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FUZZY PAIRWISE GAMMA SEMICONTINUITY AND FUZZY PAIRWISE GAMMA SEMICOMPACTNESS IN FUZZY BITOPOLOGICAL SPACES

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Abstract

The purpose of this paper is to introduce the concept of fuzzy pairwise gamma semi-continuous and fuzzy pairwise gamma semi-open mappings in fuzzy bitopological spaces and to discuss the essential properties related to these concepts. This paper focuses on the notion of fuzzy pairwise gamma semi-compactness in fuzzy bitopological spaces and examines the relationship between fuzzy pairwise gamma semi-compact spaces and fuzzy pairwise α, β, γ compact spaces. This article also analyses the characteristics of fuzzy pairwise gamma semi-compact spaces involving the concept of fuzzy pairwise gamma semi-continuity.

I. Introduction

Azad [1] studied the notion of fuzzy semi open and fuzzy semi continuous mappings in fuzzy topological spaces. Fuzzy γ-open sets and Fuzzy γ-continuity in fuzzy topological spaces was discussed by Hanafy [6]. In 1989,

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Kandil and El-Shafee [10] introduced the notion of fuzzy bitopological spaces (Fbts, in short) as an extension of fuzzy topological spaces. Since then many authors continued an immense study in the theory of fuzzy bitopological spaces. F. S. Mahmoud et.al [15] introduced and studied Fuzzy-γ-open sets and fuzzy-γ-continuity in fuzzy bitopological spaces. The notion of semi open and semi continuous mappings in fuzzy bitopological spaces was examined by S. Sampath Kumar [25]. Kandil et.al has discussed Fuzzy P^*- compactness in fuzzy bitopological space considering the supra fuzzy topology.

In this paper, we combine the concept of fuzzy pairwise semi continuous (semi open) and fuzzy-γ-continuous (γ-open) mappings to obtain the notion of fuzzy pairwise-γ-semi continuous (γ-semi open) mappings in fuzzy bitopological spaces. Further in this paper, we have introduced the concept of fuzzy pairwise γ semi-compactness considering fuzzy compactness in the sense of Chang [5].

In Section 3, we generalise the concept of fuzzy pairwise-γ-semi continuous (γ-semi open, γ-semi closed) mappings in fuzzy bitopological setting. Various properties and attributes characterizing fuzzy pairwise-γ-semi continuous mappings and fuzzy pairwise-γ-semi open mappings have been discussed along with necessary examples.

In Section 4, we introduce the notion of fuzzy pairwise γ semi-compact spaces using the approach of fuzzy pairwise γ compact spaces. We investigate some of the properties of fuzzy pairwise γ semi-compact spaces along with fuzzy pairwise continuous, fuzzy pairwise gamma semi-continuous mappings and fuzzy pairwise homeomorphism.

II. Preliminaries

Definition 2.1. Let X be a nonempty set and $I = [0, 1]$. A fuzzy set (briefly F-set) A in X is a mapping from X to I. A fuzzy set A of X is contained in a fuzzy set B of X denoted by $A \leq B$ if and only if $A(x) \leq B(x)$ for each $x \in X$.

A fuzzy point (briefly F-point) [28] with singleton support $x \in X$ and the value $\alpha \in [0, 1]$ is denoted by x_α. The complement A' of a fuzzy set X is

$1 - A$ defined by $(1 - A) = 1 - A(x)$ for each $x \in X$. A fuzzy point $x_\beta \in A$ if and only if $\beta \leq A(x)$.

Definition 2.2 [20]**.** A F-set A is the union of all F-points which belong to A. A F-point x_β is said to be quasi coincident with the fuzzy set A denoted by $x\beta qA$ if and only if $\beta + A(x) > 1$.

A F-set A is said to be quasicoincident [20] with B denoted by AqB if and only if there exists $x \in X$ such that $A(x) + B(x) > 1$. $A \leq B$ if and only if $I(AqB')$.

Definition 2.3 [10]**.** A set X on which are defined two (arbitrary) F-topologies δ_1 and δ_2 is called a F-bitopological space (briefly F-bts) and denoted by (X, δ_1, δ_2). As to the notions, we shall write δ_i-int(λ) and δ_i-cl(λ) to mean respectively the interior and closure of a F-set λ with respect to the F-topology δ_i in F-bts (X, δ_i, δ_j), with δ_i-F-o set and δ_i-F-c set, we mean respectively δ_i-F- open and δ_i-F- closed set. The indices i and j take values $\{1, 2\}$ throughout this paper and $i \neq j$, $i = j$ gives the known results in F-ts.

Definition 2.4 [25]**.** Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called

(a) a (δ_i, δ_j) F semiopen (briefly (δ_i, δ_j) F-so) set of X if δ_j-cl $(\delta_i$-int$(\lambda))$;

(b) a (δ_i, δ_j) F semiclosed (briefly (δ_i, δ_j) F-sc) set of X if $\lambda \geq \delta_j$-int$(\delta_i$-cl $(\lambda))$; The set of all (δ_i, δ_j) F-so, (resp. (δ_i, δ_j) F-sc) sets of a F-bts X will be denoted by (δ_i, δ_j) FSO(X), (resp. $(\delta_i, \delta_j) FSC(X)$.

Definition 2.5 [24]**.** Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called

(a) a $(\delta_i, \delta_j) F\alpha$ open (briefly $(\delta_i, \delta_j) F$- $\alpha 0$) set of X if $\lambda \leq \delta_i$-int$(\delta_j$-cl$(\delta_i$-int$(\lambda)))$;

(b) a $(\delta_i, \delta_j)F\alpha$ closed (briefly $(\delta_i, \delta_j)F$- αc) set of X if $\lambda \geq \delta_i$-cl $(\delta_j$-int $(\delta_i$-cl $(\lambda)))$;

(c) a $(\delta_i, \delta_j)F$ preopen (briefly $(\delta_i, \delta_j)F$-po) set of X if $\lambda \leq \delta_i$-int $(\delta_j$-cl $(\lambda))$;

(d) a $(\delta_i, \delta_j)F$ preclosed (briefly $(\delta_i, \delta_j)F$-pc) set of X if $\lambda \geq \delta_i$-cl $(\delta_j$-int$(\lambda))$;

The set of all $(\delta_i, \delta_j)F$-αo, $(\delta_i, \delta_j)F$-αc, $(\delta_i, \delta_j)F$-po, $(\delta_i, \delta_j)F$-pc, sets of a F-bts X will be denoted by $(\delta_i, \delta_j)F\alpha O(X)$, $(\delta_i, \delta_j)F\alpha C(X)$, $(\delta_i, \delta_j)FPO(X)$ and $(\delta_i, \delta_j)FPC(X)$ respectively.

