. In many real systems, however, the entrance of a negative customer can result in server failure and the destruction of work. Few studies have been conducted on repairing queueing systems with negative clients. The $Geo/Geo/1$ retrial queue with negative consumers was studied by Wang and Zhang [16]. unreliable server, where the server fails at any time customers that are unhappy arrive at a system. Lee et al. [17] presented the results of repairable $Geo/G/1$ queues with catastrophes, in which if a disaster strikes a system, all current customers (i.e., a queue) will be affected. Customers in service (as well as those in line) are compelled toget out of the system. The authors defined a disaster in as a server failure that results in the loss of all work in progress in a system. Neither of them, however, thought the negative customers in the case of repairable queueing systems. This results in server failure as well as the cancellation of a single transaction a typical consumer.

We need to find configurations and rules that will optimise a queueing system before we can construct it. To accomplish so, we must first comprehend how the queueing system would work under various configurations and rules, such as the length of the queue, which relates to the number of items in the queue or consumers who must wait in a queue or in a buffer to be dealt with. This is frequently a useful measure of a queueing system's performance. From the user's perspective, the longer the line, the poorer the performance. Another indicator of the system's behaviour is the length of time it takes for a client to receive service of course, the longer this performance metric is, the poorer the customer's perception of the system in terms of service time becomes.

Atenciaanalyzied a discrete time system with service control and repairs in a detailed manner along with this a threshold policy is introduced and a new queueing model is developed namely. A randomized threshold discrete time system with service control and repairs.

2. Model description

Let the time axis be divided into equal intervals of unit length called slots. Since in discrete queue many events occur simultaneously, so the order of these must be stated. There are two arrivals namely Late Arrival system (LAS) and early Arrival system (EAS). In this model we follow LAS.

In this study, we look at a discrete-time queueing system with starting failures in which a client arrives and uses one of three techniques. With displacements and expulsions, the first and second follow the LCFS discipline, respectively. The third one acts as a negative customer, i.e., it kicks the client who is now being served out of the system has no effect on the system any longer. Customers arrive using a Bernoulli arrival process with rate, where a represents the likelihood of a client arriving at a specific slot. When a customer enters the server, the service station must be turned on. If the server is successfully launched (with probability α), the customer can begin using his or her service right away and if the server is busy at the time of his/her arrival, the client who is currently being serviced gets displaced to the first place of the queue with probability p_1 the customer who was in service is expelled from the system with probability p_2, and the arriving customer becomes a negative customer with a probability of p_3 i.e., kills the customer in service while having no effect on an empty room, otherwise, if the server fails to start (with a complementary chance $\overline{\alpha}$), it is sent directly to repair, and the customer is placed at the front of the line. Customers who arrive at the system during a repair time are placed at the back of the line and also the server provides the service only when N customers present in the system otherwise the service idle. i.e., The (P, N) policy is introduced. Clearly $p_1 + p_2 + p_3 = 1$.

The service times are identically independent random variables and are generally distributed with generating function

$$S_1(x) = \sum_{i=1}^{\infty} S_{1,i} x^i$$

The served consumer will, without a doubt, depart the system after the service is completed and will have no further impact on the system.

The repair times are identically independent random variables and are generally distributed with generating function

$$S_2(x) = \sum_{i=1}^{\infty} S_{2,i} x^i$$

and the n^{th} factorial moments $\beta_{2,n}$ Naturally, the service station is as good as new following repairs.

The load of the system is given by $\rho_1 + \rho_2$, where

$\rho_1 = \lambda(1 - \rho_3)\overline{b}$

$$\rho_2 = \frac{\lambda\bar{\lambda}\bar{\alpha}(1-p_3)\beta_{2,1}}{\alpha(p_1 S_1(\bar{\lambda}) + \bar{\lambda}(1-p_1))}$$

where $\bar{b}$ is the mean sojourn time of the customer in the server 3. Markov chain

3. Markov chain

The state of the system at time n^+ is defined as

$$C_n = \{\phi^{(n)}, \xi_{1,n}, \xi_{2,n}, N_n\}$$

where $\phi^{(n)} = 0, 1, 2$ denotes the state of the server

when $\phi^{(n)} = 0$, the server is free

$= 1$, the server is busy and $\xi_{1,n}$, represents remaining service time of the customer

$= 2$, the server is under repair $\xi_{2,n}$, represents remaining repair time and N_n represent number of customers in the queue, It can be shown that $\{C_n; n \geq 1\}$ is the markov process of our queuing system.

