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ON $\theta g^{*}s$- CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract

We explore a modern class of generalization of closed sets called nano θ-generalized star semi-closed (briefly. $N\theta g^{*}s$- closed) sets in nano topological spaces in this paper and also its basic properties are analysed. Besides the view for $N\theta g^{*}s$- continuous functions and $N\theta g^{*}s$- irresolute functions are also initiated and their properties are examined. Also, distinct illustrations are rendered to interpret the behavior of new sets.

1. Introduction

Earlier in 1968, Velicko [19] raised the thought of θ-closed sets and Levine [7] instigated the idea of generalized closed sets as a generalization of closed sets in topological spaces. Recently Sathishmohan et al [16] have initiated the concept of $\theta g^{*}s$- closed sets in topological spaces. The notation of nano topology was commenced by Lellis Thivagar [5]. Sathishmohan et al [17] explored the idea of nano $\beta\theta$- closed set and Rajendran [13] raised the new class of nano set called $g^{*}s$- closed set. Besides we initiate a new class of sets called $N\theta g^{*}s$-closed set in nano topological spaces and also, further we analyse the basic properties and characterizations in this paper.

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2. Preliminaries

In this segment, we recall some known definitions which has initiated by various research persons, i.e., Definitions of Apprximationa space [11], nano topology [5], some open sets [5, 15], generalized nano closed sets, nano $\beta\theta$- closed sets, [1, 2, 4, 8, 13, 14, 17] and their continuous functions [2, 3, 6, 9, 10, 12, 14, 18] and $\theta g^{*}s$- closed set [16]. Further, we denote the closed as cld and open as open.

3. Nano $\theta g^{*}s$- closed Sets

In this segment, we initiate and analyse the thought of $\theta g^{*}s$- closed sets via nano topological spaces and brought its basic properties.

Definition 3.1. A point x of a space $(U, \tau R(X))$ is called nano semi θ-cluster point of A if $A \cap Nscl\,(V)\,6 = \varphi$, for every nano semi-open V containing x.

The set of all nano semi θ-cluster points of A is called nano semi θ-closure of A and is denoted by $Nscl\ \theta(A)$. Hence, a subset A is called nano semi θ-cld if $Nscl\ \theta(A) = A$. The complement of a nano semi θ-cld is called nano semi θ-open.

Definition 3.2. A subset G of a nano topological space $(U, \tau R(X))$ is called nano θ-generalized star semi-cld (briefly $N\theta g^{*}s$- cld) if $Nscl\ \theta(G) \subseteq D$ whenever $G \subseteq D$ and D is nano g-open. The complement of $N\theta g^{*}s$- cld is called $N\theta g^{*}s$- open.

Example 3.3. Let $U = \{m, n, o, p\}$ with $U/R = \{\{n\}, \{p\}, \{n, 0\}\}$ and $X = \{m, p\}$. Then $\tau_R(X) = \{U, \varphi, \{p\}, \{m, o, p\}, \{m, o\}\}$ which are nano opens.

The nano $cld = \{U, \varphi, \{n\}, \{m, n, o\}, \{n, p\}\}$.

The nano $g\text{-}cld = \{U, \varphi, \{n\}, \{m, n\}, \{n, o\}, \{n, p\}, \{m, n\, o\}, \{m, n, p\}, \{n, o, p\}\}$.

The nano $\theta\text{-}cld = \{U, \varphi\}$.

The nano semi θ-$cld = \{U, \varphi, \{n\}, \{p\}, \{m, o\}, \{n, p\}, \{m, n, o\}\}$. The nano $\theta g^{*}s$-$cld = \{U, \varphi, \{n\}, \{p\}, \{m, n\}, \{m, o\}, \{n, o\}, \{n, p\}, \{m, n, o\}, \{m, n, p\}, \{n, o, p\}\}$.

Theorem 3.4. *If a nano cld set G in* $(U, \tau R(X))$, *then G is* $N\theta g^{*}s$- *cld set*.