Definition 2.6 [19]**.** Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called

(a) a $(\delta_i, \delta_j)F\beta$ open (briefly $(\delta_i, \delta_j)F$-βo) set of X if $\lambda \leq \delta_{r}$-cl $(\delta_j$-int $(\delta_i$-cl $(\lambda)))$;

(b) a $(\delta_i, \delta_j)F\beta$ closed (briefly $(\delta_i, \delta_j)F$-βc) set of X if $\lambda \geq \delta_{r}$-int $(\delta_j$-cl $(\delta_i$-int $(\lambda)))$;

The set of all $(\delta_i, \delta_j)F$-βo, (resp. $(\delta_i, \delta_j)F$-βc) sets of a F-bts X will be denoted by $(\delta_i, \delta_j)F\beta O(X)$, (resp. $(\delta_i, \delta_j)F\beta C(X)$).

Definition 2.7 [15]**.** Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called a $(\delta_i, \delta_j)F\gamma$ open (resp. $(\delta_i, \delta_j)F\gamma$ closed), briefly $(\delta_i, \delta_j)F$-γo (resp. $(\delta_i, \delta_j)F$-γc) if $\lambda \leq \delta_i$-int $(\delta_j$-cl $(\lambda)) \vee \delta_j$-cl $(\delta_i$- int $(\lambda))$, resp. $\lambda \geq \delta_i$-cl $(\delta_j$-int $(\lambda)) \wedge \delta_j$-int $(\delta_i$-cl $(\lambda))$. The family of all $(\delta_i, \delta_j)F$-γo (resp. $(\delta_i, \delta_j)F$-γc) sets of X is denoted by $(\delta_i, \delta_j)F$-$\gamma O(X)$) and (resp. $(\delta_i, \delta_j)F$-$\gamma C(X)$).

Definition 2.8 [17]**.** Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then A is called a (δ_i, δ_j) Fuzzy-γ-semiopen (briefly $(\delta_i, \delta_j)F$-γ-so) set if $A \leq \delta_j$-cl$(\delta_i$-γ int $(A))$ and (δ_i, δ_j) Fuzzy-γ-semiclosed (briefly

$(\delta_i, \delta_j)F$-γ-sc) set if $A \geq \delta_j$-int$(\delta_i$-γ cl $(A))$. The family of all $(\delta_i, \delta_j)F\gamma$ so (resp. $(\delta_i, \delta_j)F$-γ-sc) sets of X is denoted by $(\delta_i, \delta_j)F$-γ-$SO(X)$ and (resp. $(\delta_i, \delta_j)F\gamma SC(X)$).

Remark 2.9 [17]**.** (i) The union of $(\delta_i, \delta_j)F$-γ-SO sets is a $(\delta_i, \delta_j)F$-γSO set.

(ii) The intersection of $(\delta_i, \delta_j)F$-γSC sets is a $(\delta_i, \delta_j)F$-γ-SC set.

Definition 2.10 [17]**.** Let (X, δ_i, δ_j) be a fuzzy bitopological space and x_β is a *f* point of *X*. A fuzzy set *A* of *X* is called

(a) $(\delta_i, \delta_j)F\gamma$ semi neighbourhood (briefly $(\delta_i, \delta_j)F$-γ semi nbhd) of x_β if there exists a $(\delta_i, \delta_j)F\gamma SO$ set *O* such that $x_\beta \in O \leq A$

(b) $(\delta_i, \delta_j)F\gamma$ semi *q* neighbourhood (briefly $(\delta_i, \delta_j)F$-γ semi *q* nbhd) of x_β if there exists a $(\delta_i, \delta_j)F\gamma SO$ set *O* such that $x_\beta q O \leq A$.

Remark 2.11. 1. Every $\delta_i FO$ is $(\delta_i, \delta_j)F\alpha O, (\delta_i, \delta_j)F\gamma SO, (\delta_i, \delta_j)F\gamma SO$ and $(\delta_i, \delta_j)F\beta O$.

2. Every $(\delta_i, \delta_j)F\alpha O$ is $(\delta_i, \delta_j)F\gamma O$ and every $(\delta_i, \delta_j)F\alpha O$ is $(\delta_i, \delta_j)F\beta O$.

3. Every $(\delta_i, \delta_j)F\alpha O$ is $(\delta_i, \delta_j)F\gamma SO$.

4. Every $(\delta_i, \delta_j)FPO$ or $(\delta_i, \delta_j)FSO$ or $(\delta_i, \delta_j)F\gamma SO$ is $(\delta_i, \delta_j)F\gamma O$ and $(\delta_i, \delta_j)F\beta O$.

Definition 2.12 [5]**.** A mapping $f : (X, \tau) \to (Y, \eta)$ is called a fuzzy continuous mapping if $f^{-1}(\lambda) \in \delta$, for each $\lambda \in \eta$.

Definition 2.13 [5]**.** A mapping $f : X \to Y$ is called f open if $f(A)$ is *f* open set in *Y*, for each *f*-open set in *X*.

Definition 2.14 [25]**.** Let $f : X \to Y$ be a mapping from fbts *X* to another fbts *Y*. *f* is called a fuzzy pairwise continuous (resp. fuzzy pairwise open and fuzzy pairwise closed) mapping if and only if the induced mappings

$f: (X, \tau_k) \to (Y, \eta_k)(k = 1, 2)$ are fuzzy continuous (resp. fuzzy open and fuzzy closed).

Definition 2.15 [25]. Let $f: X \to Y$ be a mapping from fbts X to another fbts Y. f is called a fpsc mapping, if $f^{-1}(\lambda) \in (\tau_i, \tau_j)$ so set of X for each η_i fo set λ of Y, equivalently $f^{-1}(\mu)$ is a (τ_i, τ_j) fsc set of X for each η_i fc set μ of Y.

Definition 2.16 [25]. Let $f: X \to Y$ be a mapping from fbts X to another fbts Y. f is called a fps open (fps closed) mapping, if $f(\lambda)$ is a (η_i, η_j)- fso (resp. (η_i, η_j)- fsc) set of Y for each τ_i- fo (resp. τ_i- fc) λ set of X.

Remark 2.17. (i) A collection μ of fuzzy sets in a Fts X is a cover of a fuzzy set u of X iff $\vee_{A \in \mu} A = 1_X$ for every $x \in S(u)$.

(ii) A fuzzy cover μ of a fuzzy set u in a Fts is said to have a finite subcover iff there exists a finite subcollection $\eta = \{A_1, \ldots, A_n\}$ of μ such that $\bigcup_{j=1}^{n} A_j(x) \geq u(x)$, for every $x \in S(u)$.

Definition 2.18 [23]. Let (X, δ_i, δ_j) be a bfts. A F-set μ is called $\delta_i\delta_j$- open, $\delta_i\delta_j$- closed provided $\mu \in \delta_i \vee \delta_j$ resp. $\mu' \in \delta_i \vee \delta_j$.

Definition 2.19 [18]. A family U of (δ_i, δ_j) fuzzy sets is a FP cover of a (δ_i, δ_j) F-set B iff $B \subset \{A/A \in U\}$.

Definition 2.20 [18]. A FP cover U of a fbts (X, δ_i, δ_j) is a FP open cover (briefly FPO cover or (δ_i, δ_j) FPO cover) of iff $U \subset \delta_i \vee \delta_j$ and $\vee_{\lambda \in U} A(x) = \tilde{1}$ for every $x \in X$ and each member of U is a $(\delta_i, \delta_j)F$ open set.