The limiting probabilities are defined as

$$\pi_0 = \lim_{n\to\infty} \Pr\{\phi_1^{(n)} = 0\},\ \pi_{1,i,k} = \lim_{n\to\infty} \Pr\{\phi_1^{(n)} = 1,\ \xi_{1,n} = i,\ N_n = k\}$$

$$\pi_{2,i,k} = \lim_{n\to\infty} \Pr\{\phi_1^{(n)} = 2,\ \xi_{2,n} = i,\ N_n = k\}$$

The Kolmogorov equations for the stationary distribution of the system under consideration are

$$\pi_0 = (\bar{\lambda} + \lambda p_3)\pi_0 + \bar{\lambda}\pi_{1,1,0} + \lambda p_3 \sum_{i=1}^{\infty} \pi_{1,i,0} \tag{1}$$

$$\pi_{1,j,1} = \delta_{0,k}\lambda(1-p_3)\alpha s_{1,j}\pi_0 + \lambda p_1 \alpha s_{1,j}\pi_{1,1,1} + \bar{\lambda}\alpha s_{1,j+1,1}$$

$$+ (1-\delta_{0,k})\lambda p_1 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{2,j,0} + \lambda p_2 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,1} + \lambda p_3 \alpha s_{1,i} \sum_{j=1}^{\infty} \pi_{1,j,2}$$

$$+ (1-\delta_{0,k})\lambda(1-p_3)\alpha s_{1,i}\pi_{2,1,1} + (\bar{\lambda} + \lambda p_3)\alpha s_{1,i}\pi_{2,1,2} \tag{2}$$

$$\pi_{1,i,k} = \delta_{0,k}\lambda(1-p_3)\alpha s_{1,j}\pi_0 + \lambda p_1 \alpha s_{1,j}\pi_{1,1,k} + \bar{\lambda}\alpha s_{1,j}\pi_{1,1,k+1}$$

$$+ \bar{\lambda}\pi_{1,j+1,k} + \lambda\pi_{1,j+1k-1} + (1-\delta_{0,k})\lambda p_1 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,jk-1}$$

$$+ \lambda p_2 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,k} + \lambda p_3 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,k+1} + (1-\delta_{0,k})\lambda(1-p_3)\alpha s_{1,i}\pi_{2,1,k}$$

$$+ (\bar{\lambda} + \lambda p_3)\alpha s_{1,i}\pi_{2,1,k+1} \tag{3}$$

$$\pi_{1,N} = \delta_{0,k}\lambda(1-p_3)\alpha s_{1,j}\pi_0 + \lambda p_1 \alpha s_{1,j}\pi_{1,1,N} + \bar{\lambda}\alpha s_{1,j}\pi_{1,1,N+1}$$

$$+ \bar{\lambda}\pi_{1,j+1,N} + \lambda\pi_{1,j+1,N-1} + (1-\delta_{0,k})\lambda p_1 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,N-1}$$

$$+ \lambda p_2 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,N} + \lambda p_3 \alpha s_{1,i} \sum_{j=2}^{\infty} \pi_{1,j,N+1} + (1-\delta_{0,k})\lambda(1-p_3)\alpha s_{1,i}\pi_{2,1,N}$$

$$+ (\bar{\lambda} + \lambda p_3)\alpha s_{1,i}\pi_{2,1,N+1} \tag{4}$$

$$\pi_{2,i,1} = \delta_{1,k}\lambda(1-p_3)\bar{\alpha}s_{2,i}\pi_0 + \lambda p_1 \bar{\alpha}s_{2,i}\pi_{1,1,0} + (\bar{\lambda} + \lambda p_3)\bar{\alpha}s_{2,i}\pi_{1,j,1}$$