Proof. Let a nano cld set G of U and $G \subseteq D$, D is Ng-open in U. Since G is nano *cld*, $Ncl(G) = G \subseteq D$. In addition, $Nscl\theta(G) \subseteq Ncl(G) \subseteq D$ where D is Ng-open in U. Consequently G is a nano $\theta g^{*}s$- cld.

Theorem 3.5. *If a nano generalized cld set G in* $(U, \tau R(X))$, *then G is* $N\theta g^{*}s$- *cld set*.

Proof: Let G be Ng-cld set, then $Ncl(A) \subseteq D$ whenever $G \subseteq D$, D is nano open in U. As every nano open is Ng-open and $Nscl\theta(G) \subseteq Ncl(G)$ which indicate that $Nscl\theta(G) \subseteq D$, $G \subseteq D$, D is Ng-open in U. Hence G is $N\theta g^{*}s$- cld set.

Theorem 3.6. *If a nano regular cld set G in* $(U, \tau R(X))$, *then G is* $N\theta g^{*}s$- *cld set*.

Proof. Let G be nano regular cld set, then $A = Nrcl\,(A)$. As every Nropn is Ng-opn. Therefore $Nscl\theta(G) \subseteq Nrcl(G) = D$ then $Nscl\theta(G) \subseteq D$, whenever D is nano g-open. Thus every nano r-cld set is $N\theta g^{*}s$- cld set.

Remark 3.7. Reverse part of the above theorems is not true from the below illustrations.

Example 3.8. Let $U = \{m, n, o, p\}$ with $U/R = \{\{n\}, \{p\}, \{m, o\}\}$ and $X = \{m, p\}$. Then $\tau R(X) = \{U, \varphi, \{p\}, \{m, o, p\}, \{m, o\}\}$. Let $A = \{p\}, D = \{m, o, p\}$ whenever $A \subseteq D$, D is Ng-open. Now $Nscl\theta(G) = \{p\} \subseteq D$.

Hence $A = \{p\}$ is $N\theta g^{*}s$- cld set. But $Ncl(A) = \{n, p\}^{*} D$. However the subset $A = \{p\}$ is not a nano cld set. Thus every $N\theta g^{*}s$- cld set need not to be a nano cld set.

Example 3.9. Let $U = \{m, n, o, p\}$ with $U/R = \{\{n, p\}, \{m\}, \{o\}\}$ and $X = \{o, p\}$. Then $\tau R(X) = \{U, \varphi, \{o\}, \{n, o, p\}, \{n, p\}\}$. Let $A = \{o\}$, $D = \{n, o, p\}$ whenever $A \subseteq D$, D is Ng-open. Now $Nscl\theta(A) = \{o\} \subseteq D$. Hence $A = \{o\}$ is $N\theta g^{*}s$- cld set. But $Ncl\theta(A) = \{m, o\}^{*} D$. However the subset $A = \{o\}$ is not Ng-cld set. Thus every $N\theta g^{*}s$- cld set need not to be a Ng-cld set.

Example 3.10. Let $U = \{m, n, o, p\}$ with $U/R = \{\{n, o\}, \{m\}, \{p\}\}$ and $X = \{n, p\}$. Then $\tau R(X) = \{U, \varphi, \{p\}, \{n, o, p\}, \{n, o\}\}$. Let $A = \{p\}$, $D = \{o, p\}$ whenever $A \subseteq D$, D is Ng-open. Now $Nscl\theta(A) = \{p\} \subseteq D$. Hence $A = \{p\}$ is $N\theta g^{*}s$- cld. But $Nrcl(A) = \{m, p\}^{*} D$. Hence the subset $A = \{p\}$ is not Nr-cld. Hence every $N\theta g^{*}s$- cld set need not to be a Nr-cld set.

Theorem 3.11. *In a space* $(U, \tau R(X))$, *the following holds*,

(1) *Every nano semi-θ-cld set is* $N\theta g^{*}s$- *cld set.*

(2) *Every* Ng^{*}-*cld set is* $N\theta g^{*}s$- *cld set.*

(3) *Every* $Ng\alpha$-*cld set is* $N\theta g^{*}s$- *cld set.*

(4) *Every* $N\alpha g$-*cld set is* $N\theta g^{*}s$- *cld set.*

Reverse of the implications need not be true as seen from the following examples.