A subcover of U is a subfamily of U which is also a cover.

Definition 2.21 [18]. A fbts (X, δ_i, δ_j) said to be FP compact iff each FPO cover of X has a finite subcover.

Definition 2.22 [18]. A fbts (X, δ_i, δ_j) is said to be fuzzy pairwise γ (resp α, β) compact (FP-γC) (resp. FP-αC, FP-βC) iff for every family μ of

(δ_i, δ_j) fuzzy γ (resp. α, β) open sets such that $\vee_{A \in \mu} A = 1_x$ there is a finite subfamily $\eta \subseteq \mu$ such that $\vee_{A \in \eta} A = 1_x$ for every $x \in S(u)$.

Definition 2.23 [18]. A f set U in a fbts (X, δ_i, δ_j) is said to be FP-γ compact relative to X if for every family μ of $(\delta_i, \delta_j)F\gamma O$ sets such that $\vee_{A \in \mu} A \geq U(x)$ there exists a finite subfamily $\lambda \subseteq \mu$ such that $\vee_{A \in \lambda} A > U(x)$ for every $x \in S(u)$.

Definition 2.24 [7]. A family A of fuzzy sets has the finite intersection property iff the intersection of the members of each finite subfamily of A is non empty.

Definition 2.25 [7]. A collection of fuzzy subsets ξ of a fts X is said to form a filterbases iff for every finite collection $\{A_j : j = 1, \ldots, n\}$, $A_{j=1}^n \neq 0_X$.

Theorem 2.26 [18]. *A fbts (X, δ_i, δ_j) is FPβ compact iff for every FP filterbases Γ in X, $\wedge_{F \in \Gamma} (\delta_i, \delta_j)\beta cl(F) \neq \tilde{0}_X$.*

Theorem 2.27 [18]. *A fuzzy set W in a fbts (X, δ_i, δ_j) is FP-β compact relative to X if and only if for every FP filterbases Γ with the property that every finite member of Γ is quasi coincident with W, $\wedge_{F \in \Gamma} (\delta_i, \delta_j)\beta cl(F) \wedge W \neq \tilde{0}_X$.*

Definition 2.28 [9]. A fbts (X, δ_1, δ_2) is called a pairwise T_2-space or a pairwise Hausdorff space provided that for all $x, y \in X$ with $x \neq y$, we have $N_\sigma(x) \wedge N_\sigma(y)$ does not exist.

III. Fuzzy Pairwise Gamma Semi-Continuity

Definition 3.1. Let $f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ be a mapping from a f-bts (X, δ_i, δ_j) to another fbts (Y, η_i, η_j). Then f is called a F pairwise γ semi-continuous (briefly FPγSC) mapping if $f^{-1}(\nu)$ is a $(\delta_i, \delta_j)F\gamma SO$ set in X for each $\eta_i FO$ set ν in Y. Equivalently, $f^{-1}(\mu)$ is a $(\delta_i, \delta_j)F\gamma SC$ set in X for each $\eta_i FC$ set μ in Y.

Definition 3.2. Let $f : (X, \delta_i, \delta_j) \to (Y, \delta_i, \delta_j)$ be a mapping from a *f*-bts (X, δ_i, δ_j) to another fbts (Y, η_i, η_j). Then *f* is called a *F* pairwise γ semi open (briefly FPγSO) (resp. FPγS closed) mapping if $f(\lambda)$ is a $(\eta_i, \eta_j)F\gamma SO$ (resp. FPγSC) set of *Y* for each $\delta_i FO$ (resp. $\delta_i FC$) set λ of *X*.

Remark 3.3. Every *F*-PC(resp FP open and FP closed) mapping is FPγSC (resp. FPγS open and FPγS closed). But the converse need not be true as given below.

Example 3.4. Let μ_1, μ_2, μ_3 be *F*-sets of $X = \{a, b, c\} = Y = I$ defined as $\mu_1 = \{(a_{0.5}, b_{0.6})\}, \mu_2 = \{(a_{0.6}, b_{0.7})\}, \mu_3 = \{(a_{0.8}, b_{0.1})\}$. Consider the *F*-bts (X, δ_1, δ_2) and (Y, η_1, η_2) where $\delta_1 = \{\widetilde{0}, \widetilde{1}, \mu_1, \mu_3, \mu_1 \vee \mu_3, \mu_1 \wedge \mu_3\}$, $\delta_2 = \{\widetilde{0}, \widetilde{1}, \mu_3\}$, $\eta_1 = \{\widetilde{0}, \widetilde{1}, \mu_2\}$ and $\eta_2 = \{\widetilde{0}, \widetilde{1}, \mu_3\}$. Let $f : (X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$ be an identity mapping. Here f is FPγSC.

To check $f : (X, \delta_1) \to (Y, \eta_1)$ and $f : (X, \delta_2) \to (Y, \eta_2)$ are *F* continuous. Here μ_2 is $\eta_1 FO$ and $f^{-1}(\mu_2) = \mu_2$ is not $\delta_1 FO$ and μ_1 is $\eta_2 FO$ and $f^1(\mu_1) = \mu_1$ is not $\delta_2 FO$. Thus, *f* is FPγSC but not FPC.

Example 3.5. Let $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ be *F*-sets of the fbts (X, δ_1, δ_2) and (Y, η_1, η_2) with $X = \{a, b, c\} = Y = I$ defined as $\mu_1 = \{(a_{0.1}, b_{0.3}, c_{0.2})\}$, $\mu_2 = \{(a_{0.2}, b_{0.4}, c_{0.3})\}$, $\mu_3 = \{(a_{0.1}, b_{0.2}, c_{0.3})\}$, $\mu_4 = \{(a_{0.2}, b_{0.4}, c_{0.5})\}$, $\mu_5 = \{(a_{0.3}, b_{0.5}, c_{0.4})\}$. Here $\delta_1 = \{\widetilde{0}, \widetilde{1}, \mu_2, \mu_5\}$, $\delta_2 = \{\widetilde{0}, \widetilde{1}, \mu_3, \mu_4\}$, $\eta_1 = \{\widetilde{0}, \widetilde{1}, \mu_1, \mu_3, \mu_1 \vee \mu_3, \mu_1 \wedge \mu_3\}$ and $\eta_2 = \{\widetilde{0}, \widetilde{1}, \mu_1, \mu_2\}$. Let the mapping $f : (X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$ be an identity mapping. Here *f* is a FPγS open mapping.

Consider the mappings $f : (X, \delta_1) \to (Y, \eta_1)$ and $f : (X, \delta_2) \to (Y, \eta_2)$. Then μ_2 and μ_5 are $\delta_1 FO$ but $f(\mu_2)$ and $f(\mu_5)$ are not $\eta_1 FO$. Similarly, μ_3 and μ_4 are $\delta_2 FO$ but $f(\mu_3)$ and $f(\mu_4)$ are not $\eta_2 FO$. Then *f* is not FP open mapping. Thus, *f* is FPγS open mapping but not FP open mapping. Also, *f* is not (η_1, η_2) FPP open mapping and *f* is not (η_1, η_2) FPSS open mapping.