$$+ \lambda p_2 \bar{\alpha}s_{2,i} \sum_{j=2}^{\infty} \pi_{1,j,0} + \lambda p_3 \alpha s_{2,i} \sum_{j=1}^{\infty} \pi_{1,j,1} + (1+\delta_{1k})\lambda(1+p_3)\bar{\alpha}s_{2,i}\pi_{2,1,0}$$

$$+ (\bar{\lambda} + \lambda_{p_3})\bar{\alpha}s_{2,i}\pi_{2,1,1} + (1-\delta_{1,k})\lambda(1-p_3)\bar{\alpha}\pi_{2,1+1,0} + (\bar{\lambda} + \lambda_{p_3})\pi_{2,j=1,1} \tag{5}$$

$$\pi_{2,j,k} = \delta_{1,k}\lambda(1-p_3)\bar{\alpha}s_{2,i}\pi_0 + \lambda p_1 \bar{\alpha}s_{2,i}\pi_{1,1,k-1} + (\bar{\lambda} + \lambda p_3)\bar{\alpha}s_{2,i}\pi_{1,i,k}$$

$$+\lambda p_2\overline{\alpha}s_{2,i}\sum_{j=2}^{\infty}\pi_{1,j,k-1}+\lambda p_3\overline{\alpha}s_{2,i}\sum_{j=1}^{\infty}\pi_{1,j,k}+(1+\delta_{1k})\lambda(1+p_3)\overline{\alpha}s_{2,i}\pi_{2,1,k-1}$$

$$+(\overline{\lambda}-\lambda p_3)\overline{a}S_{2,i}\pi_{2,i,k}+(1-\delta_{0,k})\lambda(1-p_3)\overline{\alpha}\pi_{2,1+1,k-1}+(\overline{\lambda}-\lambda p_3)\pi_{2,i=1,N} \tag{6}$$

$$\pi_{2,i,k}=\delta_{1,k}\lambda(1-p_3)\overline{\alpha}s_{2,i}\pi_0+\lambda p_1\overline{\alpha}s_{2,i}\pi_{1,1,N-1}+(\overline{\lambda}+\lambda p_3)\overline{\alpha}s_{2,i}\pi_{1,i,N}$$

$$+\lambda p_2\overline{\alpha}s_{2,i}\sum_{j=2}^{\infty}\pi_{1,j,N-1}+\lambda p_3\overline{\alpha}s_{2,i}\sum_{j=1}^{\infty}\pi_{1,j,N}+(1+\delta_{1k})\lambda(1+p_3)\overline{\alpha}s_{2,i}\pi_{2,1,N-1}$$

$$+(\overline{\lambda}-\lambda_{p_3})\overline{\alpha}s_{2,i}\pi_{2,1,N}+(1-\delta_{0,k})\lambda(1+p_3)\overline{\alpha}\pi_{2,1+1,N-1}(\overline{\lambda}-\lambda_{p_3})\pi_{2,i=1,N} \tag{7}$$

where $\delta_{0,k}$ and $\delta_{1,k}$ the Kronecker's symbol, and the normalization condition is

$$\pi_0+\sum_{i=1}^{\alpha}\sum_{k=0}^{N}\pi_{1,i,k}+\sum_{i=1}^{\alpha}\sum_{k=0}^{N}\pi_{1,i,k}=1$$

We introduce following generating function to solve the above system of equations

$$\Phi_1(x,z)=\sum_{i=1}^{\infty}\sum_{k=1}^{N}\pi_{1,i,k}x^iz^k\Phi_2(x,z)=\sum_{i=1}^{\infty}\sum_{k=1}^{N}\pi_{2,i,k}x^iz^k$$

Also, we introduce auxiliary generating function

$$\Phi_{1,i}(z)=\sum\nolimits_{k=1}^{N}\pi_{1,i,k}z^k\Phi_{2,i}(z)=\sum_{k=1}^{N}\pi_{2,i,k}z^k$$