Example 3.12. Let $U = \{m, n, o, p\}$ with $U/R = \{\{m, p\}, \{n\}, \{o\}\}$ and $X = \{m, o\}$. Then $\tau R(X) = \{U, \varphi, \{o\}, \{m, o, p\}, \{m, p\}\}$. Let $C = \{m, n\}$. Then C is $N\theta g^{*}s$- cld but not nano semi-θ-cld.

Example 3.13. Let $U = \{m, n, o, p\}$ with $U/R = \{\{n, p\}, \{m\}, \{o\}\}$ and $X = \{m, p\}$. Then $\tau R(X) = \{U, \varphi, \{m\}, \{m, n, p\}, \{n, p\}\}$. Let $C = \{n, p\}$ Then C is $N\theta g^{*}s$- cld but not Ng-cld, $Ng\alpha$-cld and $N\alpha g$-cld.

Theorem 3.14. *If a* $N\theta g^{*}s$- *cld set* G *in* $(U, \tau R(X))$, *then* G *is nano generalized semi cld set*.

Proof. Let G be $\theta g^{*}s$- cld set. Let $G \subseteq D$ and D be nano open in U. Then $Nscl\,(G) \subseteq Nscl\theta(G) \subseteq D$. Hence G is a Ngs-cld set.

Example 3.15. Let $U = \{m, n, o, p\}$ with $U/R = \{\{m, o\}, \{n\}, \{p\}\}$ and $X = \{m, o\}$. Then $\tau R(X) = \{U, \varphi, \{n\}, \{m, n, o\}, \{m, o\}\}$. Let $C = \{o\}$. Then C is Ngs-cld set but not $N\theta g^{*}s$- cld set.

Theorem 3.16. *In a space* $(U, \tau R(X))$, *the following hold*

(1) *Every* $N\theta g^{*}s$- *cld set is* $N\beta$-*cld set*.

(2) *Every* $N\theta g^{*}s$- *cld set is Nsg-cld set*.

(3) *Every* $N\theta g^{*}s$- *cld set is* $Ng^{*}p$- *cld set*.

Reverse of the implications need not be true as seen from the following example.

Example 3.17. Let $U = \{m, n, o, p\}$ with $U/R = \{\{m, o\}, \{n\}, \{p\}\}$ U/R = $\{\{m, o\}, \{n\}, \{p\}\}$ and $X = \{m, o\}$. Then $\tau R(X) = \{U, \varphi, \{n\}, \{m, n, o\}, \{m, o\}\}$. Let $C = \{m\}$. Then C is $N\beta$-cld set, Nsg-cld set and $Ng^{*}p$- cld set but not $N\theta g^{*}s$- cld set.

Theorem 3.18. *The union of any two* $N\theta g^{*}s$- *clds in* $(U, \tau R(X))$, *is also a* $N\theta g^{*}s$- *clds in* $(U, \tau R(X))$.

Proof. Let P and Q be any two $N\theta g^{*}s$- cld sets in $(U, \tau R(X))$. Let D be a Ng-open in U such that $P \subseteq D$ and $Q \subseteq D$. Then we have $P \cup Q \subseteq D$. P and Q are $N\theta g^{*}s$- cld sets in $(U, \tau R(X))$, $Nscl\theta(P) \subseteq D$ and $Nscl\theta(Q) \subseteq D$. Now $Nscl\theta(P \cup Q) = Nscl\theta(P) \cup Nscl\theta(Q) \subseteq D$. Thus we have $Nscl\theta(P \cup Q) \subseteq D$ whenever $(P \cup Q) \subseteq D$, D is Ng-opn in U. This implies $(P \cup Q)$ is a $N\theta g^{*}s$- cld set in $(U, \tau R(X))$.