Theorem 3.6. *Let* (X, δ_i, δ_j) *and* (Y, η_i, η_j) *be two fbts and*

$f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ *be a mapping. Then the following conditions are equivalent.*

(a) *f is FPγS continuous.*

(b) *For every set A in* X, $f((\delta_i, \delta_j)\gamma scl(A)) \leq \eta_i cl(f(A))$

(c) *For every set B in* Y, $((\delta_i, \delta_j)\gamma scl(f^{-1}(B)) \leq f^{-1}(\eta_i cl(B))$

(d) *For every set B in* Y, $f^{-1}(\eta_i \operatorname{int}(B)) \leq (\delta_i, \delta_j)\gamma \operatorname{sint}(f^{-1}(B))$

(e) *For every set A in* X, $\eta_i \operatorname{int}(A) \leq f(\delta_i, \delta_j)\gamma \operatorname{sint}(A)$

(f) *The inverse image of every* $\eta_i FO$ *set of Y is a* $(\delta_i, \delta_j)F\gamma SO$ *set in X.*

(g) *The inverse image of every* $\eta_i FC$ *set of Y is a* $(\delta_i, \delta_j)F\gamma SC$ *set in X.*

Proof: a $\Leftrightarrow$ b a $\Rightarrow$ b Let f be FPγSC. Then by definition $f^{-1}(\mu)$ is a $(\delta_i, \delta_j)F\gamma SC$ set of X for each $\eta_i FC$ set μ of Y. Let A be any f. set in X, then $f(A) \in Y$ and $\eta_i cl(f(A))$ is a $\eta_i FC$ set in Y. Then $f^{-1}(\eta_i cl(f(A)))$ is a $(\delta_i, \delta_j)F\gamma SC$ set in X (1). That is, $A \leq f^{-1}(\eta_i cl(f(A)))$. Then $(\delta_i, \delta_j)\gamma scl(A) \leq (\delta_i, \delta_j)\gamma scl\,[f^{-1}(\eta_i cl(f(A))]) = f^{-1}(\eta_i cl(f(A))$ by (1). Thus, $f[(\delta_i, \delta_j)\gamma scl(A) \leq f[f^{-1}(\eta_i cl(f(A))] \leq \eta_i cl(fg(A))$.

(b) $\Rightarrow$ (a) Conversely, suppose the given condition is true. Let B be any $\eta_i FC$ set in Y. Then $f[(\delta_i, \delta_j)\gamma scl(f^{-1}(B))] \leq \eta_i cl\,(f(f^{-1}(B))) \leq \eta_i cl(f(B))$. That is $(\delta_i, \delta_j)\gamma scl(A) \leq f^{-1}(\eta_i cl(B))$ (2) which is $f^{-1}(B)$. Then $f^{-1}(B)$ is a $(\delta_i, \delta_j)F\gamma SC$ set in X. Then f is FPγS continuous.

(b) $\Rightarrow$ (c) and (c) $\Rightarrow$ (a) Follows from (2)

(a) $\Leftrightarrow$ (d) (a) $\Rightarrow$ (d) Suppose f is FPγS continuous. Let B be any f set in Y. Then $f^{-1}(B) \in X$ and $\eta_i \operatorname{int}(B)$ is a $\eta_i FO$ set in Y. Then $f^{-1}(\eta_i \operatorname{int}(B))$ is $(\delta_i, \delta_j)F\gamma SO$ set in X and $f(f^{-1}(B)) \geq B$. Then $f^{-1}(B) \geq f^{-1}(\eta_i \operatorname{int}(B))$. Then $(\delta_i, \delta_j)\gamma \operatorname{sint}[f^{-1}(\eta_i \operatorname{int}(B))] \leq (\delta_i, \delta_j)\gamma \operatorname{sint}[f^{-1}(B)]$. Thus, $[f^{-1}(\eta_i \operatorname{int}(B))] \leq (\delta_i, \delta_j)\gamma \operatorname{sint}[f^{-1}(B)]$.

(d) $\Rightarrow$ (a) Suppose (d) holds, let B be a fuzzy set in X. Then $f(B)$ is a $\eta_i FO$ set in Y. Let $A = f(B)$. Then by hypothesis, $[f^{-1}(\eta_i \text{ int}(B)] \leq (\delta_i, \delta_j)\gamma\text{sint}\,[f^{-1}(B)]$. That is $f^{-1}(f(B)) \leq (\delta_i, \delta_j)\gamma\text{sint}\,[f^{-1}(B)]$ which implies $B \leq (\delta_i, \delta_j)\gamma\text{sint}\,[f^{-1}(B)]$. Then B is $(\delta_i, \delta_j)F\gamma SO$ in X. That is $f^{-1}(A)$ is $(\delta_i, \delta_j)F\gamma SO$ in X. Thus, f is FPγS continuous.

(a) $\Leftrightarrow$ (e) (a) $\Rightarrow$ (e) Let A be any f. set in X, then $f(A) \in Y$ and $\eta_i \text{ int}(f(A))$ is a $\eta_i FO$ set in Y. By definition 3.1, $f^{-1}(\eta_i \text{ int}(f(A)))$ is $(\delta_i, \delta_j)F\gamma SO$ set in X. Now, $A \geq f^{-1}(f(A))$. Then $A \geq f^{-1}(\eta_i \text{ int}(f(A)))$. That is, $(\delta_i, \delta_j)\gamma\text{sint}[f^{-1}(\eta_i \text{ int}(f(A))] \leq (\delta_i, \delta_j)\gamma\text{sint}(A)$. Then $f^{-1}(\eta_i \text{ int}(f(A)) \leq (\delta_i, \delta_j)\gamma\text{sint}(A)$. (3). Then, $\eta_i \text{ int}(f(A)) \leq f[(\delta_i, \delta_j)\gamma\text{sint}(A)]$.

(e) $\Rightarrow$ (a), (d) $\Leftrightarrow$ (e) (f) $\Rightarrow$ (d) and (g) $\Leftrightarrow$ (a) is obvious.

Theorem 3.7. *Let* $f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ *be a mapping where* (X, δ_i, δ_j) *and* (Y, η_i, η_j) *are fbts. Then the following are equivalent.*

(a) *f is FPγS continuous.*

(b) *For every point* x_α *of X and every* $\eta_i FO$ *set V with* $f(x_\alpha) \in V$, *there exists a* $(\delta_i, \delta_j)F\gamma SO$ *set U in X such that* $x_\alpha \in U$ *and* $f(U) \leq V$.

(c) *For every point* x_α *of X and every* $\eta_i F$ *nbhd V of* $f(x_\alpha)$, $f^{-1}(V)$ *is a* $(\delta_i, \delta_j)F\gamma$ *semi-nbhd of* x_α.

(d) *For every point* x_α *of X and every* $\eta_i F$ *nbhd V of* $f(x_\alpha)$, *there is a* $(\delta_i, \delta_j)F\gamma$ *semi-nbhd W of* x_α *in X such that* $f(W) \leq V$.