In equations (2), (3) and (4) multiply both sides by z^k and taking summation over k and use equation (1), we get,

$$\phi_{1,i}(z)=\overline{\lambda}\phi_{1,j+1}(z)+\frac{[\overline{\lambda}+\lambda p_1 z(1-2)]}{z}\alpha s_{1,i}\phi_{1,1}(z)$$

$$+\frac{(\bar{\lambda}+\lambda z)+\lambda p_3(1-z)}{z}\alpha s_{1,i}\phi_{2,1}(z)-\frac{(1-z)}{z}\lambda(1-p_3)\alpha s_{1,i}\pi_0$$

$$+\lambda\alpha s_{1,i}\phi_1(1, z)\left[\frac{p_1z^2+p_zz+p_3}{z}\right] \tag{8}$$

In equations (5), (6) and (7) multiply both sides by z^k and taking summation over k and use equation (1), we get,

$$\phi_{2,i}(z)=\phi_{2,j+1}(z)[\bar{\lambda}+\lambda+p_3z(1-z)]+\phi_{1,1}(z)\bar{\alpha}s_{1,i}[\bar{\lambda}+\lambda zp_1+(1-z)]$$

$$+\lambda\bar{\alpha}s_{2,i}\phi_1(1, z)[pz^2+p_zz+p_3]+\bar{\alpha}s_{2,i}\phi_{21}(z)[\lambda z+\bar{\lambda}+\lambda p_3(1-z)]$$

$$-(1-z)\bar{\alpha}s_{2,i}\lambda(1-p_3)\pi_0 \tag{9}$$

In equation (8) multiply both sides by x^i and taking summations over I and after some algebraic simplifications, we get

$$\frac{x-\bar{\lambda}}{x}\phi_1(x, z)=\left[\frac{\lambda+\lambda p_1z(1-z)}{z}\alpha S_1(x)-\bar{\lambda}\right]\phi_{1,1}(z)$$

$$+\frac{p_1z^2+p_zz+p_3}{z}\lambda\alpha S(x)\phi(1, z)$$

$$+\frac{\bar{\lambda}+\lambda z+\lambda p_3(1-z)}{z}\alpha S_1(x)\phi_{2,1}(z)-\frac{1-z}{z}\lambda(1-p_3)\pi_0 \tag{10}$$

In equation (9) multiply both sides by x^i and taking summations over i and after some algebraic simplifications, we get

$$x-\frac{[\bar{\lambda}+\lambda z+p_3(1-z)]}{x}\phi_2(x, z)=[\bar{\lambda}+\lambda zp_1(1-z)]\bar{\alpha}S_2(x)\phi_{1,1}(z)$$

$$+[p_1z^2+p_2z+p_{3\lambda}]\lambda\bar{\alpha}S_2(x)\phi_1(1, z)+[\bar{\lambda}+\lambda z+\lambda p_3(1-z)]\bar{\alpha}S_2(x)$$

$$-[\bar{\lambda}+\lambda z+\lambda p_3(1-z)]\phi_{2,1}(z)-\lambda(1-z)(1-p_3)\alpha S_2(x)\pi_0 \tag{11}$$

Put $x=1$ in (10) and on simplifications, we get

$$\lambda\phi_1(1, z) = \frac{1}{z - \alpha[p_1 z^2 + p_2 z + p_3]}\begin{bmatrix} [(\bar{\lambda} + \lambda p_1 z(1-z))\alpha - \overline{\lambda z}]\phi_{1,1}(z) \\ + [\bar{\lambda} + \lambda z + \lambda p_3(1-z)]\alpha\phi_{2,1}(z) \\ -(1-z)\lambda(1-p_3)\lambda\pi_0 \end{bmatrix} \tag{12}$$

Using (12) in (10) and on simplification, we get

$$\frac{x - \bar{\lambda}}{x}\phi_1(x, z) =$$

$$\frac{1}{z - \alpha[p_1 z^2 + p_2 z + p_3]}\begin{bmatrix} [p_1 z\bar{\lambda}(1-p_3)](1-z)\alpha S_1(x) \\ -\bar{\lambda}[z - \alpha[z - \alpha[p_1 z^2 + p_2 z + p_3]]\phi_{1,1}(z) \\ + \bar{\lambda}\lambda z + \lambda p_3(1-z)\alpha S_1(x)\phi_{2,1}(z) \\ -(1-z)(1-p_3)\bar{\alpha}S_2(x)\pi_0 \end{bmatrix} \tag{13}$$