Remark 3.19. In general intersection of two $N\theta g^{*}s$- clds need not be $N\theta g^{*}s$- cld in general from the below illustrations.

Example 3.20. Let $U = \{m, n, o, p\}$ with $U/R = \{\{m, n\}, \{o\}, \{p\}\}$ and $X = \{m, p\}$. Then $\tau R(X) = \{U, \varphi, \{p\}, \{m, n, p\}, \{m, n\}\}$. The subsets $\{m, n\}$ and $\{m, o\}$ are $N\theta g^{*}s$- cld but their intersection $\{m, n\} \cap \{m, o\} = \{m\}$ is not $N\theta g^{*}s$- cld in U.

Theorem 3.21. *Let P be a $N\theta g^{*}s$- cld subset if $(U, \tau R(X))$. If $P \subseteq Q \subseteq Nscl\theta(P)$, then Q is also a $N\theta g^{*}s$- cld subset of $(U, \tau R(X))$.*

Proof. Let D be a Ng-open of a $N\theta g^{*}s$- cld subset of $\tau R(X)$ such that $Q \subseteq D$. As $P \subseteq Q$, we have $P \subseteq D$. As P is a $N\theta g^{*}s$- cld, $Nscl\theta(P) \subseteq D$. Given $Q \subseteq Nscl\theta(P)$, we have $Nscl\theta(Q) \subseteq Nscl\theta(P)$. As $Nscl\theta(Q) \subseteq Nscl\theta(P)$ and $Nscl\theta(P) \subseteq D$, we have $Nscl\theta(Q) \subseteq D$ whenever $Q \subseteq D$ and D is Ng-open. Hence Q is also a $N\theta g^{*}s$- cld subset of $\tau R(X)$.

Theorem 3.22. *A subset P is $N\theta g^{*}s$- cld if and only if $Nscl\theta(P) - P$ contains no non-empty Ng-cld.*

Proof. Necessity: Let F be a Ng-cld subset of $Nscl\theta(P) - P$. Then $P \subseteq U - F$ where P is $N\theta g^{*}s$- cld and $U - F$ is Ng-open. Since P is $Nscl\theta(P)$, then $Nscl\theta(P) \subseteq U - F$ that is $F \subseteq U - Nscl\theta(P)$. Since by assumption $F \subseteq Nscl\theta(P)$ then $F \subseteq (U - Nscl\theta(P)) \cap (Nscl\theta(P)) = \varphi$. This proves that F is empty.

Sufficiency: Suppose that $P \subseteq D$ and D is Ng-open. If $Nscl\theta(P) \subseteq D$, then $Nscl\theta(P) \cap U - D \subset Ncl\theta(P) \cap (U - D)$ is non-empty Ng-cld subset of $Nscl\theta(P) - P$.

Remark 3.23. For a subset G of a nano topological space $(U, \tau R(X))$

(1) $SN \sin t_{\theta}(G) = Nscl_{\theta}(U - G)$

(2) $SNscl_{\theta}(G) = N \sin t_{\theta}(U - G)$.

Theorem 3.24. *A subset $G \subseteq U$ is $N\theta g^{*}s$- open iff $F \subseteq N \sin t_{\theta}(G)$ whenever F is a Ng-cld and $F \subseteq G$.*

Proof. Necessity: Let G be a $N\theta g^{*}s$- open and suppose $F \subseteq G$, where F is Ng-cld. Then $U - G$ is $N\theta g^{*}s$- cld contained in Ng-opn of $U - F$. Hence $Nscl\theta(U - G) \subseteq (U - F)$ and $U - N\sin t_{\theta}(G) \subseteq U - F$. Thus $F \subseteq N\sin t_{\theta}(G)$. Sufficiency: If F is Ng-cld with $F \subseteq N\sin t_{\theta}(G)$ and $F \subseteq G$. Then $U - N\sin t_{\theta}(G) \subseteq U - F$. Thus $Nscl\theta(U - G) \subseteq I - F$. Hence $U - G$ is a $N\theta g^{*}s$- cld and G is $N\theta g^{*}s$- open.