(e) *For every point* x_α *of X and every* $\eta_i FO$ *set V with* $f(x_\alpha)$ *of V, there exists a* $(\delta_i, \delta_j)F\gamma SO$ *set U such that* $x_\alpha q U$ *and* $f(U) \leq V$.

(f) *For every point* x_α *of X and every* $\eta_i Fq$ *nbhd V of* $f(x_\alpha)$, *there is a* $(\delta_i, \delta_j)F\gamma$ *semi-q nbhd of* x_α.

Proof. (a) $\Leftrightarrow$ (b) (a) $\Rightarrow$ (b) Let f be FPγS continuous. Let x_α be a f point and V be $\eta_i FO$ in Y with $f(x_\alpha) \in f(U)$. Let $U = f^{-1}(V)$ where $f^{-1}(V)$ is a $(\delta_i, \delta_j)F\gamma SO$ set in X since f is FPγS continuous. Let $x_\alpha \in U$. Then $f(x_\alpha) \in f(U)$. That is $f(x_\alpha) \in f(f^{-1}(V))$. Then $f(U) \leq f(f^{-1}(V))$ which implies $f(U) \leq V$.

(b) ⇒ (a) This is obvious.

(a) ⇒ (c) Suppose f is FPγS continuous. Let x_α be a f point of X and V be $\eta_i F$ nbhd of $f(x_\alpha)$ in Y. Then there exists a $\eta_i FO$ set W in Y such that $f(x_\alpha) \in W \leq V$ which implies $x_\alpha \in f^{-1}(W) \leq f^{-1}(V)$ (1). $f^{-1}(V)$ is $(\delta_i, \delta_j)F\gamma SO$ since f is FPγS continuous. Then $f^{-1}(W)$ is $(\delta_i, \delta_j)F\gamma SO$ in X. From (1) and by definition 2.10, $f^{-1}(V)$ is a $(\delta_i, \delta_j)F\gamma$ semi nbhd of x_α in X.

(c) ⇒ (d) This is obvious.

(d) ⇒ (b) Let $x_\alpha \in X$ and V be $\eta_i FO$ with $f(x_\alpha) \in V$. Since a f. open set is a nbhd of each of its points V is a $\eta_i F$ nbhd of $f(x_\alpha)$. By hypothesis, there is a $(\delta_i, \delta_j)F\gamma$ semi-nbhd W of x_α in X such that $f(W) \in V$. Then there exists a $(\delta_i, \delta_j)F\gamma SO$ set U in X such that $x_\alpha \in U \leq W$. That is, $f(x_\alpha) \in f(U) \leq f(W)$. Then $f(U) \leq V$.

(a) ⇔ (e) (a) ⇒ (e) Assume that f is FPγS continuous. Let $x_\alpha \in X$ and V be a $\eta_i F$ nbhd of $f(x_\alpha)$. Then by definition there exists a $\eta_i FO$ set W such that $f(x_\alpha) q W \in V$. That is $x_\alpha q f^{-1}(W) \leq f^{-1}(V)$ and $f^{-1}(W)$ is $(\delta_i, \delta_j)F\gamma SO$. Let $U = f^{-1}(W)$. Then $f(U) \leq V$.

(e) ⇒ (a) Let V be $\eta_i FO$ in Y. Let $x_\alpha \in f^{-1}(V)$. Then $f(x_\alpha) \in V$. Choose a f. point $x'_\alpha(x) = 1 - x_\alpha(x)$. Then $f(x'_\alpha) q V$. By hypothesis, there is a $(\delta_i, \delta_j)F\gamma SO$ set U such that $x'_\alpha q U$ and $f(U) \leq V$. That is $x'_\alpha(x) + U(x) = 1\text{-}\alpha + U(x) > 1$. Then $U(x) > \alpha$ which implies $x_\alpha \in U$. That is $x_\alpha \in U \leq f^{-1}(V)$. Then $f^{-1}(V)$ is $(\delta_i, \delta_j)F\gamma SO$. Hence f is FPγS continuous.

(e) $\Rightarrow$ (f) Let $x_\alpha \in X$ and V be a $\eta_i FO$ set such that $f(x_\alpha)qV$. Then there exists a $\eta_i FO$ set W such that $f(x_\alpha)qW \leq V$. By assumption, there is a $(\delta_i, \delta_j)F\gamma SO$ set U such that $x'_\alpha qU$ and $f(U) \leq W$. That is, $x_\alpha qU \leq f^{-1}(W) \leq f^{-1}(V)$. Thus, $f^{-1}(V)$ is a $(\delta_i, \delta_j)F\gamma$ semi-q nbhd of x_α.

Theorem 3.8. *Let* $(X, \delta_i, \delta_j), (Y, \eta_i, \eta_j)$ *and* (Z, τ_i, τ_j) *be fbts's. If* $f : X \to Y$ *is FPγS continuous,* $g : Y \to Z$ *is FP continuous then* $gof : X \to Z$ *is FPγS continuous.*

Proof. Given $f : X \to Y$ is FPγS continuous. Then for each $\eta_i FO$ set λ in Y, $f^{-1}(\lambda)$ is a $(\delta_i, \delta_j)F\gamma SO$ set in X. Also, $g : Y \to Z$ is FP continuous. Then $g : (Y, \eta_i) \to (Z, \tau_i)$ and $g : (Y, \eta_j) \to (Z, \tau_j)$ are F continuous. That is, for each $\mu \in \tau_i$, $f^{-1}(\mu) \in \eta_i$ and for each $\nu \in \tau_j$, $f^{-1}(\nu) \in \eta_j$. Now consider $\mu \in \tau_i$. Then $g^{-1}(\mu) \in \eta_i$, then $f^{-1}(g^{-1}(\mu))$ is a $(\delta_i, \delta_j)F\gamma SO$ set in X. This shows $gof : X \to Z$ is FPγS continuous.

Remark 3.9. Composition of two FPγS continuous mappings need not be FPγS continuous which is shown in the following example.