Using (12) in (11) and on simplification, we get

$$\frac{x - [\bar{\lambda} + \lambda z + \lambda p_3(1-z)]}{x}\phi_2(x, z) =$$

$$\frac{1}{z - \alpha[p_1 z^2 + p_2 z + p_3]}\begin{bmatrix} (p_1 z + \bar{\lambda}(1-p_3))(1-z)z\alpha S_2(x)\phi_{1,1}(z) \\ + [\bar{\lambda} + \lambda p_3(1-z)]\begin{bmatrix} z\bar{\alpha}S_2(x) - z \\ + \alpha[p_1 z^2 + p_2 z + p_3] \end{bmatrix}\phi_{2,1}(z) \\ -\lambda z(1-p_3)\bar{\alpha}S_2(x)\pi_0 \end{bmatrix} \tag{14}$$

Put $x = \bar{\lambda}$ in (13) and on simplification, we get

$$\phi_{1,1}(z) = \frac{(1-z)(1-p_3)\lambda\alpha S_1(\bar{\lambda})}{D(z)}\pi_0$$

Put $x = [\bar{\lambda} + \lambda z + \lambda p_3(1-z)]$ in (14) and on simplification, we get

$$\phi_{2,1}(z) = \frac{1}{[\bar{\lambda} + \lambda z + \lambda p_3(1-z)]\alpha S_1(\bar{\lambda})D(z)}$$

$$\{\lambda\bar{\lambda}\varepsilon z(1-z)(1-p_3)S_2(\bar{\lambda} + \lambda z + \lambda p_3(1-z))\}\pi_0$$

Where

$$D(z) = [p_1 z + \bar{\lambda}(1 - p_3)](1 - z)\alpha S_1(\bar{\lambda}) + \overline{\lambda\bar{\alpha}} z S_2(\bar{\lambda} + \lambda z + \lambda p_3(1 - z))$$

$$- \underline{\bar{\lambda}}(z - \alpha(p_1 z^2 + p_2 z + p_3))$$

Substituting the above equations in (10) and (11), we get following generating function after some algebraic simplifications,

$$\phi_1(x, z) = \frac{S_1(x) - S_1(\bar{\lambda})}{x - \bar{\lambda}} \frac{\lambda\bar{\lambda}\alpha x(1 - z)(1 - p_3)}{D(z)} \pi_0$$

$$\phi_2(x, z) = \frac{S_2(x) - S_2(\bar{\lambda} + \lambda z + \lambda p_3(1 - z))}{x - (\bar{\lambda} + \lambda z + \lambda p_3(1 - z))} \frac{\lambda\bar{\lambda}\bar{\alpha} xz(1 - z(1 - p_3))}{D(z)} \pi_0$$

The marginal generating function of the number of customers in the queue when the server is busy is given by,

$$\phi_1(1, z) = \frac{1 - S_1(\bar{\lambda})\bar{\lambda}\alpha(1 - z)(1 - p_3)}{D(z)} \pi_0$$

The marginal generating function of the number of customers in the queue when the server is down is given by,

$$\phi_2(1 \cdot z) = \frac{[1 - S_2(\bar{\lambda} + \lambda z + \lambda p_3(1 - z))]\bar{\lambda}\bar{\alpha} z}{D(z)} \pi_0$$

By using the normalizaton condition

$$\pi_0 + \phi_1(1, 1) + \phi_2(1, 1) = 1$$

We obtain the unknown quantity namely steady state condition is derived which is given below,

$$\pi_0 = \frac{1}{\alpha[p_1 S_1(\bar{\lambda}) + \bar{\lambda}(1 - p_1)}[[p_1 + \bar{\lambda}(1 - p_3)]]\alpha S_1(\bar{\lambda}) - \lambda\bar{\lambda}\alpha(1 - p_3)\beta_{2,1} - \bar{\lambda}\alpha(P_1 - p_3)]$$

Which is obviously less than one.