Theorem 3.25. *If* $N\sin t_{\theta}(P) \subseteq Q \subseteq P$ *and if P is* $N\theta g^{*}s$- *open, then Q is* $N\theta g^{*}s$- *open.*

Proof. Let $N\sin t_{\theta}(P) \subseteq Q \subseteq P$, then $P^{c} \subseteq Q^{c} \subseteq Nscl_{\theta}(P^{c})$, where P^{c} is $N\theta g^{*}s$- cld and hence Q^{c} is $N\theta g^{*}s$- cld by Remark 3.20. Therefore Q is $N\theta g^{*}s$- open.

4. $N\theta g^{*}s$- Continuous Functions and $N\theta g^{*}s$- Irresolute Functions

In this segment, we establish the idea of $N\theta g^{*}s$- continuous functions and $N\theta g^{*}s$- irresolute functions in nano topological spaces and also its revised properties are given.

Definition 4.1. Let $(U, \tau R(X))$ and $(V, \tau_{R}0(Y))$ be a nano topological spaces. Then the function $k : (U, \tau R(X)) \to (V, \tau_{R}0(Y))$ is said to be $N\theta g^{*}s$- continuous function (denoted by $N\theta g^{*}s$- cnts) on U, if the inverse image of every nano cld in V is $N\theta g^{*}s$- cld in U.

Example 4.2. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{m, n\}, \{o\}, \{p\}\}$ and $X = \{m, p\}$. Then $\tau_{R}(X) = \{U, \varphi, \{p\}, \{m, n, p\}, \{m, n\}\}$. Let $V/R^{0} = \{\{m, o\}, \{n\}, \{p\}\}$ and $Y = \{m, p\}$. Then $\tau_{R^{0}}(Y) = \{U, \varphi, \{p\}, \{m, o, p\}, \{m, o\}\}$. Define $k : (U, \tau_{R}(X)) \to (\tau_{R}0(Y))$ as $k(m) = m$, $k(n) = o$, $k(o) = n$, $k(p) = p \Rightarrow k$ is $N\theta g^{*}s$- cnts.

Theorem 4.3. *A function* $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$, *then the following holds*

(1) *If k is nano cnts then it is* $N\theta g^* s$*- cnts.*

(2) *If k is Ng-cnts then it is* $N\theta g^* s$*- cnts.*

(3) *If k is Nr-cnts then it is* $N\theta g^* s$*- cnts.*

(4) *If k is Ng*α*-cnts then it is* $N\theta g^* s$*- cnts.*

(5) *If k is N*α*g-cnts then it is* $N\theta g^* s$*- cnts.*

(6) *If k is* Ng^**-cnts then it is* $N\theta g^* s$*- cnts.*

Proof. (1) Let $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ be nano cnts and B be a nano cld in V. Then $k^{-1}(B)$ is nano cld in U. Since every nano cld is $N\theta g^* s$- cld.

Therefore $k^{-1}(B)$ is $N\theta g^* s$- cld.

The proof of (2) to (6) is as follows from (1).

Examples given below shows that the converse part for the above theorem need not be true in general.

Example 4.4. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{m, p\}, \{n\}, \{o\}\}$ and $X = \{m, o\}$. Then $\tau_R(X) = \{U, \varphi, \{o\}, \{m, o, p\}, \{m, p\}\}$. Let $V/R^0 = \{\{n, o\}, \{m\}, \{p\}\}$ and $Y = \{n, p\}$. Then $\tau_{R^0}(Y) = \{U, \varphi, \{p\}, \{n, o, p\}, \{n, o\}\}$. Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = n, k(n) = m, k(o) = o, k(p) = p$. Then k is $N\theta g^* s$- cnts. But $k^{-1}\{m, p\} = \{n, p\}$ is not nano cld. So k is not nano-cnts.