Example 3.10. Let $(X, \delta_1, \delta_2), (Y, \eta_1, \eta_2)$ and (Z, τ_1, τ_2) be fbts's with $X = \{a, b, c\} = Y = Z = I$. Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 be fuzzy sets defined as: $\lambda_1 = \{a_{0.5}, b_{0.6}, c_{0.4}\}, \lambda_2 = \{a_{0.2}, b_{0.3}, c_{0.5}\}, \lambda_3 = \{a_{0.2}, b_{0.1}, c_{0.5}\}$, $\lambda_4 = \{a_{0.3}, b_{0.2}, c_{0.4}\}$ and $\lambda_5 = \{a_{0.7}, b_{0.9}, c_{0.6}\}$. Here $\delta_1 = \{\tilde{0}, \tilde{1}, \lambda_2\}$, $\delta_2 = \{\tilde{0}, \tilde{1}, \lambda_1\}$, $\eta_1 = \{\tilde{0}, \tilde{1}, \lambda_3\}$, $\eta_2 = \{\tilde{0}, \tilde{1}, \lambda_2\}$, $\tau_1 = \{\tilde{0}, \tilde{1}, \lambda_4\}$ and $\tau_2 = \{\tilde{0}, \tilde{1}, \lambda_5\}$. Consider the identity mappings $f : (X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$ and $g : (X, \eta_1, \eta_2) \to (Z, \tau_1, \tau_2)$. Here $\delta_2 cl(\delta_1 \gamma \operatorname{int}(\lambda_3) = \lambda'_2$. This implies λ_3 is $(\delta_i, \delta_j)F\gamma SO$. Thus, f is FPγS continuous. Also $\eta_2 cl(\eta_1 \gamma \operatorname{int}(\lambda_4)) = \eta_2 cl(\lambda_3) = \lambda'_1$. Then λ_4 is $(\delta_i, \delta_j)F\gamma SO$. which shows that g is FPγS continuous. But g of is not FPγS continuous as $\delta_2 cl(\delta_1 \gamma \operatorname{int}(\lambda_4) = 0$.

Theorem 3.11. *Let* $f : (X, \delta_i, \delta_j) \to (Y, \eta_j, \eta_j)$ *be a mapping from a* *fbts* (X, δ_i, δ_j) *to another fbts* (Y, η_i, η_j). *If g is a mapping defined as* $g:(X, \delta_i, \delta_j) \to (XxY, \beta_i, \beta_j)$ *where* β_k *is the fuzzy product topology generated by* δ_k *and* $\eta_k (k = 1, 2)$ *defined by* $g(x) = (x, f(x))$ *is FPγS continuous, then f is FPγS continuous.*

Proof. Let V be a η_i FO set of Y. Consider $f^{-1}(V) = \tilde{1} x f^{1}(V) = g^{-1}(\tilde{1} x V)$. Given g is FPγS continuous and $\tilde{1} x V$ is a β_i FO set of $X \times Y$ then $g^{-1}(\tilde{1} \times V)$ is a $(\delta_i, \delta_j) F\gamma SO$ set of X. Thus, f is FPγS continuous.

Theorem 3.12. *Let* (X, δ_i, δ_j) *and* (Y, η_i, η_j) *be two fbts and* $f:(X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ *be a mapping. Then the following statements are equivalent.*

(a) *f is FPγS open.*

(b) $f(\delta_i \operatorname{int}(A)) \leq \eta_j cl(\eta_i \gamma \operatorname{int}(f(A)))$ *for each f set A of X.*

(c) $f(\delta_i \operatorname{int}(B)) \leq ((\eta_i, \eta_j)\gamma \operatorname{sint}(f(B)))$ *for each f set B of Y.*

Proof. (a) ⇒ (b) Let A be any $\delta_i FO$ set of X. Then $\delta_i \operatorname{int}(A)$ is $\delta_i FO$ in X. This implies $f(\delta_i \operatorname{int}(A))$ is $(\eta_i, \eta_j) FSO$ in Y. Then $f(\delta_i \operatorname{int}(A)) \leq \eta_j cl(\eta_i \gamma \operatorname{int}(f(\delta_i \operatorname{int}(A)))) \leq \eta_j cl(\eta_i \gamma \operatorname{int}(f(A)))$ for any set f set A in X. (b) ⇒ (a) and (a) ⇒ (c) This is obvious.

Theorem 3.13. *Let* (X, δ_i, δ_j) *and* (Y, η_i, η_j) *be two fbts and* $f:(X, \delta_i, \delta_j) \to (Y, \eta_j, \eta_j)$ *be a mapping. Then the following statements are equivalent.*

(a) *f is FPγS closed.*

(b) $f(\delta_i cl(A)) \geq \eta_j \operatorname{int} cl(\eta_i \gamma cl(f(A)))$ *for each f set A of X.*

(c) $f(\delta_i cl(B)) \leq ((\eta_i, \eta_j)\gamma scl(f(B)))$ *for each f set B of Y.*

Theorem 3.14. *Let* $f:(X, \delta_i, \delta_j) \to (Y, \eta_j, \eta_j)$ *be a FPγSO mapping. Let B be a* $\eta_i FO$ *set and A be a* $\delta_i FC$ *set containing* $f^{-1}(B)$. *Then there exists a* $(\eta_i, \eta_j) F\gamma SC$ *set M of Y containing B such that* $f^{-1}(M) \leq A$.

Theorem 3.15. *Let* $f:(X, \delta_i, \delta_j) \to (Y, \eta_j, \eta_j)$ *be a FPO mapping and* $g:(Y, \eta_i, \eta_j) \to (Z, \tau_i, \tau_j)$ *be a FPγSO mapping. Then* $gof:(X, \delta_i, \delta_j) \to (Z, \tau_i, \tau_j)$ *is FPγSO.*

Proof. $f:(X) \to (Y)$ is a FPO mapping implies that for a $\delta_i FO$ set M in X, $f(M)$ is $\delta_j FO$ in Y. Now since $g:(Y) \to (Z)$ is a FPγSO mapping, $g[f(M)] = (gof)(M)$ is a (δ_i, δ_j) is FPγSO set in Z. Thus, gof is FPγSO.

Theorem 3.16. *Let* $f:(X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ *be a FPγSC mapping which is onto and* $g:(Y, \eta_i, \eta_j) \to (Z, \tau_i, \tau_j)$ *be a FPγSO mapping. If gof is FPγS closed then g is FPγS closed.*

Remark 3.17. It is obvious that every FPγC, FPPC, FPγC (resp. FPβO, FPPO, FPγO) mappings are F-PγSC mapping and every FPγSC mapping is a F-PβC mapping. But the converses are not true which is illustrated in the following examples.

Example 3.18. Let $X = \{a, b, c\} = Y = I$ be fbts with topologies $\delta_1, \delta_2, \eta_1$ and η_2 defined as follows $\delta_1 = \{\tilde{0}, \tilde{1}, \lambda_1\}$, $\delta_2 = \{\tilde{0}, \tilde{1}, \lambda_2\}$, $\eta_1 = \{\tilde{0}, \tilde{1}, \lambda_3\}$ and $\eta_2 = \{\tilde{0}, \tilde{1}, \lambda_4\}$ where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are fuzzy sets defined as follows: $\lambda_1 = \{(a_{0.5}, b_{0.2}, c_{0.1})\}$, $\lambda_2 = \{(a_{0.5}, b_{0.3}, c_{0.5})\}$, $\lambda_3 = \{(a_{0.6}, b_{0.4}, c_{0.3})\}$, $\lambda_4 = \{(a_{0.5}, b_{0.7}, c_{0.6})\}$. Let $f:(X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$ be an identity mapping. Here λ_3 is $\eta_1 FO$. Also λ_3 is $(\delta_1, \delta_2)F\gamma SO$ but not $(\delta_1, \delta_2)F\alpha O$ and $(\delta_1, \delta_2)FPO$. Thus, *f* is *F*-PγS continuous but not *F*-Pα continuous and *F*-PP continuous.