The Probability generating function PGF of an queue size at an arbitrary time is given by,

$$\phi(z) = \pi_0 + \phi_1(1, z) + \phi_2(1, z)$$

$$\phi[z] = \frac{(1 - z)[p_1 z S_1(\bar{\lambda}) + \bar{\lambda}(1 - p_3 z)]}{D(z)} \alpha\pi_0$$

4. Performance Measure

Mean Queue Length

The average number of customers in the queue i.e., mean queue length E(N) is obtained by differentiating PGF $\phi(z)$ with respect to z and then put $\phi(z)$ which is given below,

$$E(N) = \phi'(1) = \frac{\begin{bmatrix} 2p_1\alpha(\bar{\lambda} - S_1(\bar{\lambda}))(1 - S_1(\bar{\lambda})) + 2\lambda\bar{\lambda}\overline{\bar{\alpha}}\beta_{2,1} \\ + [\bar{\lambda} - p_1(\bar{\lambda} - S_1(\bar{\lambda}))\lambda^2\bar{\alpha}(1 - p_3)\beta_{2,2}]\bar{\lambda}(1 - p_3) \end{bmatrix}\left[2\begin{bmatrix} \alpha S(\bar{\lambda})(p_1 + \bar{\lambda}(1 - p_3)) \\ - \lambda\overline{\bar{\lambda}\bar{\alpha}}\beta_{2,1}(1 - p_3) - \bar{\lambda}\alpha(p_1 - p_3) \end{bmatrix}\right]}{[p_1 S_1(\bar{\lambda}) + \bar{\lambda}(1 - p_1)}$$

5. Numerical examples

In this section we presented numerical examples in two cases. In both cases we analysed the following concepts.

(i) The effect on mean queue length when customer arrival increases

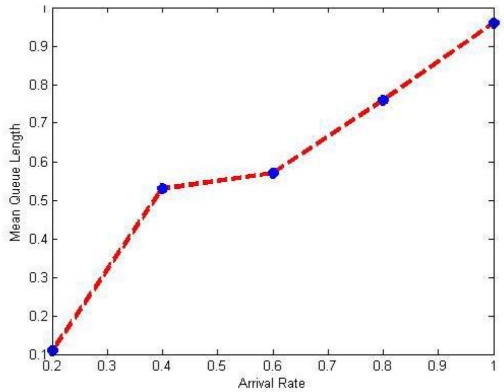
(ii) The effect on mean queue length when service rate increases.

Case (i). In case (I) inter arrival times follow geometrical distribution. Service times, repair times, are also geometrically distributed.

(1). When the arrival rate increases the effect on mean queue length is discussed below with the following graph and table.

Table 1.1. Arrival rate Vs Mean queue length.

Arrival rate	π_0	Mean queue length
0.2	0.22	.11
0.4	0.35	53
0.6	0.43	.57
0.8	0.59	.76
1.0	0.72	.96



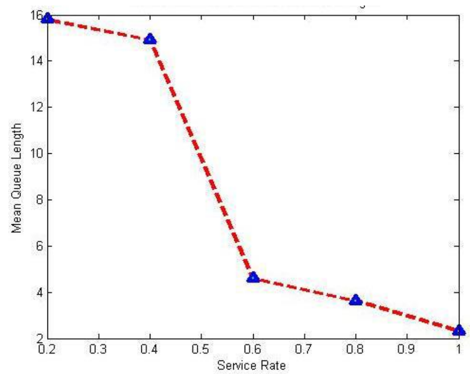
Graph 1.1. Arrival rate Vs mean queue length.

From the above graph and table we conclude that mean queue length increase as customer arrival increases.

(2). When the server time increases the effect on mean queue length is discussed below with the following graph and table.

Table 1.2. Service rate Vs Mean queue length.

Service rate	π_0	Mean queue length
0.2	0.93	15.9
0.4	0.75	14.8
0.6	0.58	4.57
0.8	0.42	3.61
1.0	0.36	2.34



Graph 1.2. Service rate Vs Mean queue length.

From the above graph and table we conclude that mean queue length decrease as service rate increases.

6. Conclusion

In this article a randomized threshold discrete time system with service control and repairs is discussed. In this model an analytical expression for probability generating function is derived by using generating function technique and in performance measure an analytical expression for mean queue length is obtained. In numerical examples the influence of mean queue length is discussed at various arrival and service rates. This model is most suitable for many situations of our real life.

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