Example 4.5. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{m, p\}, \{o\}, \{p\}\}$ and $X = \{m, p\}$. Then $\tau_R(X) = \{U, \varphi, \{p\}, \{m, n, p\}, \{m, n\}\}$. Let $V/R^0 = \{\{m, o\}, \{n\}, \{p\}\}$ and $Y = \{m, p\}$. Then $\tau_{R^0}(Y) = \{U, \varphi, \{p\}, \{m, o, p\}, \{m, o\}\}$. Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = m, k(n) = 0,$

$k(0) = n, k(p) = p$. Then k is $N\theta g^{*}s$- cnts. But $k^{-1}\{m, o\} = \{m, n\}$ is not Ng-cld. So k is not Ng-cnts.

Example 4.6. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{o, p\}, \{m\}, \{n\}\}$ and $X = \{m, p\}$. Then $\tau_R(X) = \{U, \varphi, \{m\}, \{m, o, p\}, \{o, p\}\}$. Let $V/R^0 = \{\{n, p\}, \{m\}, \{o\}\}$ and $Y = \{m, p\}$. Then $\{m, n, p\}, \{n, p\}\}$. $\tau_{R^0}(Y) = \{U, \varphi, \{m\}$, Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = n, k(n) = o$, $k(o) = m, k(p) = p$. Then k is $N\theta g^{*}s$- cnts. But $k^{-1}\{m, o\} = \{n, o\}$ is not Nr-cld. So k is not Nr-cnts.

Theorem 4.7. *A function* $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$, *then the following holds*

(1) *Every* $N\theta g^{*}s$- *cnts function is Ngs-cnts.*

(2) *Every* $N\theta g^{*}s$- *cnts function is* $N\beta$-*cnts.*

(3) *Every* $N\theta g^{*}s$- *cnts function is Nsg-cnts.*

(4) *Every* $N\theta g^{*}s$- *cnts function is* $Ng^{*}s$- *cnts.*

Example 4.8. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{n, o\}, \{m\}, \{p\}\}$ and $X = \{n, p\}$. Then $\tau_R(X) = \{U, \varphi, \{p\}, \{n, o, p\}, \{n, o\}\}$. Let $V/R^0 = \{\{m, n\}, \{o\}, \{p\}\}$ and $Y = \{m, p\}$. Then $\tau_{R^0}(Y) = \{U, \varphi, \{p\}$, $\{m, n, p\}, \{m, n\}\}$. Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = m, k(n) = p$, $k(o) = o, k(p) = n$. Then k is Ngs-cnts $N\beta$-cnts, Nsg-cnts and $Ng^{*}p$- cnts. But $k^{-1}\{o\} = \{o\}$ k−1{o} = {o} is not $N\theta g^{*}s$- cld. So k is not $N\theta g^{*}s$- cnts.

Remark 4.9. The composition of two $N\theta g^{*}s$- cnts functions is again a $N\theta g^{*}s$- cnts as shown in the below illustration.

Example 4.10. Let $U = V = W = \{m, n, o, p\}$, $U/R = \{\{o, p\}, \{m\}, \{n\}\}$ and $X = \{n, o\}$. Then $\tau_R(X) = \{U, \varphi, \{n\}, \{n, o, p\}, \{o, p\}\}$. Let $V/R^0 = \{\{m, o\}, \{n\}, \{p\}\}$ and $Y = \{m, o\}$. Then $\tau_{R^0}(Y) = \{U, \varphi, \{n\}$,

$\{m, n, o\}, \{m, o\}\}$. Let $W/R^{00} = \{\{n, p\}, \{m\}, \{o\}\}$ and $Z = \{o, p\}$. Then $\tau_{R^{00}}(Z) = \{U, \varphi, \{o\}, \{n, o, p\}, \{n, p\}\}$. Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = p$, $k(n) = n$, $k(o) = o$, $k(p) = m$. Defin $q : (V, \tau_R 0(Y)) \to (W, \tau_R 00(Z))$ as $q(m) = p$, $q(n) = n$, $q(o) = o$, $q(p) = m$. Since $\{m, n, p\}$ is cld in $(W, \tau_R 00(Z))$. Since $(q^0 k)^{-1}(\{m, n, p\}) = k^{-1}(q^{-1}\{m, n, p\}) = k^{-1}(\{m, n, p\})$ $= \{m, n, p\}$ which is $N\theta g^* s$- cld in U. Hence $k \circ q$ is $N\theta g^* s$- cnts.