Example 3.19. Let $\lambda_1, \lambda_2, \lambda_3$ and λ_4 be *F*-sets of $X = \{a, b, c\} = Y = I$ with topologies $\delta_1, \delta_2, \eta_1, \eta_2$ such that (X, δ_1, δ_2) and (Y, η_1, η_2) are fbts. Define $f:(X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$ as an identity mapping. Let $\lambda_1 = \{(a_{0.2}, b_{0.4}, c_{0.5})\}$, $\lambda_2 = \{(a_{0.2}, b_{0.1}, c_{0.3})\}$, $\lambda_3 = \{(a_{0.7}, b_{0.3}, c_{0.4})\}$, $\lambda_4 = \{(a_{0.2}, b_{0.5}, c_{0.6})\}$ where $\delta_1 = \{\tilde{0}, \tilde{1}, \lambda_3\}$, $\delta_2 = \{\tilde{0}, \tilde{1}, \lambda_2\}$, $\eta_1 = \{\tilde{0}, \tilde{1}, \lambda_1\}$ and $\eta_2 = \{\tilde{0}, \tilde{1}, \lambda_4\}$. Here λ_1 is $\eta_1 FO$ and $(\delta_1, \delta_2)F\beta O$. But not $(\delta_1, \delta_2)F\gamma O$ and $(\delta_1, \delta_2)F\gamma SO$ thus *f* is F-Pβ continuous but not *F*-Pγ continuous and *F*-PγS continuous.

Example 3.20. Let $X = \{a, b, c\} = Y = I$ and let $\lambda_1, \lambda_2, \lambda_3$ and λ_4 be F-sets of X and Y defined as follows:

$$\lambda_1 = \{(a_{0.2}, b_{0.5}, c_{0.6})\}, \lambda_2 = \{(a_{0.5}, b_{0.4}, c_{0.3})\}, \lambda_3 = \{(a_{0.1}, b_{0.4}, c_{0.5})\}.$$

Let (X, δ_1, δ_2) and (Y, η_1, η_2) be fbts with $\delta_1 = \{\widetilde{0}, \widetilde{1}, \lambda_1\}, \delta_2 = \{\widetilde{0}, \widetilde{1}, \lambda_2\}, \eta_1 = \{\widetilde{0}, \widetilde{1}, \lambda_3\}$ and $\eta_2 = \{\widetilde{0}, \widetilde{1}, \lambda_2\}$. Consider the identity mapping $f : (X, \delta_1, \delta_2) \to (Y, \eta_1, \eta_2)$. Here λ_3 is $(\delta_1, \delta_2)F\gamma O$ and f is FPγ continuous. But λ_3 is not $(\delta_1, \delta_2)F\gamma SO$ as $\lambda_3 > \delta_2 cl(\delta_1 \gamma \operatorname{int}(\lambda_1)) = 0$. Hence f is not FPγS continuous.

IV. Fuzzy Pairwise Gamma Semicompact

Definition 4.1. A FP cover U of a fbts (X, δ_i, δ_j) is a FPγS open cover (briefly FPγSO cover or $(\delta_i, \delta_j)FP\gamma SO$ cover) of iff $U \subset \delta_i \vee \delta_j$ and $\vee_{\lambda \in U} A(x) = \widetilde{1}$ for every $x \in X$ and each member of U is a $(\delta_1, \delta_2)F\gamma S$ open set. A subcover of U is a subfamily of U which is also a cover.

Definition 4.2. A fbts (X, δ_i, δ_j) said to be FPγS compact iff each FPγSO cover of X has a finite subcover.

Theorem 4.3. *Let (X, δ_i, δ_j) be a fbts. Every $(\delta_j, \delta_j)F$ closed subset of a FPγS compact space is FPγS compact.*

Theorem 4.4. *Suppose the fbts (X, δ_i, δ_j) is FP Hausdorff. Let $U \subset X$ be a FPγS compact set, then U is $(\delta_i, \delta_j)F\gamma SC$ in X.*

Proof. Let $u \in U$. The fbts (X, δ_i, δ_j) is FP Hausdorff implies that for every f point $x \in X\text{-}U$, there exist disjoint $(\delta_i, \delta_j)F$ nbhds $V_{u,x}$ and $W_{x,u}$ of u and x respectively. Then the family $\{V_{u,x} \wedge U / u \in U\}$ is a FPγSO cover of U. Since U is FPγS compact, this has a finite subcover $\{V_{u_1,x}, \ldots, V_{u_n,x}\}$ for some $u_1, \ldots, u_n \in U$. Let $W_x = \bigcap_{i=1}^{n} W_{x,u_i}$. Here W_x is a $(\delta_i, \delta_j)F\gamma S$ nbhd of x, also W_x and U are disjoint. If not, then there exists a f point $v \in W_x \wedge U$. That is $v \in V_{u_j,x}$ for some $j \in \{1, 2, \ldots, n\}$ and

$v \in W_x \subset W_{x,u_i}$ which cannot happen as $V_{u_j,x} \wedge W_{x,u_i} = \phi$. Hence $X\text{-}U = \bigcup_{x \in X-U} W_x$, which shows $X - U$ is $(\delta_i, \delta_j)F\gamma SO$ which implies U is $(\delta_i, \delta_j)F\gamma SC$.

Theorem 4.5. *A fbts* (X, δ_i, δ_j) *is* $(\delta_i, \delta_j)F\gamma S$ *compact iff for every collection* $\{A_\lambda : \lambda \in \Delta\}$ *of* $(\delta_i, \delta_j)F\gamma SC$ *sets of X having the finite intersection property has a nonempty intersection.*

Theorem 4.6. *If A and B are two FPγS compact subsets of a fbts* (X, δ_i, δ_j) *then* $A \vee B$ *is FPγS compact.*

Proof. Follows from Remark 2.11.

Remark 4.7. Arbitrary union of FPγS compact sets is FPγS compact.

Theorem 4.8. *Let* (X, δ_i, δ_j) *be a fbts*

(a) *Every FPγS compact space is* $\delta_i F$ *compact.*

(b) *Every FPγS compact space is FPα compact.*

Proof. (a) Let U be a $\delta_i FC$ set of X. Then $X\text{-}U$ is $\delta_i FO$ in X. By remark 2.11(1), $X\text{-}U$ is $(\delta_i, \delta_j)F\gamma SO$ in X. Let A be a FPO cover of X. Then A is FPγSO cover of X. Since X is FPγS compact, A has a finite subcollection covering X. Thus, X is $\delta_i F$ compact.

(b) By remark 2.11(3), similar to Theorem 4.8 (a)

Theorem 4.9. *Let* (X, δ_i, δ_j) *be a fbts*

(a) *Every FPγ compact space is FPγS compact.*

(b) *Every FPβ compact space is FPβS compact.*

Proof. (a) Follows from remark 2.11 (4)

Remark 4.10. The concepts of FPγS compact and FPP compact are independent.