Theorem 4.11. *A function $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ is $N\theta g^* s$- cnts if and only if the inverse image of every nano cld in V in $N\theta g^* s$- cld in U.*

Proof. Let k be $N\theta g^* s$- cnts and H be nano cld in V. That is $V - G$ is nano opn in V. Since k is $N\theta g^* s$- cnts, $k^{-1}(V - H)$ is $N\theta g^* s$- open in U. That is $k^{-1}(V) - k^{-1}(H) = U - k^{-1}(V - H)$ is $N\theta g^* s$- opn in U. Hence $k^{-1}(H)$ is $N\theta g^* s$- cld in U, if k is $N\theta g^* s$- cnts on U.

Conversely, Let the inverse image of every nano cld in V is $N\theta g^* s$- cld in U. Let G be a nano opn in V. Then $V - G$ is nano cld is V. Then $k^{-1}(V - G)$ is $N\theta g^* s$- cld in U. Therefore, $k^{-1}(G)$ is $N\theta g^* s$- opn in U. Thus the inverse image of every nano opn in V in $N\theta g^* s$- opn in U. That is, k is $N\theta g^* s$- cnts in U.

Theorem 4.12. *A function $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ is $N\theta g^* s$- cnts if and only if $k(N\theta g^* scl(G) \subseteq Ncl\theta(k(G))$ for every subset G of U.*

Proof. Let k be $N\theta g^* s$- cnts and $G \subseteq U$. Then $k(A) \subseteq V$. Since k is $N\theta g^* s$- cnts and $Ncl\theta(k(G))$ is nano cld in V, $k^{-1}(Ncl\theta(k(G)))$ is $N\theta g^* s$- cld in U. Since $k(G) \subseteq Ncl\theta(k(G))$, $k^{-1}(k(G)) \subseteq k^{-1}(Ncl\theta(k(G)))$. Thus $N\theta g^* scl(G) \subseteq k^{-1}(Ncl\,(k(G)))$. Therefore, $(N\theta g^* scl(G)) \subseteq Ncl\theta(k(G))$ for every subset G of U.

Conversely, Let $f(N\theta g^{*}scl(G)) \subseteq Ncl\theta(k(G))$ for every subset G of U. If H is nano cld in V, since $k^{-1}(H) \subseteq U, k(N\theta g^{*}scl(k^{-1}(H))) \subseteq Ncl\theta(k^{-1}(H)) = Ncl\theta(H)$. That is, $N\theta g^{*}scl(k^{-1}(H)) \subseteq k^{-1}(H)$. But $k^{-1}(H) \subseteq N\theta g^{*}scl(k^{-1}(H))$. Thus $N\theta g^{*}scl(k^{-1}(H)) = k^{-1}(H)$. Therefore $k^{-1}(H)$ is $N\theta g^{*}s$-cld in U for every nano cld H in V. That is, k in $N\theta g^{*}s$- cnts.

Definition 4.13. Let $(U, \tau_R(X))$ and $(V, \tau_R 0(Y))$ be a nano topological spaces. Then the function $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ is said to be $N\theta g^{*}s$- ires on U, if the inverse image of every $N\theta g^{*}s$- cld in V is $N\theta g^{*}s$- cld in U.

Theorem 4.14. *If a function* $(V, \tau_R 0(Y))$ *is* $N\theta g^{*}s$- *ires, then it is* $N\theta g^{*}s$- *nts but not conversely*.

Proof. Let $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ is $N\theta g^{*}s$- ires function. Then the inverse image $k^{-1}(G)$ of every $N\theta g^{*}s$- cld G in V is $N\theta g^{*}s$- cld in U. Since every nano cld is $N\theta g^{*}s$- cld, the inverse image of every nano cld in V is $N\theta g^{*}s$- cld in U whenever the inverse image of every $N\theta g^{*}s$- cld is $N\theta g^{*}s$- cld. Hence $N\theta g^{*}s$- ires function is $N\theta g^{*}s$- cnts.

The inverse part need not be true from the below illustration.

Example 4.15. Let $U = V = \{m, n, o, p\}$ with $U/R = \{\{m, p\}, \{n\}, \{o\}\}$ and $X = \{m, o\}$. Then $\tau_R(X) = \{U, \varphi, \{o\}, \{m, o, p\}, \{m, p\}\}$. Let $V/R^{0} = \{\{n, o\}, \{m\}, \{p\}\}$ and $Y = \{n, p\}$. Then $\tau_R 0(Y) = \{U, \varphi, \{p\}, \{n, o, p\}, \{n, p\}\}$. Define $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ as $k(m) = n, k(n) = m, k(o) = o, k(p) = p$. Then k is $N\theta g^{*}s$- cnts since the inverse image of every nano cld in V is $N\theta g^{*}s$- cld in U. But k is not $N\theta g^{*}s$- ires since $k^{-1}\{n, o\} = \{m, o\}$ is not $N\theta g^{*}s$- cld in U even though $\{n, o\}$ is $N\theta g^{*}s$- cld in V. Hence a $N\theta g^{*}s$- cnts function in not $N\theta g^{*}s$- ires.

Theorem 4.16. *Let* $(U, \tau_R(X)) \to (\tau_R 0(Y))$ *and* $(W, \tau_R 00(Z))$ *be nano topological spaces. If the functions* $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ *and* $q : (V, \tau_R(Y)) \to (W, \tau_R 00(Z))$ *are both* $N\theta g^* s$- *ires.*

Proof. As the function $q : (V, \tau_R(Y)) \to (W, \tau_R 00(Z))$ is $N\theta g^* s$- ires, the inverse image $q^{-1}(G)$ of every $N\theta g^* s$- open G in W is $N\theta g^* s$- open in V. Hence $q^{-1}(G)$ is a $N\theta g^* s$- open in V and $k : (U, \tau_R(X)) \to (\tau_R 0(Y))$ being $N\theta g^* s$- sires implies that $k^{-1}[q^{-1}(G)]$ is $N\theta g^* s$- opn in U. Thus $(q \circ k)^{-1}(G) = k^{-1}[q^{-1}(G)]$ is $N\theta g^* s$- open in U for every $N\theta g^* s$- open $q^{-1}(G)$ in V. Hence $q \circ k : (U, \tau_R(X)) \to (W, \tau_R 00(Z))$ is $N\theta g^* s$- ires.

Theorem 4.17. Let $(U, \tau_R(X)), (V, \tau_R 0(Y))$ and $(W, \tau_R 00(Z))$ be nano topological spaces. For any $N\theta g^* s$- ires function $k : (U, \tau_R(X)), (V, \tau_R 0(Y))$ and any $N\theta g^* s$- cnts function $q : (V, \tau_R(Y)), (W, \tau_R 00(Z))$, the composition $q \circ k : (U, \tau_R(X)), (W, \tau_R 00(Z))$ is $N\theta g^* s$- cnts.

Proof. Let G be a nano cld in W. Since the function $q : (V, \tau_R(X)) \to (W, \tau_R 00(Z))$ is $N\theta g^* s$- cnts, the inverse image $q^{-1}(G)$ is $N\theta g^* s$- cld in V. Since the function $k : (U, \tau_R(X)), (V, \tau_R 0(Y))$ is Nθg∗s-ires, the inverse image $k^{-1}[q^{-1}(G)]$ of $N\theta g^* s$- cld $q^{-1}(G)$ in V is $N\theta g^* s$- cld in U. Thus the inverse image $(q \circ k)^{-1}(G)$ is $N\theta g^* s$- cld in U for every $N\theta g^* s$- cld G in W. Hence the composition $q \circ k : (U, \tau_R(X)) \to (W, \tau_R 00(Z))$ is $N\theta g^* s$- cnts.

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