Theorem 4.11. *Every FPγS continuous image of a FPγS compact space is* $(\eta_i, \eta_j)F\gamma S$ *compact.*

Proof. Let (X, δ_i, δ_j) and (Y, η_i, η_j) be fbts. Define $f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ be a FPγSO mapping. Suppose X is FPγS compact. Define $U_{f(x)} = \{A_i, i \in I\}$ to be a $(\eta_i, \eta_j)FP\gamma SO$ cover of $f(x)$. Let $(\delta_i, \delta_j)F\gamma SO$ cover of X be $U_x = \{f^{-1}(A_i), i \in I\}$. This has a finite subcover $U'_x = \{f^{-1}(A_{i_1}), \ldots, f^{-1}(A_{i_n})\}$ for some $i_1, \ldots, i_n \in I$, as X is FPγS compact. Then $U'_{f(x)} = \{A_{i_1}, \ldots, A_{i_n}\}$ is a $(\eta_i, \eta_j)F\gamma SO$ finite subcover of $U_{f(x)}$. Hence $f(x)$ is $(\eta_i, \eta_j)F\gamma S$ compact.

Theorem 4.12. *Let (X, δ_i, δ_j) and (Y, η_i, η_j) be fbts where X is FPγS compact. Let $f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ be a surjective FPγS continuous function from X to Y. Then Y is FPγS compact.*

Proof. Let BY be $(\eta_i, \eta_j)F\gamma SO$ cover of Y. Since f is a surjective FPγS continuous function, for a $\eta_i FO$ set U in Y, $f^{-1}(U)$ is $(\delta_i, \delta_j)F\gamma SO$ set in X. Let $B_X = \{f^{-1}(U)/U \in B_Y\}$. Then B_X is a $(\delta_i, \delta_j)F\gamma SO$ cover of X. Since, X is FPγS compact this has a finite subcover, $B'_x = \{f^{-1}(U_1), \ldots, f^{-1}(U_n)\}$ where $U_1, \ldots, U_n \in B_Y$. Thus, $B'_Y = \{U_1, \ldots, U_n\}$ becomes a finite subcover of B_Y. Hence (Y, η_i, η_j) is FPγS compact.

Definition 4.13. Let $f : (X, \mathfrak{J}_1, \mathfrak{J}_2) \to (Y, \delta_1, \delta_2)$ be a function. f is called fuzzy pairwise gamma semi homeomorphism iff f is a bijection, FPγS continuous and $f^{-1} : (Y, \delta_1, \delta_2) \to (X, \mathfrak{J}_1, \mathfrak{J}_2)$ is FPγS continuous.

Theorem 4.14. *Let (X, δ_i, δ_j) be a FPγS compact space and (Y, η_i, η_j) be a FP Hausdorff space. If $g : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ be a FPγS continuous mapping which is a bijection, then g is a FPγS homeomorphism.*

Proof. To show g is a FPγS homeomorphism it is enough to show that g^{-1} is FPγS continuous. Let C be a $\delta_i FO$ set in X. Define $U = X - C$. Then U is $\delta_i FC$ in X. By theorem 4.3, U is FPγS compact in X. Since g is FPγS continuous, by theorem 4.4, $g(U)$ is $(\eta_i, \eta_j)FP\gamma S$ compact in Y. Now since Y

is FP Hausdorff, by theorem 4.4, $g(U)$ is $(\eta_i, \eta_j)F\gamma SC$ in Y. Consider $Y\text{-}g(U) = g(X)\text{-}g(U) = g(X - U) = g(C)$. Then $g(C)$ is $(\eta_i, \eta_j)F\gamma SO$ in Y. Thus, g^{-1} is FPγS continuous.

Definition 4.15. A f set U in a fbts (X, δ_i, δ_j) is said to be FP-γS compact relative to X if for every family μ of $(\delta_i, \delta_j)F\gamma SO$ sets such that $\vee_{A\in\mu} A \geq U(x)$ there exists a finite subfamily $\lambda \subseteq \mu$ such that $\vee_{A\in\lambda} A > U(x)$ for every $x \in S(u)$.

Theorem 4.16. *A fbts (X, δ_i, δ_j) is FP-γS compact iff for every FP filterbases Γ in X, $\wedge_{F\in\Gamma} (\delta_i, \delta_j)\gamma scl(F) \neq \tilde{0}_X$.*

Proof. Follows from Theorem 2.37 and Theorem 4.9(b)

Theorem 4.17. *A fuzzy set V in a fbts (X, δ_i, δ_j) is FPγS compact relative to X if and only if for every FP filterbases Γ with the property that every finite member of Γ is quasi coincident with V, $\wedge_{F\in\Gamma} (\delta_i, \delta_j)\gamma scl(F) \wedge V \neq \tilde{0}_X$.*

Theorem 4.18. *If (X, δ_i, δ_j) is a fbts, every FP-γS closed subset of a FP-γS compact space X is FP-γS compact relative to X.*

Proof. Let Γ be a FP filterbases in X and ν be its finite sub collection. Let M be a FP-γSC set such that $Mq \wedge \{H : H \in \nu\}$ holds for any finite sub collection ν. Define $\Gamma^* = \{M\} \vee \Gamma$. Then for any finite sub collection ν^* of Γ^*, if M is not in ν^*, then $\wedge \nu^* \neq \tilde{0}_X$. If $M \in \nu^*$ and $Mq \wedge \{H : H \in \upsilon^* - M\}$, then $\wedge \nu^* \neq \tilde{0}_X$. This implies ν^* is a FP filterbases in X. Also, $\wedge_{H\in\Gamma^*} (\delta_i, \delta_j)\gamma scl(H) \neq \tilde{0}_X$ as X is FP-γS compact. That is, $\wedge_{H\in\Gamma} (\delta_i, \delta_j)\gamma scl(H) \wedge M \neq \tilde{0}_X$ and every finite member of Γ is quasi coincident with M. Hence by Theorem 4.16, M is FP-γS compact relative to X.

Theorem 4.19. *If a mapping $f : (X, \delta_i, \delta_j) \to (Y, \eta_i, \eta_j)$ is FPγS continuous and M is FP-γS compact relative to X, then $f(M)$ is FP-γS compact relative to Y.*

Proof. Let $\{U_j : j \in \Delta\}$ be a $FP\gamma SO$ cover of $S(f(M))$ in Y. Let $x \in f(M)$ then $f(x) \in f(S(M))$. Then the collection $\{f^{-1}(U_j) : j \in \Delta\}$ is a $FP\gamma SO$ cover of $S(M)$ in X as f is $FP\gamma S$ continuous. Then there is finite sub collection $\{f^{-1}(U_1), \ldots, f^{-1}(U_n)\}$ satisfying $S(M) \leq \bigvee_{j=1}^{n} f^{-1}(U_j)$ $= f^{-1}(\bigvee_{j=1}^{n} U_j)$. Then $S(f(M)) = f(S(M)) \leq f(f^{-1} \bigvee_{j=1}^{n} U_j)$. This implies $S(f(M)) \leq \bigvee_{j=1}^{n} U_j$. Thus, $f(M)$ is FP-γS compact relative to Y.

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